Memory effects in variable amplitude and multiaxial fatigue crack growth : an incremental approach

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Résumé

This presentation is dedicated to an incremental approach for modelling fatigue crack growth with memory effects due to the non-linear behavior of the material. This approach is developed at LMT since 2003, in collaboration with several industrial partners. The first part of this paper presents the context and the objectives, and the key assumptions on which the model is based. The second part presents some examples of applications of the model, fatigue crack growth in mode I conditions, under variable amplitude loading ; non-isothermal situations ; crack growth in coupled environmental and fatigue loading conditions ; extension of the model to non-proportional mixed mode loading conditions, and to short cracks. The last part presents some ongoing work, possible developments and scientific challenges that remain to be overcome.

Mots clés : fatigue ; crack growth ; memory effects ; mixed mode ;

1. Introduction

Accurate predictions of fatigue crack propagation and of the service life of critical components in operating conditions, remains difficult, for the following reasons:

- 2D / 3D: Stress concentration area where short cracks may be initiated usually display spatial gradient and a certain degree of multiaxiality over a domain which sizes is in the order of magnitude with the short crack to long crack transition length. However, data and crack propagation models are often based on mode I long crack growth behavior.
- Non-linear material behavior: The materials used for critical components are usually ductile and hence display a non-linear behavior which is at the origin of memory effects in fatigue crack growth. The importance of these history effects on fatigue crack growth has been demonstrated and explained is numerous publications. Various crack propagation models (NASGRO, PREFFAS, Strip Yield …) have been developed to account for these history effects. If the material is non-linear, the entire load history (not only the peak to peak loads), is to be considered to predict the fatigue crack growth rate. However, since most models are predicting a fatigue crack growth rate per cycle, they require the use of a cycle counting method (e.g. rainflow) so as to be applied to load sequences in operating conditions that might be quite far from being cyclic. When the load sequence stemming from the cycle reconstruction differs significantly from the original one, the life prediction may be questionable. The incremental approach, which we have developed at LMT, avoids the use of a cycle reconstruction method by predicting a crack extent in each load increment.

- Other non-linear mechanisms may be involved (unilateral contact between the crack faces and friction, time dependent damage mechanism such as corrosion or creep …) and be coupled with pure fatigue crack growth. An incremental approach makes it easier to consider independently the effects of each non-linear mechanism in each time step and to model their synergetic effects.
- Complex loading conditions: Loading sequences in actual operating conditions can be rather complex. The mechanical loadings can be uniaxial or multiaxial, varying in space and time (variable amplitude loading in mode I, in phase or out of phase loadings in mixed modes conditions…). Similarly, it may be non-isothermal in space and time (thermal fatigue...)

2. Incremental approach

Since 2003, an incremental approach was developed at LMT, step by step, to predict fatigue crack growth in complex loading conditions and in non-linear materials. The approach is based on the basic assumption that "pure" fatigue crack growth stems from crack tip plasticity (Neumann (1969) and Li (1989)). With such an assumption, an incremental model for "pure" fatigue crack growth could be derived from an incremental plasticity model for the crack tip region. The crack growth rate per second can then be predicted from the measure of crack tip plasticity. Quite a few authors have derived successful predictions of the fatigue crack growth rate in complex loading conditions, from the analysis of the plastic strain field around the crack front obtained from non-linear finite element simulations. However, non-linear finite element simulations remains out of reach for a use in an industrial context, where cracks are usually 3D. A simplified model is thus required, but the finite element method can be used to develop a simplified model and to verify its capabilities.

The simplified model that has been developed at LMT aims at condensing all the effects of the non-linear behavior of the material in the crack tip region in a set of constitutive equations based on the minimum number of variables necessary to reasonably represent the problem of crack tip plasticity. Moreover, the simplifying assumptions in the model are chosen to be suitable with a use in mixed mode conditions.

