Analyses en fatigue à grand nombre de cycles : Apport des approches probabilistes

François HILD

LMT

école normale supérieure paris—saclay

cnrs

Outline

Damage mechanisms / examples

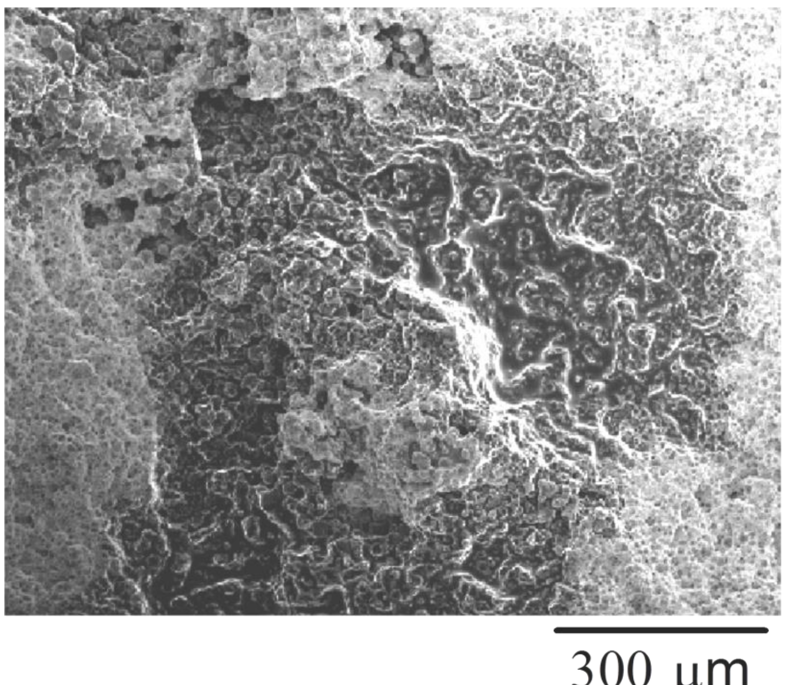
Weakest link hypothesis

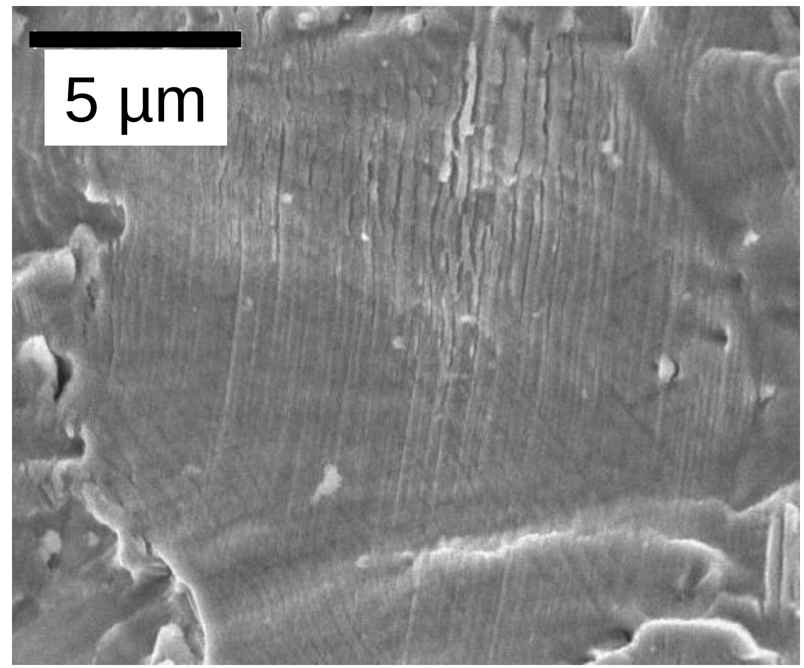
Self-heating experiments / endurance limits

