Introduction to the Modeling of Dynamic Fragmentation with Extrinsic Cohesive Elements

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Abstract

A common way to model cracks in the finite-element method framework consists in using the cohesive elements (also called interface elements). They have been widely used because they are based on simple phenomenological laws and they naturally allow modeling multiple interacting cracks. On the other hand, their inherent mesh dependency represents their most important limitation. These characteristics make the finite-element method with cohesive elements an optimal tool to model dynamic fragmentation or when statistics about a fragmentation process are sought. This handout provides the main concepts and the formulation behind the FEM with cohesive elements.

Contents

1 Cohesive zone concept

The theory on which cohesive elements are based on dates back to the 1960s, when the cohesive zone concept was introduced by Dugdale (1960)

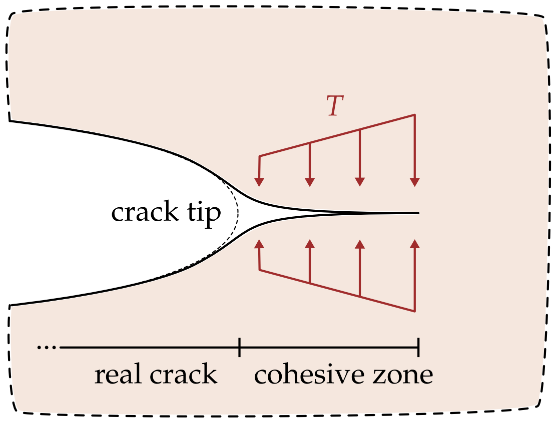


Figure 1: Cohesive zone schematics.

and Barenblatt (1962). According to this theory, the debonding process among atoms during the propagation of a crack is supposed to occur in a small zone next to the crack tip. Therefore, a way to model crack propagation consists in artificially elongating the crack ahead of the crack tip, while imposing some tractions on the so created surfaces to keep the crack closed (see figure 1). The traction T decays with the increasing opening displacement δ according to a traction separation law (or cohesive law), that can be made irreversible. Two examples of cohesive laws are shown in figure 2 on the next page and are illustrated in section 2. The non-linearity of the crack propagation process is thus limited to the cohesive law. The irreversibility of the law allows energy dissipation in proportion to the fracture toughness of the material. A damage parameter is usually associated with the cohesive law. It ranges from 0, when the opening displacement is still zero, to 1, when the cohesive traction drops to zero and the debonding is complete.

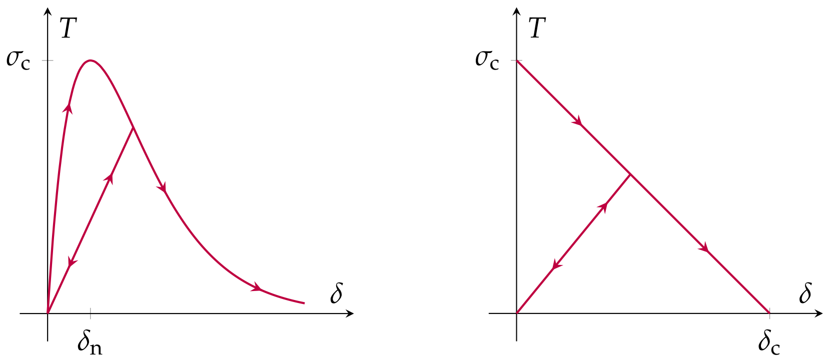
The length of the cohesive zone is an important parameter that is used to model fracture with the cohesive approach. A common way to estimate it in statics and mode I was introduced by Rice (1980) and is

$$l = \frac{9\pi}{32} \left(\frac{E}{1 - \nu^2} \right) \frac{G_c}{\sigma_c^2} \tag{1}$$

where l is the cohesive zone length, E the elastic modulus, ν the Poisson's ratio, G_c the fracture energy and σ_c the maximum traction.

2 Intrinsic and extrinsic approaches

It is evident that the quality of a model based on the cohesive zone concept strongly depends on the type of cohesive law in use. There are two main



(a) Intrinsic. **(b)** Extrinsic.