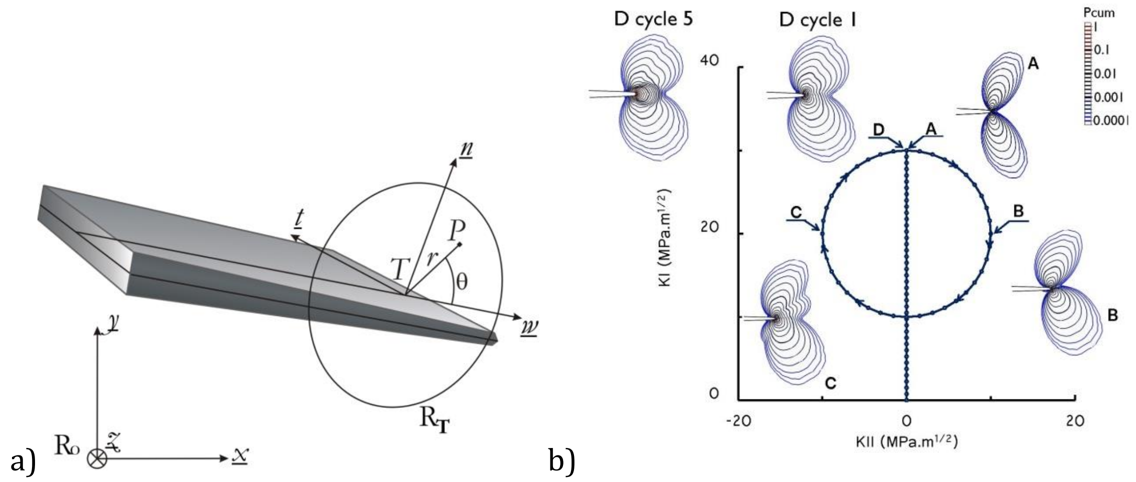

Figure 1. *(a) Local coordinate system attached in T to the local crack front and plane. (b) Illustration : Five out-of-phase mixed mode I + II loading cycles (A-B-C-D) were simulated in non-linear conditions using the finite element method. The iso-values of the equivalent plastic strain Pcum are plotted in logarithmic scale, during the first cycle in points A, B, C and D of the load path, and at the end of the fifth cycle (D).*

The approach is based on a sery of hypotheses that are briefly recalled below :

- Infinitesimal strain conditions are considered.
- The local solution is assumed to be dominated by the local geometry of the crack. The remote boundary conditions and their history are hence expected to control the intensity of the crack tip fields but not their spatial distribution, which is assumed to be given once for all, and be associated to the local crack geometry. This ensures the consistency between LEFM and the present approach.
- A curvilinear coordinate system R_T, can be attached to the local crack front and the local crack plane, considering a suitable characteristic scale. In this local coordinate system R_T, the crack is assumed to be locally planar and in generalized plane strain conditions along the locally straight crack front. This assumption allows partitioning the crack tip fields into mode I (symmetric), mode II (anti-symmetric) and mode III (anti-planar) local components (Fig. 1a).

$$H1 - \ kinematics: \ \underline{v}(P,t)_{R_0} = \underbrace{\underline{v}(T,t)_{R_0} + \omega(R_T/R_0) \wedge \underline{TP}}_{crack\ growth} + \underbrace{\underline{v}(P,t)_{R_T}}_{behaviour\ of\ the\ crack\ tip\ region} \quad (1)$$

- With respect to the local coordinate system attached to the local crack plane and crack front, the geometry of the crack is **locally scale invariant**. This implies that, in each time step, the local solution of the problem can be expressed as the product of an intensity factor and of a spatial distribution, which is also scale invariant. It implies that this spatial distribution can be expressed as the product of a function $f(r)$ of the scale (the distance to the crack tip r) and of a function $\underline{g}(\theta)$ of the angular position with respect to the crack plane. With this approach, the spatial distribution $f(r)\underline{g}(\theta)$ is given once for all and the intensity factor I(t) can be considered as a degree of freedom. The behavior of the crack tip region can be defined in a consistent framework for linear elastic conditions and for non-linear conditions.