Crack networks: thermal striping

Summary and perspectives

Damage Mechanisms

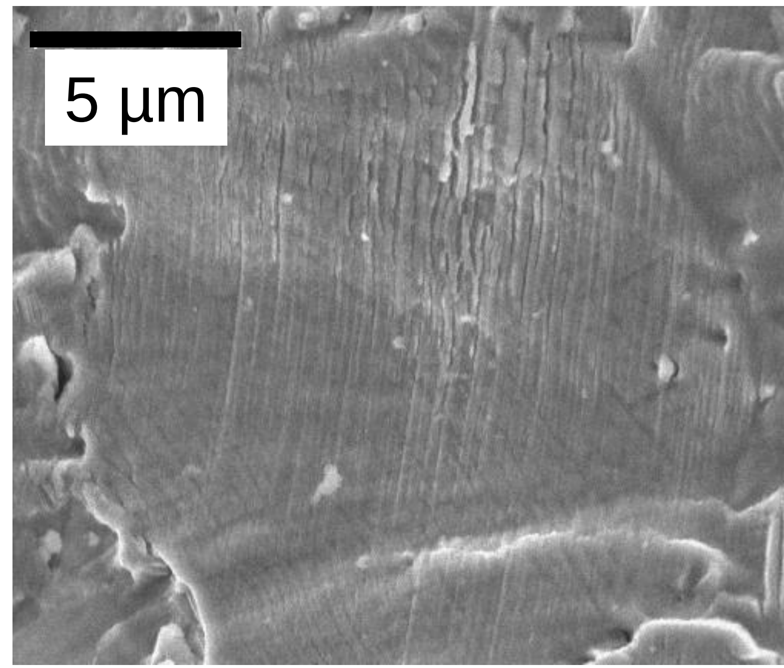


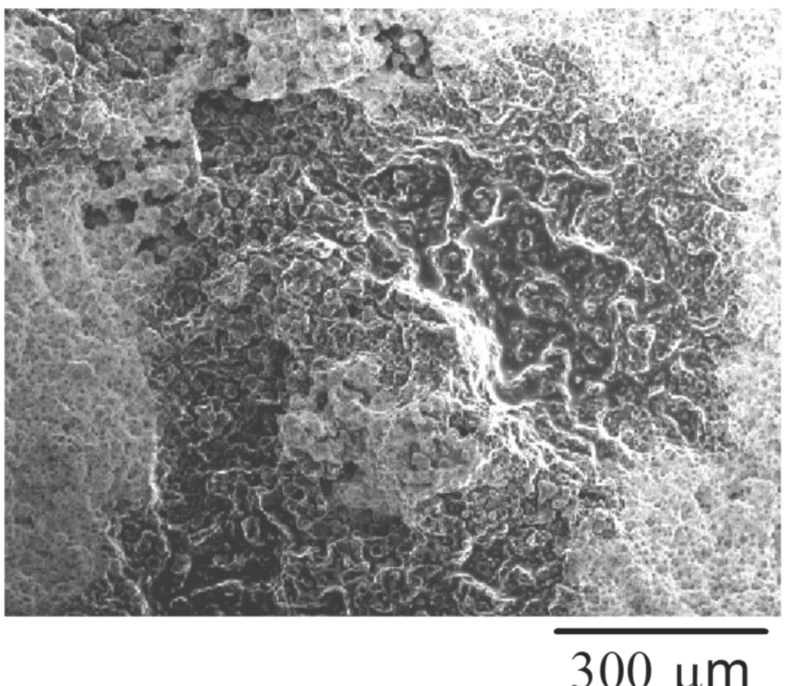


Damage Mechanisms

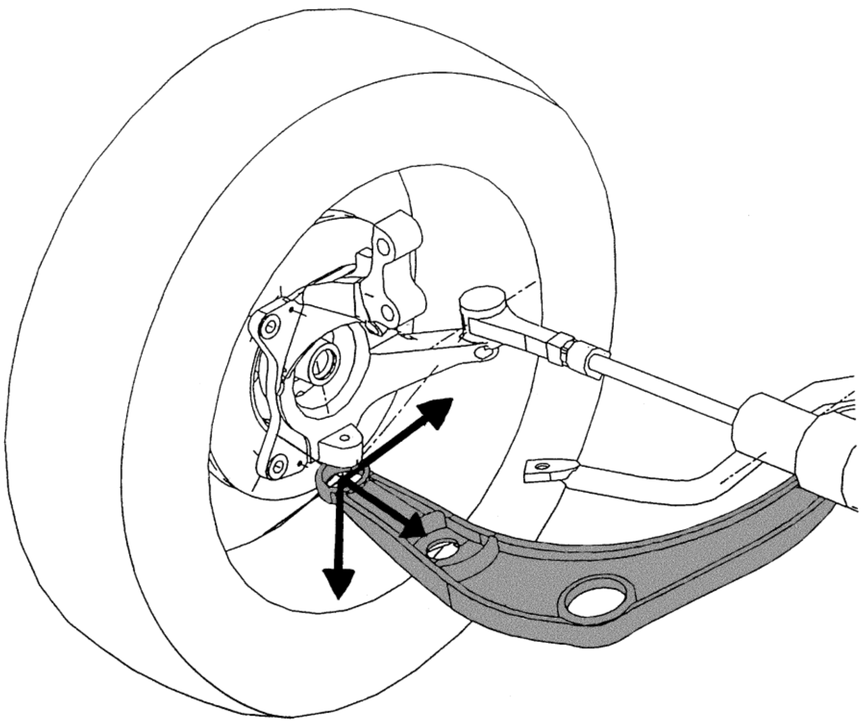
How to model distributions of initiation sites?

- points (Poisson Point Process)
  - $\Rightarrow$ no length scale
- more accurate description
  - $\Rightarrow$ length scale(s)