Figure 2: Two examples of cohesive laws. Both plots include one unloading-reloading cycle.

families of cohesive laws: intrinsic and extrinsic ones. The former is used if the location of the cohesive zone can be predetermined. The latter is used for dynamically inserted cohesive zones, i.e. any location can be a site for crack nucleation. These differences are explained in the following paragraphs.

The intrinsic approach is the oldest and simplest one. According to it the cohesive zone is permanently integrated in the model at the beginning of the simulation. A popular intrinsic cohesive law was proposed by Xu and Needleman (1993) and is reported in figure 2a. The underlying equation relating traction and opening displacement is

$$T(\delta) = \sigma_c \frac{\delta}{\delta_n} e^{1-\frac{\delta}{\delta_n}} \tag{2}$$

where δ_n is the opening displacement at which the maximum traction σ_c occurs. In the formulation proposed by Ortiz and Pandolfi (1999) the unloading-reloading phase is elastic, as shown in the picture. The area underneath the exponential curve is equal to the fracture energy G_c.

The intrinsic cohesive laws have been integrated in many commercial codes and have been successfully utilized for explicit and implicit simulations with cohesive elements (more information on them is provided in section 3 on the next page). Their implementation in FEM codes is straightforward because the geometry of the model does not change during the simulation. However there are two significant disadvantages concerning this method. The first one is the predetermination of the crack path, which is clear for problems like delamination but becomes unpredictable for dynamic fragmentation. The second disadvantage is related to the shape of the initial part of the intrinsic laws. In fact, a perfectly undamaged cohesive zone should let the material behave as if it was intact, by keeping the crack closed. This is impossible to achieve with the intrinsic laws, because they need non-zero opening displacements to generate non-zero tractions. Therefore the presence of an intrinsic cohesive zone is inevitably altering the stiffness of the material. Such issue can be limited by using a law with a steep initial slope, but in practice this sometimes leads to numerical convergence problems.

The extrinsic approach aims to overcome the limitations of the intrinsic one. According to it, the cohesive zone is dynamically created and expanded during the simulation in the areas where the critical stress is reached by the material. The extrinsic cohesive laws are monotonically decreasing functions that have the peak stress when the opening displacement is zero. This approach was pioneered by Camacho and Ortiz (1996), who also proposed the following simple linear-decreasing cohesive law:

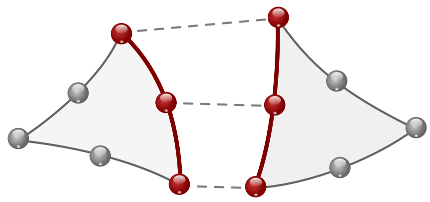
$$T(\delta) = \sigma_c \left(1 - \frac{\delta}{\delta_c}\right) \tag{3}$$

where δ_c is the critical opening displacement at which debonding is complete (see figure 2b). Also in this case unloading-reloading cycles are elastic and the area underneath the cohesive envelope is equal to the fracture energy G_c. With this approach the model is changing its geometry during the simulation and so it is possible to reproduce multiple cracks developing in any direction. Moreover the stiffness of the material is not altered, because a cohesive zone is present only where the stress is high enough to nucleate a crack. A drawback of the extrinsic approach is the complexity of its implementation. As far as cohesive elements are concerned, just some research non-commercial codes accomplished it and only with explicit time integration schemes.

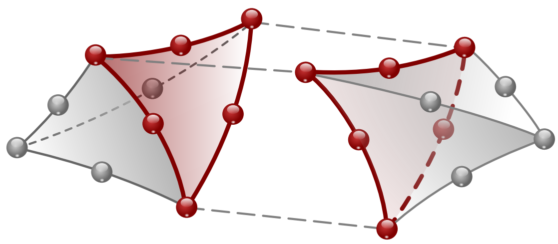
3 Cohesive elements

The cohesive elements are the most common way to introduce the cohesive zone concept in the finite-element method framework. A cohesive element is a special zero-volume element that allows a crack to propagate along the edges between two standard elements. It consists of a pair of edge-elements that are superposed one to the other when the opening displacement is zero (see figure 3 on the following page). An extensive formulation of the cohesive elements was realized by Ortiz and Pandolfi (1999). For the triangular elements the cohesive elements consist of a pair of segments, while for tetrahedra they consist of a pair of triangles. The cohesive elements use the shape functions of their sub-elements, in order to interpolate the nodal position field and compute its value on the quadrature points. The opening displacement vector field is expressed as

$$\boldsymbol{\Delta} = \sum_{n=1}^{\#\text{nodes}} \left(\mathbf{x}_n^+ - \mathbf{x}_n^-\right) N_n \tag{4}$$



(a) Triangle T6.