$$\mathrm{H2} - \ \mathrm{kinematics}: \underline{v}(P,t)_{R_T} = I(t) f(r) \underline{g}(\theta), \text{ where } \boldsymbol{f(\alpha r) = \beta f(r)} \quad (2)$$

- Since it is always possible, by reversing the loading direction, to get a linear elastic behaviour, during an infinitesimal load increment, the elastic behaviour of the crack tip region requires, for each mode, an independent degree of freedom, even if elastic-plastic conditions are considered. Therefore, the hypotheses listed above should apply independently to elastic and inelastic behaviours. In particular, an intensity factor and a spatial distribution may also be defined to characterize the non-linear part of the kinematics of the crack tip region. If crack tip plasticity is well confined, the elastic bulk constrains the development of the crack tip plastic zone and hence limits also drastically the number of useful degrees of freedom required to represent reasonably well the plastic flow obtained in the crack tip region.

$$H3 - \ kinematics: \ \underline{v}(P,t)_{R_T} = \sum_{i=1}^{3} \underbrace{\tilde{\dot{K}}_i(t).\underline{\varphi}_i^e(P)}_{\underline{v}_i^e(P,t)} + \underbrace{\dot{\rho}_i(t).\underline{\varphi}_i^c(P)}_{\underline{v}_i^c(P,t)} \quad (3)$$

This last property is illustrated in Fig. 1b, for example. The cumulated plastic strain field obtained by elastic-plastic FE simulation in out-of-phase I+II mixed mode conditions was plotted in a logarithmic scale, for different points of a circular loading path in a K_I-K_{II} plane. At each point A, B, C or D, the angular distribution of p_{cum} is obviously the same whatever the distance r to the crack tip, so that $p_{cum} \propto f(r)g(\theta)$ in each time step. In addition, it can be seen in Fig. 1b that p_{cum} decays exponentially from the crack tip, with the same decay rate throughout the mixed mode loading cycle.

As a consequence, in elastic plastic conditions, the velocity field in R_T can be approximated as the superposition of three modes, denoted by i. Each mode requires a degree of freedom $\dot{\tilde{K}}_i$ for the elastic response and another degree of freedom $\dot{\rho}_i$. for the inelastic one. Both the elastic and the inelastic part are expressed as the product of a spatial distribution and of an intensity factor, used as a degree of freedom. The spatial distribution is constructed a priori and is the result of the different constraints (local crack geometry, symmetry, scale independence…).
$\underline{v}_i^c(P,t)$ stands for the non-elastic part of the velocity field, while $\underline{v}_i^e(P,t)$ represents the elastic part.

If the material behaviour is linear elastic, then the intensity factor $\dot{\tilde{K}}_i$ of the elastic part of the velocity field is equal to the nominal applied stress intensity factor $\dot{K}_i^{\infty}$. Otherwise, these two quantities are slightly different, because elastic strain may arise from **applied stresses** (and therefore from $\dot{K}_i^{\infty}$) but also from **internal stresses** arising from crack tip plasticity and from the confinement of the plastic zone. The difference ($\dot{\tilde{K}}_i - \dot{K}_i^{\infty}$) can be interpreted as the shielding effect of the plastic zone. As expected, ($\dot{\tilde{K}}_i - \dot{K}_i^{\infty}$) was observed to be directly proportional to $\dot{\rho}_i$. by post-treatment of FE simulations.

The elastic reference fields $\underline{\varphi}_i^e(\underline{x})$ are obtained from a linear FE computation for each mode with $\dot{K}_i^{\infty} = 1MPa\sqrt{m}$ and fit the Westergaard's solutions.

The non-elastic reference field $\underline{\varphi}_i^c(P)$ are obtained from elastic-plastic FE computations, using a model-reduction technique, as being the best possible spatial field to approximate by Eq. 3 the velocity field evolution calculated for each mode for a loading ramp from zero to 0.8 K_{Ic}. According to the hypotheses H2, $\underline{\varphi}_i^c(P)$, can be locally represented by $f_i^c(r)\underline{g}_i^c(\theta)$ where $f_i^c(\alpha r) = \beta f_i^c(r)$.