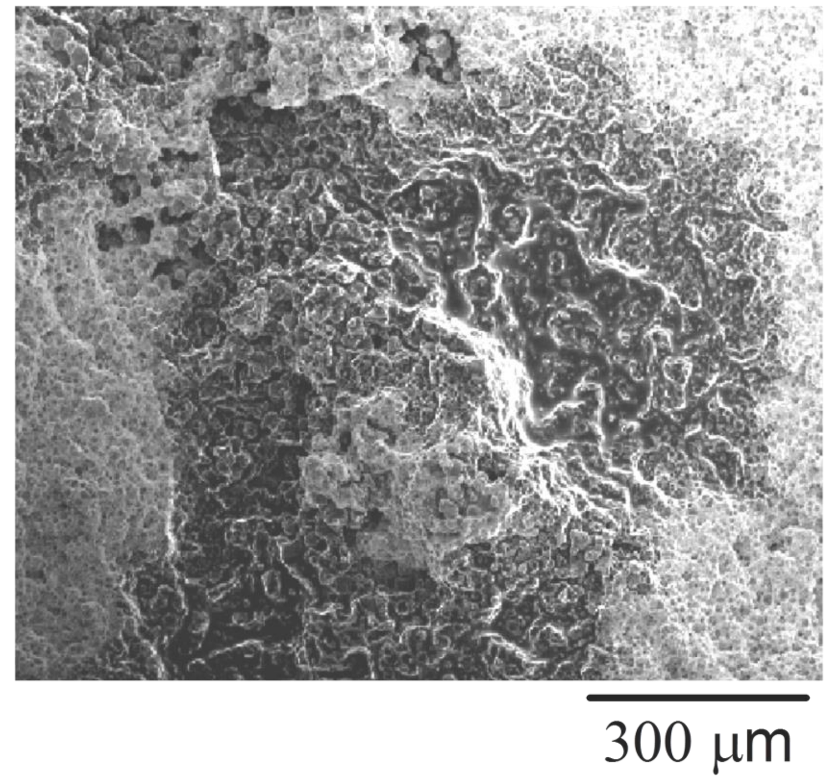




HCF of Suspension Arm



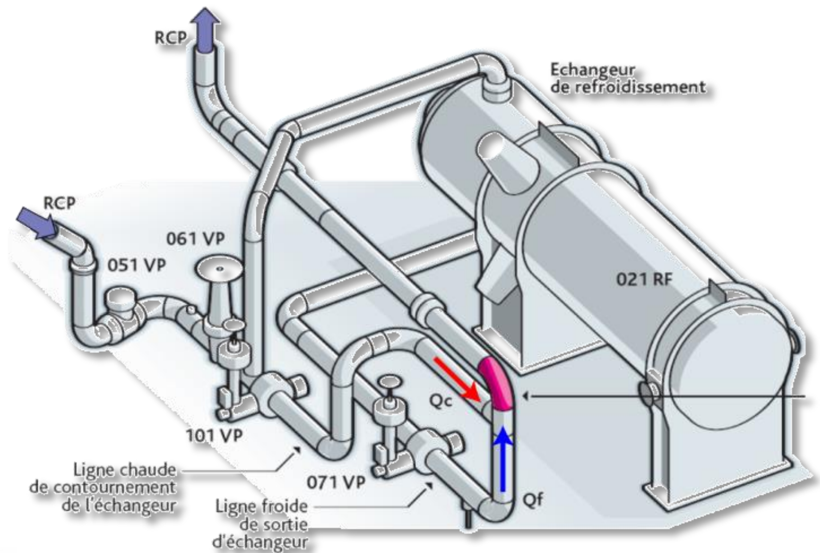
Suspension arm

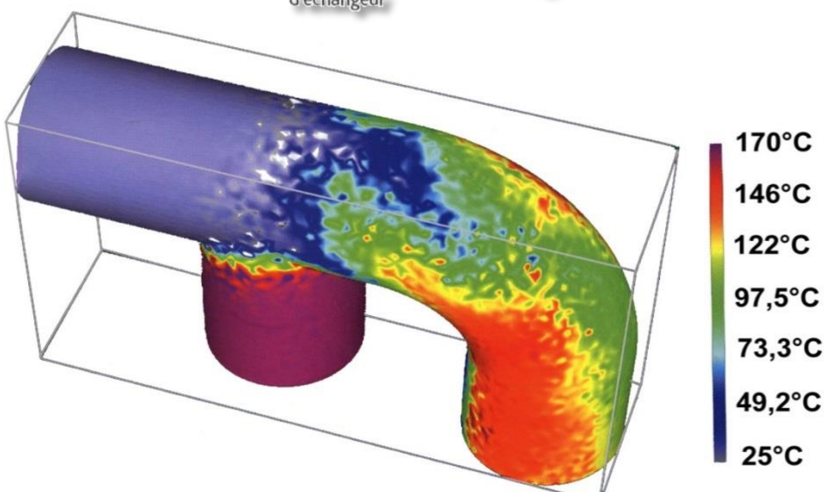


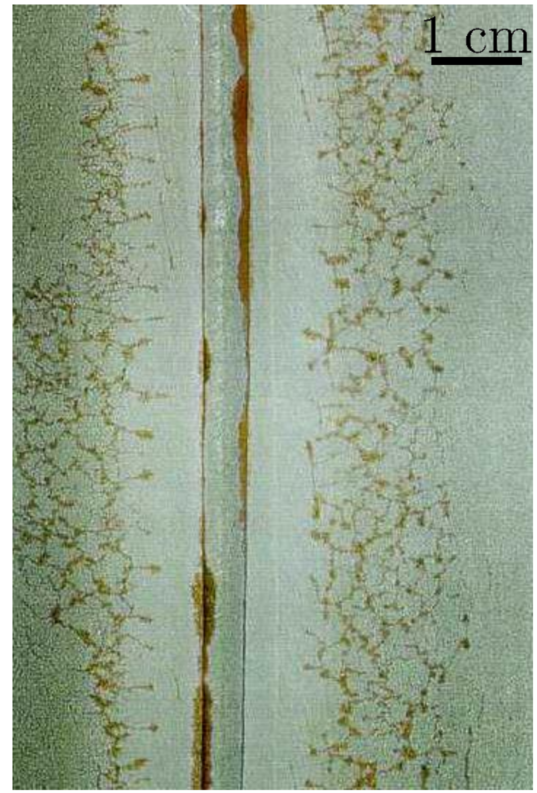
Microshrinkage cavity
in nodular graphite cast iron

[Chantier *et al.*, 2000, *Proc. ECF13* 3C.127]

Reactor Heat Removal System of Pressurized Water Reactors







observations on damaged zone

[Cipière et al., 2002, *Proc. Int. Symp. Contrib. Mat. Investig. Resol. Prob. Encount. PWRs*]
[Stéphan *et al.*, 2002, *Proc. Fatigue 2002* pp. 1707-1714]

Mécamat

2017

Outline

Damage mechanisms / examples

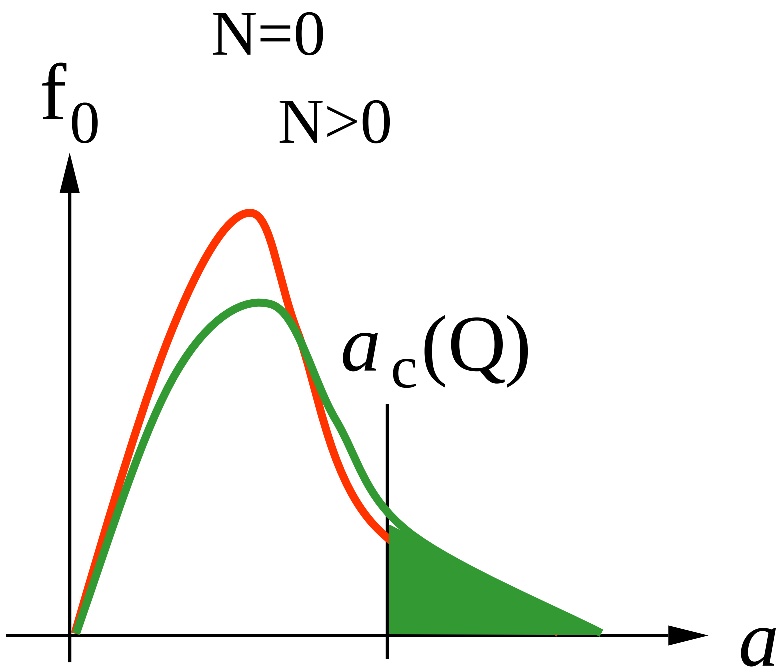
Weakest link hypothesis

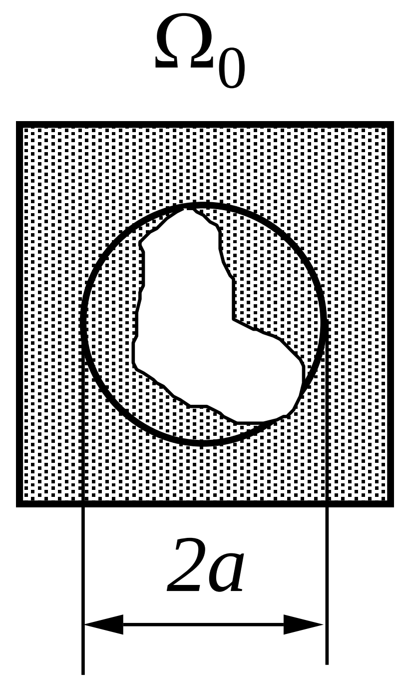
Self-heating experiments / endurance limits