(b) Tetrahedron T10.

Figure 3: Representation of 2D and 3D standard elements with their respective cohesive element.

where N_n are the shape functions of the sub-element and $\mathbf{x}_n^{\pm}$ for $n = 1, \ldots, N$ are the current coordinates of the nodes ($\pm$ refers to the two surfaces of the cohesive element). Therefore an effective opening displacement δ can be calculated also for mixed mode, by combining the tangential and normal components of $\boldsymbol{\Delta}$. At this point, the traction can be computed with a cohesive law and then integrated over the element.

The cohesive elements are compatible with both the intrinsic and extrinsic approaches. In the first case, they are created before starting the simulation and placed along the predetermined crack path. In the second case, the effective stress σ_{eff} is monitored along the elements' borders, which accounts for both normal and tangential components. Whenever $\sigma_{\text{eff}} \geq \sigma_{\text{c}}$ along an edge, a cohesive element is inserted. It is important to notice that extrinsic cohesive elements are affected by mesh dependency, because they force cracks to pass through the elemental edges. However, complex phenomena like crack branching and coalescence are naturally handled by the numerical method, without any additional parameters.

A good estimation of the element size is given by the cohesive zone length, expressed for example with equation (1). In fact the element size must be smaller than the cohesive zone length in order to properly model it.

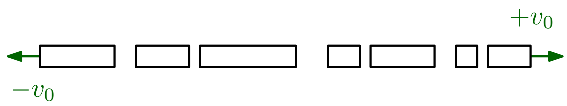


Figure 4: Quasi-1D bar subjected to uniform expansion (Vocialta and J.-F. Molinari 2015).

4 Examples

In this section two examples of dynamic fragmentation modeled with extrinsic cohesive elements are shown. They were realized with the C++ finite-element library Akantu[1], that is an open-source software containing a state-of-the-art cohesive elements' implementation.

4.1 1D fragmentation with contact

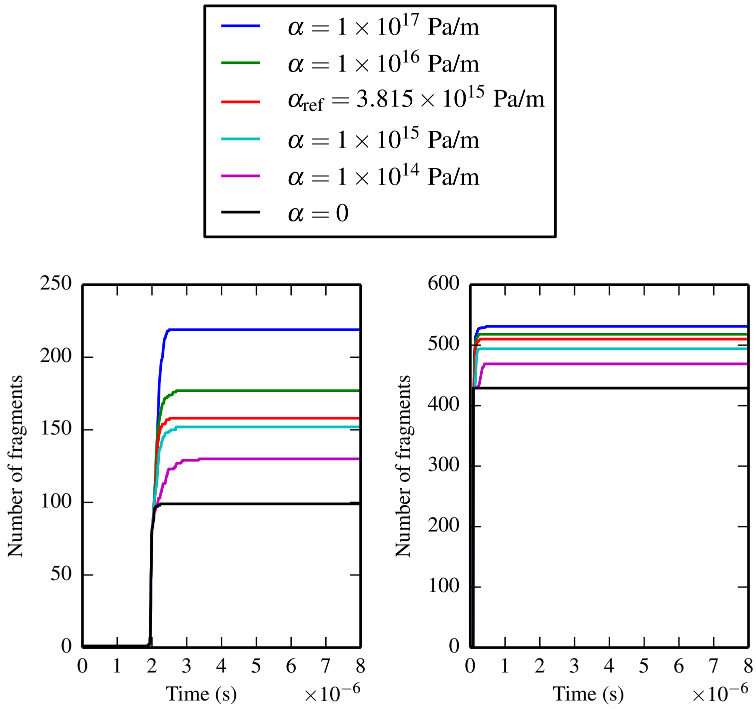
This example was developed in Vocialta and J.-F. Molinari (2015). The purpose was analyzing the influence of the contact among fragments in dynamic fragmentation processes. The model consists in a quasi-1D homogeneous bar meshed with quadrangular elements (just one element in the thickness). The bar is subjected to a uniform expansion that is high enough to cause fragmentation (see figure 4). Surprisingly, even under this pure tensile loading, enabling or disabling contact can be fundamental. In fact with contact the number of fragments can increase of more than 100%, for low strain rates and certain material properties (see figure 5 on the following page).