Assuming that the plastic zone is confined, implies that $f_i^c(r) \underset{r\to\infty}{\longrightarrow} 0$. And since $\underline{\varphi}_i^c(P)$ is the spatial distribution of the inelastic part of the velocity field at crack tip, it should be discontinuous across the crack faces and maximum at the crack front, which implies that it should decay exponentially and which was observed in FE computations.

$$\begin{cases} f_i^c(\alpha r) = \beta f_i^c(r). \\ f_i^c(r) \underset{r\to\infty}{\longrightarrow} 0 \\ f_i^c(r) \underset{r\to 0}{\longrightarrow} 1 \end{cases} \rightarrow f_i^c(r) = e^{-kr} \quad (4)$$

$f_i^c(r)$ was rescaled to 1 when $r \to 0$, by convention. In addition, $\underline{g}_i^c(\theta)$ is discontinuous across the crack faces and was rescaled so that :

$$\left[\underline{g}_i^c(\theta)(\theta = \pi) - \underline{g}_i^c(\theta)(\theta = -\pi)\right]_\theta = 1 \quad (5)$$

In practice, this post treatment is used to rescale each reference field $\underline{\varphi}_i^c(\underline{x})$ by a constant scalar value, so that the limit when *r* tends to zero of its discontinuity across the crack plane would be equal to 1:

$$\left[\underline{\varphi}_i^c(\theta = \pi, r \to 0) - \underline{\varphi}_i^c(\theta = -\pi, r \to 0)\right]_\theta = 1 \quad (6)$$

In other word, the intensity factor $\dot{\rho}_I$ of $\underline{\varphi}_I^c(\underline{x})$ can now also be viewed as the **CTOD**, the intensity factor $\dot{\rho}_{II}$ of $\underline{\varphi}_{II}^c(\underline{x})$ as the mode II **CTSD**, and $\dot{\rho}_{III}$ as the mode III **CTSD**.

Details about the reference fields $\underline{\varphi}_i^e(P)$ and $\underline{\varphi}_i^c(P)$ and their construction for each mode can be found in previous papers.

With all these assumptions, the crack tip field in non-linear mixed mode conditions can be fully characterized by only **six independent degrees of freedom** $\dot{K}_I^{\infty}$, $\dot{K}_{II}^{\infty}$, $\dot{K}_{III}^{\infty}$, and

$\dot{\rho}_I, \dot{\rho}_{II}, \dot{\rho}_{III}$ and $\underline{\dot{\rho}} = (\dot{\rho}_I, \dot{\rho}_{II}, \dot{\rho}_{III})$ represents the **discontinuity vector** of the local plastic velocity field across the crack face.

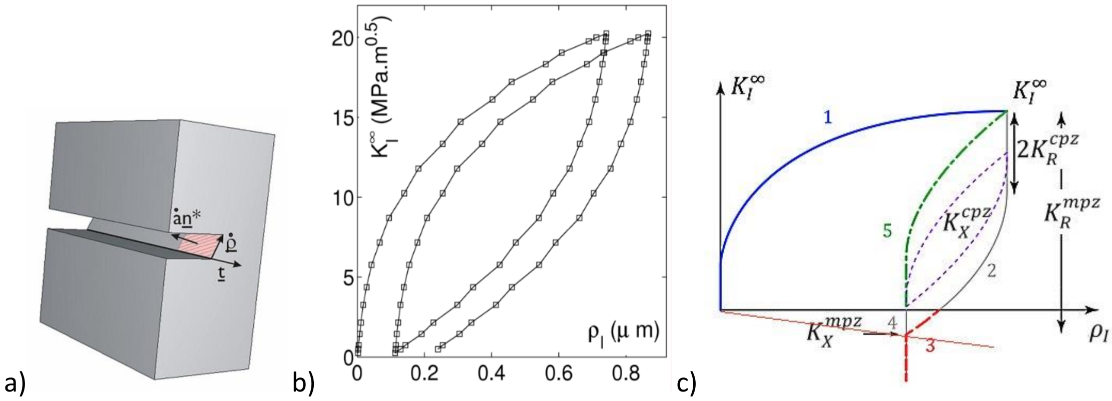

Figure 2. *a) Schematics of the process of creation of new cracked area by crack tip plasticity (Neuman's model (1969), Li's model (1989)). (b) Evolution from FE analyses of the mode I plastic intensity factor ρ_I as a function of the mode I nominal applied stress intensity factor K_I^∞ (c) Illustration of the evolution of the role of each internal variable used in the model*