Crack networks: thermal striping

Summary and perspectives

Local Failure Probability





$$P_{F0}(Q,N) = \int_{a_c(Q)}^{+\infty} f_N(a)\, da$$

Local Failure Probability

Unstable propagation: $P_{F0}(Q) = \int_{a_c}^{+\infty} f_0(a)\, da$

Stable propagation: $P_{F0}(Q,N) = \int_{a_c}^{+\infty} f_N(a)\, da$

No initiation of new defects: $f_N(a_N)\, da_N = f_0(a_0)\, da_0$

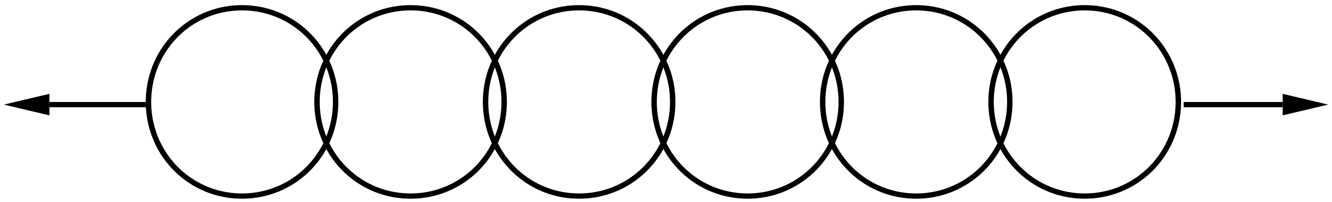
Stable propagation: $P_{F0}(Q,N) = \int_{a_{c0}}^{+\infty} f_0(a)\, da$

a_{c0}: initial size that becomes critical after N cycles ($=a_c$)

[HF and Roux, 1991, *Mech. Res. Comm.* 18 pp. 409-414]

Global Failure Probability

Weakest link hypothesis*

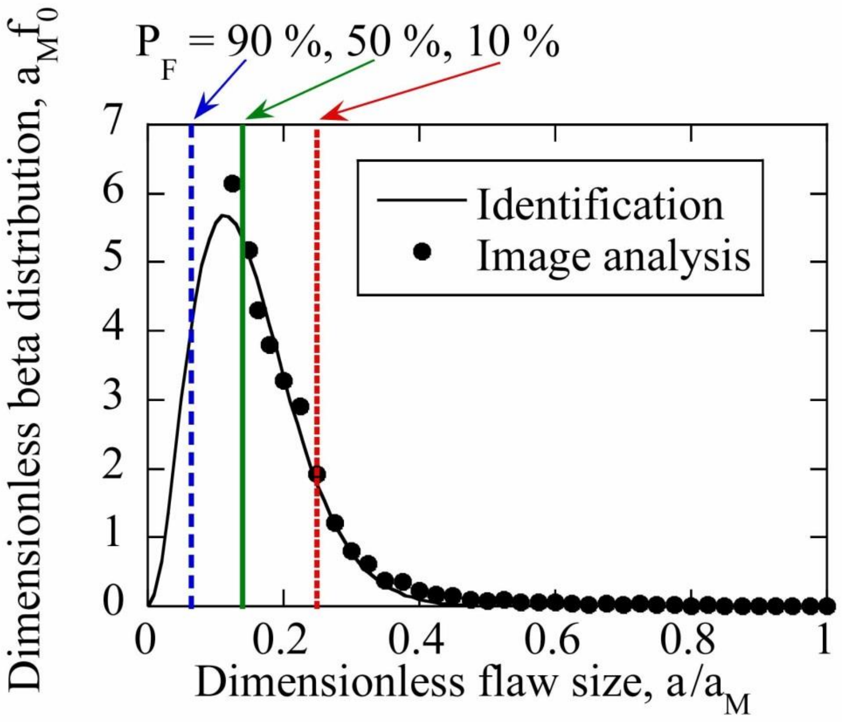


$$P_S = \prod_{i=1}^{M} P_{Si} \quad \Leftrightarrow \quad \ln(1-P_F) = \sum_{i=1}^{M} \ln(1-P_{Fi})$$

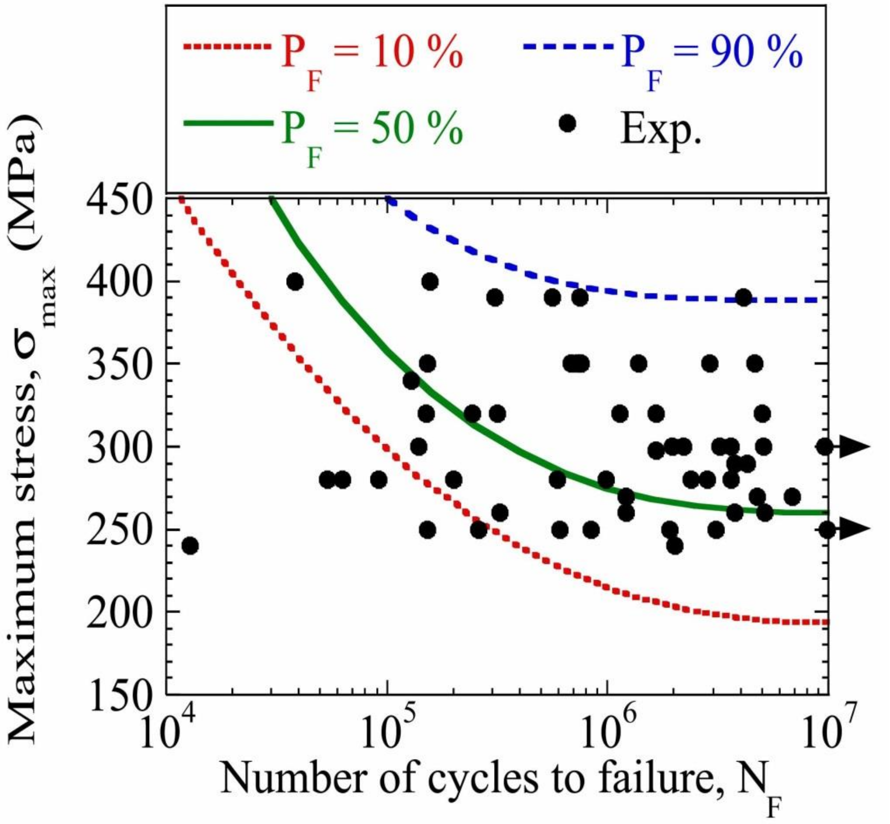
$$P_F = 1 - \exp\left\{ \frac{1}{V_0} \int_{\Omega} \ln\left(1 - P_{F0}\right) dV \right\}$$

*[Freudenthal, 1968, in *Fracture* 2 pp. 591-619]