4.2 3D fragmentation in parallel

This example shows the possibilities offered by an optimized implementation of the extrinsic cohesive elements (Vocialta, Richart, and J.-F. Molinari 2015). The algorithms behind them have been coded in parallel to take advantage of the computational capabilities offered by modern clusters containing thousands of cores. In fact the mesh is partitioned and distributed among processors while some communications are synchronizing the insertion of cohesive elements (figure 6a on the next page). Partitions are created in such way to homogenize the number of elements per processor (load balances) and minimize their borders (minimum communications). Communications are realized with layers of read-only elements that are used to store data coming from neighboring partitions.

Thanks to this implementation it was possible to use fine meshes containing a number of elements of the order of millions. Figure 6b shows some results. The model consists of a brittle homogeneous hollow sphere

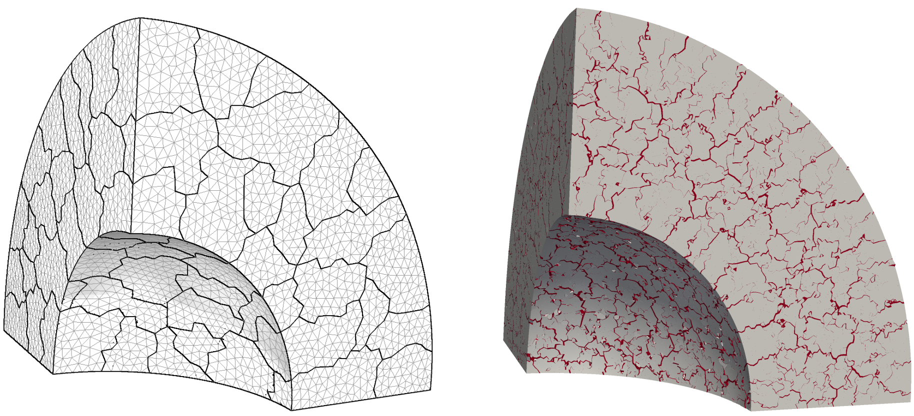
[1] http://lsms.epfl.ch/akantu



(a) $\dot{\varepsilon}_0 = 1 \times 10^3\,\mathrm{s}^{-1}$

(b) $\dot{\varepsilon}_0 = 3 \times 10^4\,\mathrm{s}^{-1}$

Figure 5: Evolution of the number of fragments with time. The coefficient α is the penalty parameter of the contact law (Vocialta and J.-F. Molinari 2015).



(a) Processors' partitions (69 573 tetrahedra).

(b) Cracks' patterns (1 696 951 tetrahedra).

Figure 6: Results of a simulation of a homogeneous hollow sphere under uniform expansion (Vocialta, Richart, and J.-F. Molinari 2015). The simulation was run in parallel on 96 processors and is based on extrinsic cohesive elements.

subjected to uniform radial expansion. Cracks nucleate at the material defects and propagate. Eventually cracks merge and form fragments. Since statistics about fragment sizes are sought, mesh dependence does not constitute a significant drawback for the extrinsic cohesive elements.

4.3 Other examples and outlook

Another application related to fragmentation is Levy, J.-F. Molinari, and Radovitzky (2012). Applications of extrinsic cohesive elements on concrete and masonry are L. Snozzi, Gatuingt, and J. F. Molinari (2012) and Leonardo Snozzi and Jean-Francois Molinari (2013).

A significant challenge of the cohesive zone modeling is to couple it to a robust treatment of frictional contact, which is the topic of ongoing research at LSMS (Kammer et al. 2014).

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