With such an approximation, following Neuman's and Li's approaches, the crack growth rate due to the geometric process of creation of cracked area by plasticity can be estimated roughly as follows :

$$\dot{a}\underline{n}^* \propto \left(\underline{t} \wedge \underline{\dot{\rho}}\right) \Rightarrow \dot{a} \propto \sqrt{\dot{\rho}_I^2 + \dot{\rho}_{II}^2} \qquad (7)$$

Where $\underline{t}$ is the unit orientation vector of the crack front, where $\underline{n}^*$ is the unit vector normal to the cracked face oriented outward and where $\dot{a}$ is the rate of creation of cracked area per unit length of the crack front. It is worth emphasizing that with such an assumption, mode III **plastic flow** in the crack tip region has no effect on the crack growth rate. This point has been the object of special experiment conducted by Fremy (2014), on the Astree machine of LMT, showing that mode III loading steps in the fatigue cycle contribute to fatigue crack growth.

Once the suitable basis ($\left(\underline{\varphi}_I^e(\underline{x}), \underline{\varphi}_{II}^e(\underline{x}), \underline{\varphi}_{III}^e(\underline{x}), \underline{\varphi}_I^c(\underline{x}), \underline{\varphi}_{II}^c(\underline{x}), \underline{\varphi}_{III}^c(\underline{x})\right)$ of reference fields has been determined, it can be used to post-process velocity fields obtained from finite elements simulations or from experimental field measurements from digital image correlation. The velocity field $\underline{v}^{rec}(P,t)$ recorded at each time step is projected onto the six reference fields to retrieve the intensity factors related to the elastic and inelastic parts for each mode :

$$\dot{\tilde{K}}_i(t) = \frac{\int \underline{v}^{rec}(P,t).\underline{\varphi}_i^e(P)\mathrm{dP}}{\int \underline{\varphi}_i^e(P).\underline{\varphi}_i^e(P)\mathrm{dP}}, \dot{\rho}_i(t) = \frac{\int \underline{v}^{rec}(P,t).\underline{\varphi}_i^c(P)\mathrm{dP}}{\int \underline{\varphi}_i^c(P).\underline{\varphi}_i^c(P)\mathrm{dP}} \qquad (8)$$

To quantify the quality of the approximation in Eq. 3, the error $C_{2R}(t)$ is also determined in each time step:

$$C_{2R}(t) = \sqrt{\frac{\int\left(\underline{v}^{rec}(P,t) - \dot{\tilde{K}}_i(t)\underline{\varphi}_i^e(P) - \dot{\rho}_i(t)\underline{\varphi}_i^c(P)\right)^2 \mathrm{dP}}{\int\left(\underline{v}^{rec}(P,t)\right)^2 \mathrm{dP}}} \qquad (9)$$

This error is usually below 10% but increases drastically when the crack is no more in small scale yielding conditions or when a contact occurs between the crack faces. To quantify the importance of the plastic part of the approximated velocity field, the error $C_{1R}(t)$ is also determined in each time step:

$$C_{1R}(t) = \sqrt{\frac{\int\left(\underline{v}^{rec}(P,t) - \dot{\tilde{K}}_i(t)\underline{\varphi}_i^e(P)\right)^2 \mathrm{dP}}{\int\left(\underline{v}^{rec}(P,t)\right)^2 \mathrm{dP}}} \quad (10)$$