HCF of SG Cast Iron



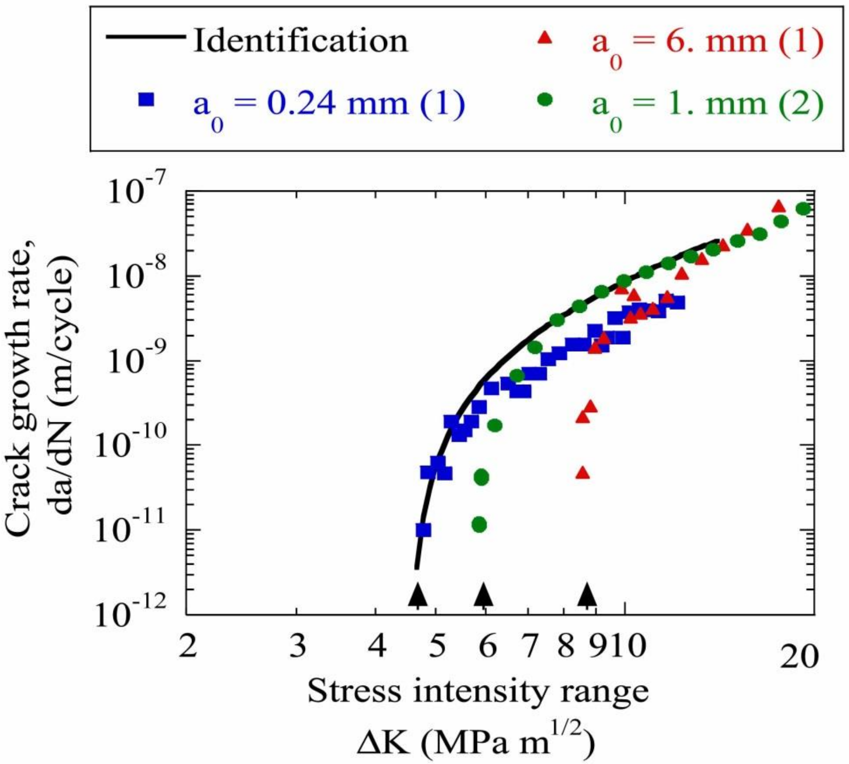
Identification of flaw size distribution



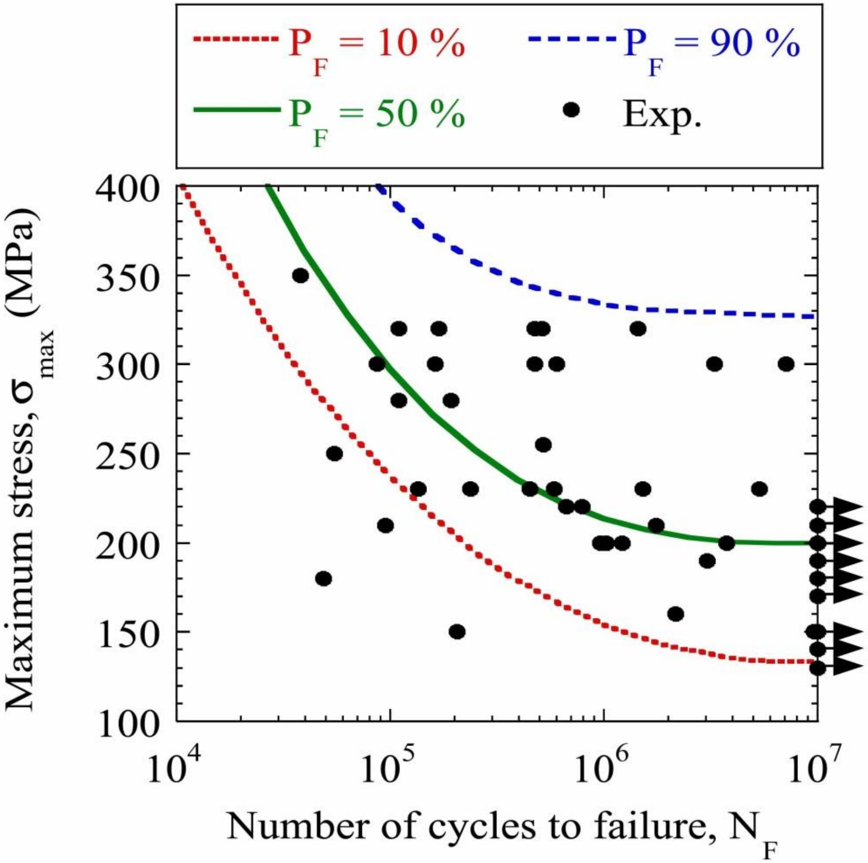
Cyclic tension test (R = 0.1)

[Yaacoub Agha *et al.*, 1998, *FFEMS* 21 pp. 287-296]

HCF of SG Cast Iron



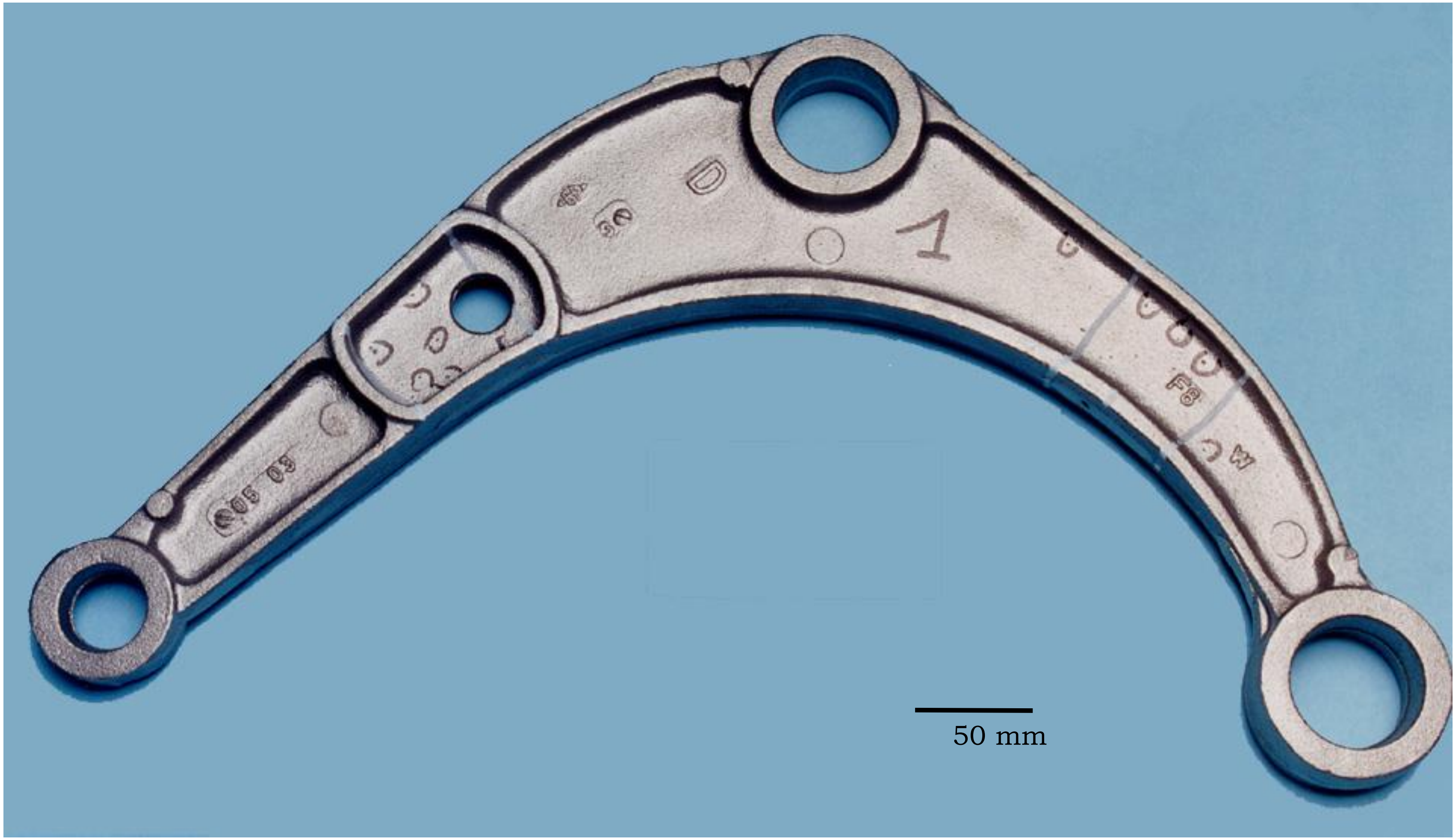
Validation of
crack propagation law



Prediction of
tension/compression
tests (R = -1)

[1][Clément *et al.*, 1984, *FFEMS* 7 pp. 251-265]
[2][Renault, 1995, internal report]

HCF of Suspension Arm



HCF of Suspension Arm

Bulk properties:

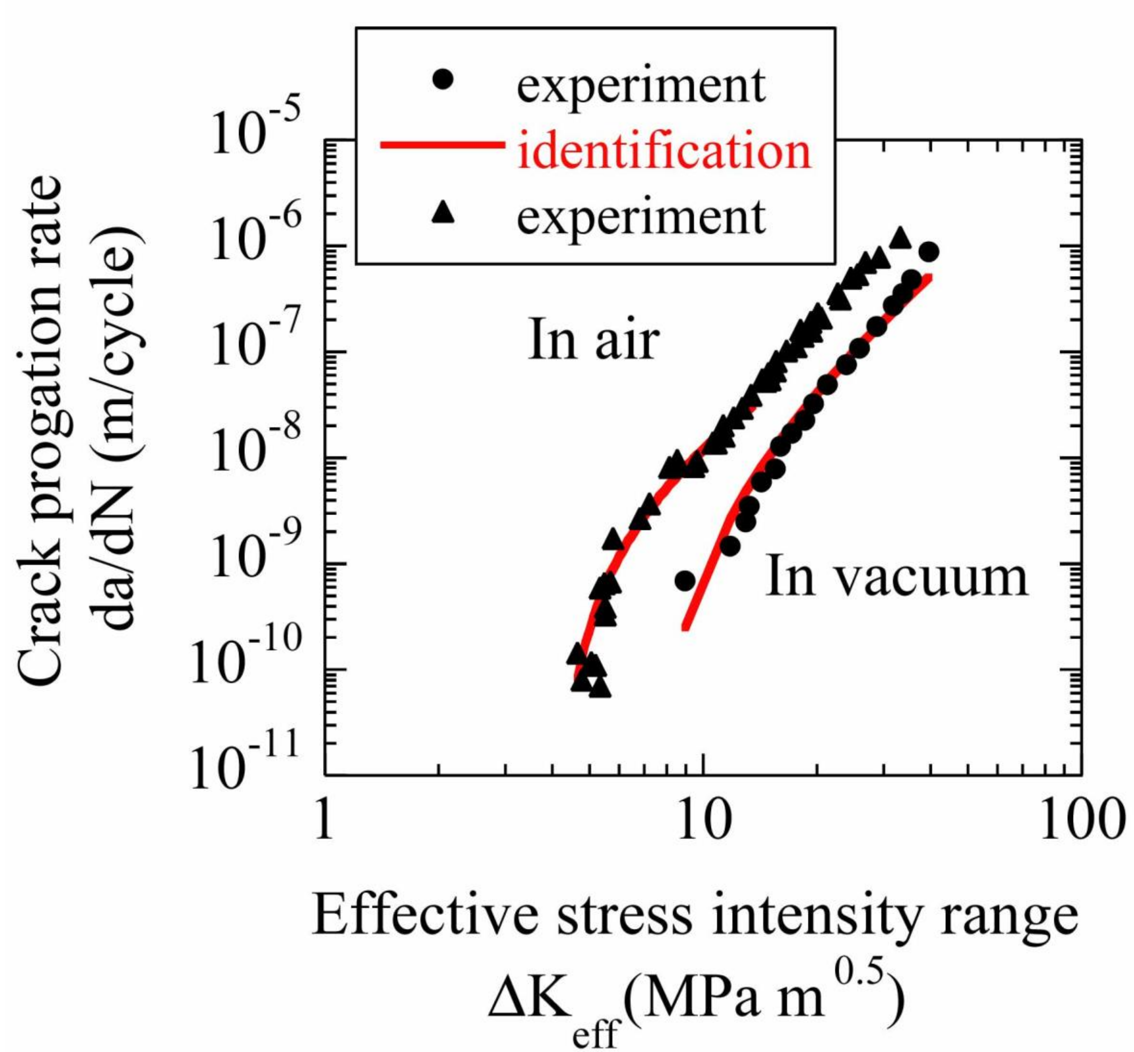
- defect distribution
- propagation law (in vacuum)
- residual stresses (neglected)
- volume elements (tetrahedra)

Surface properties:

- defect distribution
- propagation law (in air)
- residual stresses (XRD after 100 cycles)
- surface elements (shells)