A very small number of degrees of freedom can hence be used to represent reasonably well the kinematics of the crack tip region in mixed mode conditions. Numerical simulations (or experiments with full field measurement) can be used to determine the velocity field and to track the evolution of $\dot{\rho}_i$ for various loading conditions $\dot{K}_i^\infty$, so as to derive a constitutive model of the non-linear behaviour of the crack tip region. The approach used to develop the model is analogous to that used for many years by the mechanics of materials community to develop material laws with internal variables in a thermodynamic framework. The constitutive model for the plasticity of the crack tip region is then associated with a crack propagation model to get the incremental model. In "pure" fatigue, the rate $\dot{a}$ of production of cracked area per unit length of crack front is given by the plastic flow rate $\dot{\rho}_i$: $\dot{a} = \alpha\sqrt{\dot{\rho}_I^2 + \dot{\rho}_{II}^2}$.

Early work was carried out by Hamam et al (2007) on modelling fatigue crack growth in mode I at room temperature under variable amplitude loading for aircraft engine applications and then for railway applications. Then, the model was extended to model fatigue crack growth in non-isothermal conditions and in the presence of an active environment by Ruiz-Sabariego et al. (2009). Attempts have also been done to extend the model to elastic-viscoplastic materials with promising results. A set of constitutive equations was defined that allows determining $d\rho_I$, the plastic flow in mode I in the crack tip region, as a function of the mode I nominal applied stress intensity factor dK_I^∞. The model is based on two elastic domains, one for the cyclic plastic zone and the other for the monotonic plastic zone. Each of them is characterized by two internal variables that represent respectively the centre (K_X^{cpz} *and* K_X^{mpz}) and the size (K_R^{cpz} *and* K_R^{mpz}) of each elastic domain. Results such as that plotted in Fig. 2b can be obtained using the finite element method, either for a fixed position of the crack front to get $\left(\frac{\partial V_{int}}{\partial \rho_I}\right)$, or after "growing" numerically the crack without allowing plastic strain, so as to get $\left(\frac{\partial V_{int}}{\partial a}\right)$. This allows determining independently an evolution equation for each internal variable, due to plasticity $\left(\frac{\partial V_{int}}{\partial \rho_I}\right)$ or due to crack propagation $\left(\frac{\partial V_{int}}{\partial a}\right)$. The evolution equations introduced for each internal variables are empirical.

The equations were implemented and their coefficients identified using the finite element method for a low carbon steel by R. Hamam (2007). The coefficient α of the crack propagation law $\dot{a} = \alpha\sqrt{\dot{\rho}_I^2}$ was adjusted using a mode I fatigue crack growth experiment in constant amplitude fatigue at R=0. Then the model was used to simulate the stress ratio effect, the overload effect and the effects of various block loadings on fatigue crack growth. The simulations were compared to experimental results giving satisfactory results. It was shown that the model is capable of representing the stress ratio effect, the overload effect, the overload retardation effect, the higher retardation effect after 10 overloads than after one single overload etc.

3. Conclusion

An approach was proposed to model the non-local elastic-plastic behaviour of the crack tip region. This approach is based on an approximation of the kinematics of the crack tip region. To some extent it is a non-local elastic plastic constitutive model, tailored for a crack tip region.
The use of this model makes it possible to enrich the usual linear elastic fracture mechanics functions by additional terms that are capable of accounting for the cyclic elastic-plastic behaviour of the material including history effects. The model is valid only in small scale yielding conditions and is

dedicated to be used for predicting fatigue crack growth in complex loading conditions (variable amplitude loading, non-isothermal conditions etc…).
The model provides a scalar measure for each mode of the amount of plastic flow during a load step. The parameters of the model can be identified using the constitutive law of the material and non-linear finite elements computations of the behaviour of the crack tip region. This model is a non-linear constitutive law for the crack tip region, with internal variables to account for memory effects. A temperature dependency of its parameters can be defined in order to use it in non-isothermal conditions. Then, the rate of production of cracked area during a time step, due to pure fatigue, is assumed to be directly proportional to the amount of plastic flow predicted by the model. Other mechanisms (oxidation, corrosion), can be also considered, and added to the crack propagation law if necessary. The constitutive model for the crack tip region and the crack propagation law, together, are an incremental model for fatigue crack growth.

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