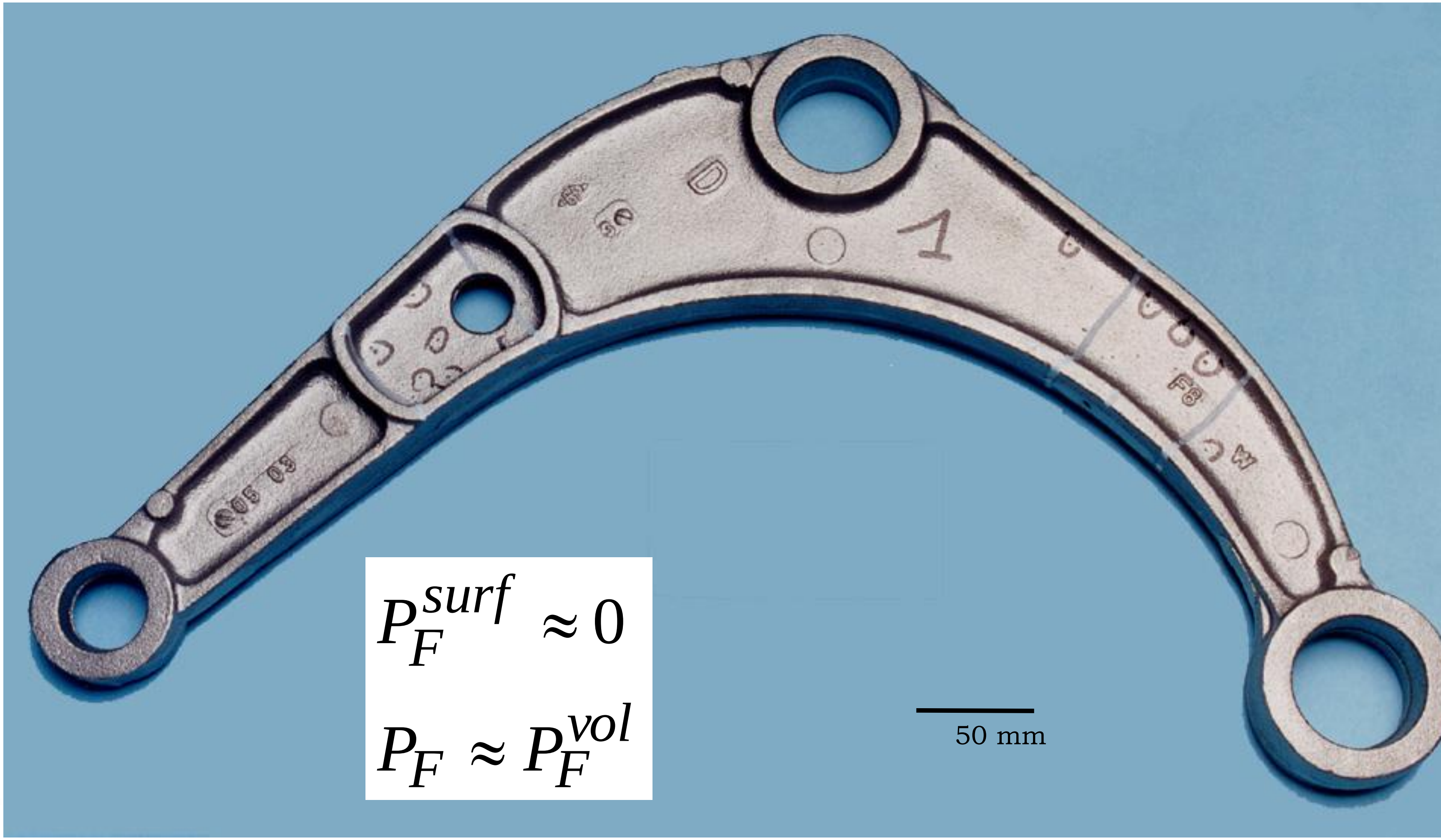
$$1 - P_F = \left(1 - P_F^{vol}\right)\left(1 - P_F^{surf}\right)$$

HCF of Suspension Arm



[Nadot *et al.*, 1998, *Scripta Mat.* 37 pp. 549-553]

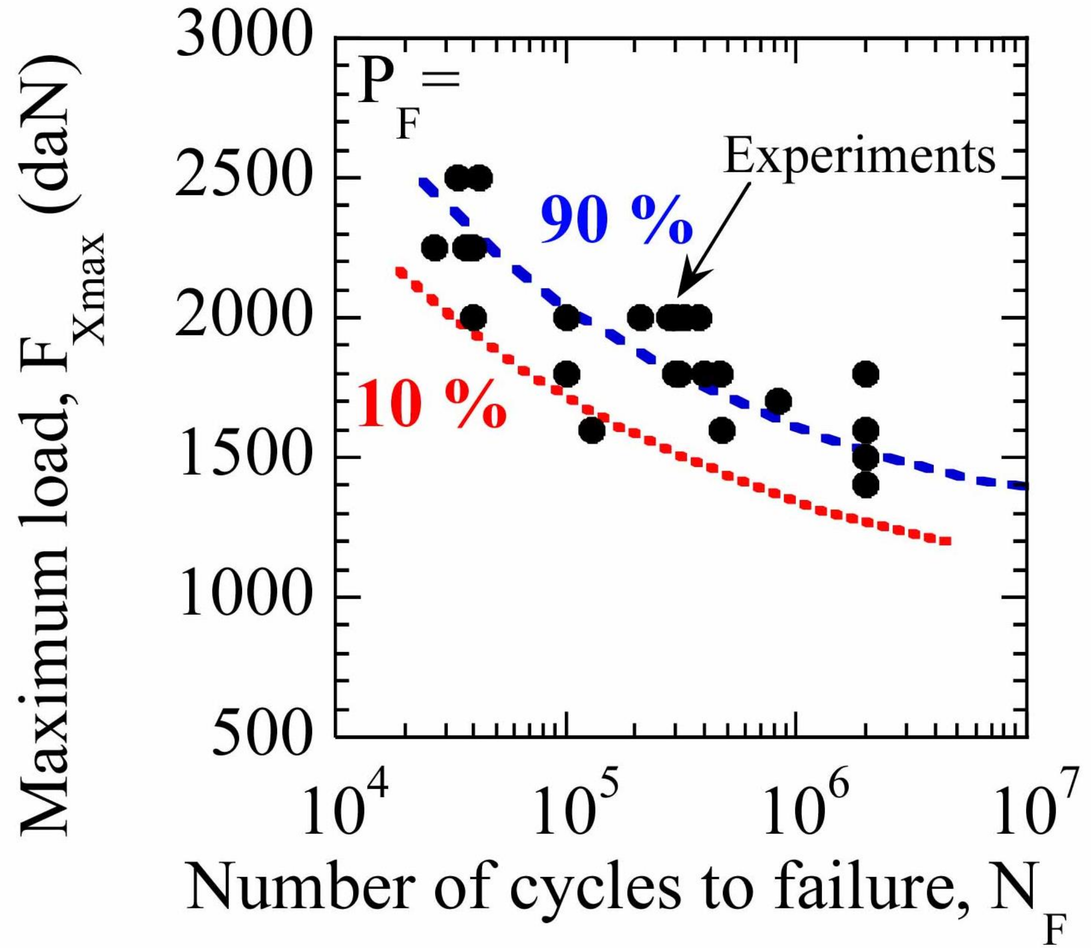
HCF of Suspension Arm



$$P_F^{surf} \approx 0$$

$$P_F \approx P_F^{vol}$$

HCF of Suspension Arm



[Chantier *et al.*, 2000, *Proc. ECF13* 3C.127]

Outline

Damage mechanisms / examples

Weakest link hypothesis

Self-heating experiments / endurance limits

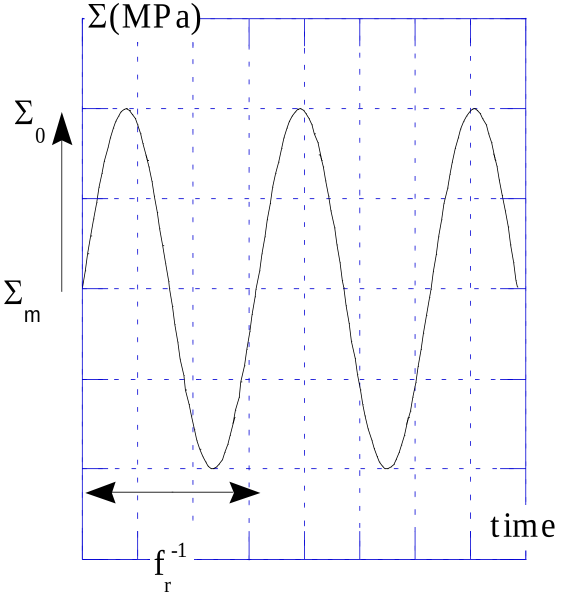
Crack networks: thermal striping

Summary and perspectives

Self-Heating Experiments

Example

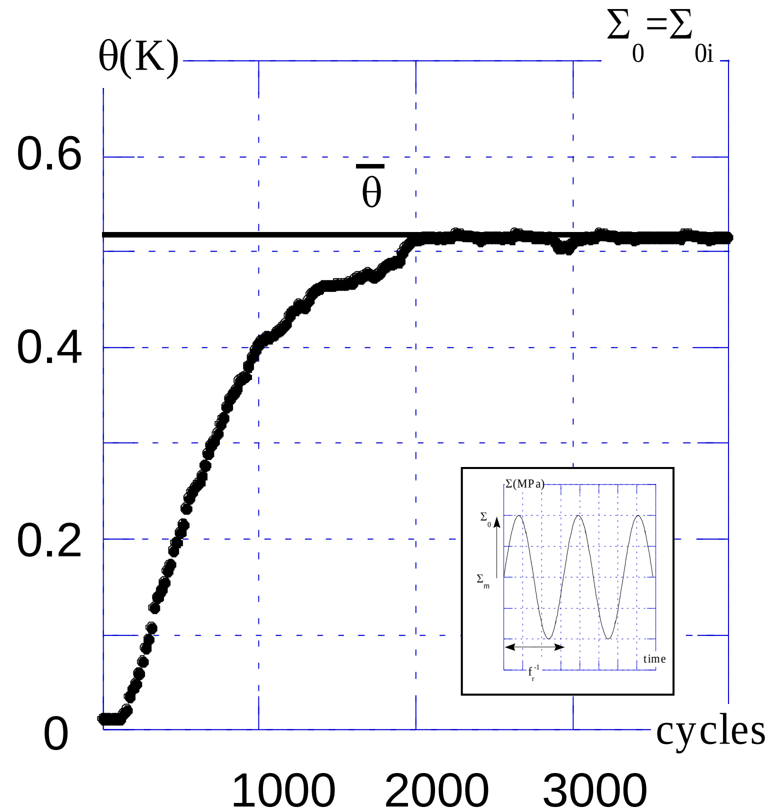
Load history



Load ratio $$R = \frac{\Sigma_{\min}}{\Sigma_{\max}}$$

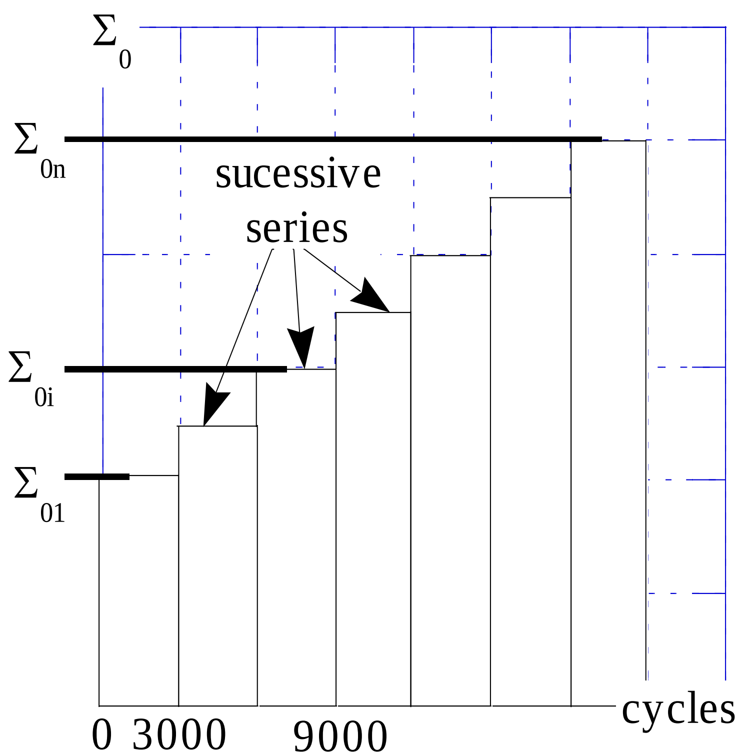
[Stromeyer, 1915, *Rep. Brit. Ass.* 638; Moore and Kommers, 1921, *Chem. Met. Eng.* 25 pp. 1141-1144; Lehr, 1926, *Glasers Ann. Gew.* 1184 pp. 109-114; Welter, 1937, *Wlad. Inst. Met.* 4 pp. 30-39; Cazaud, 1948, *La fatigue des métaux*]

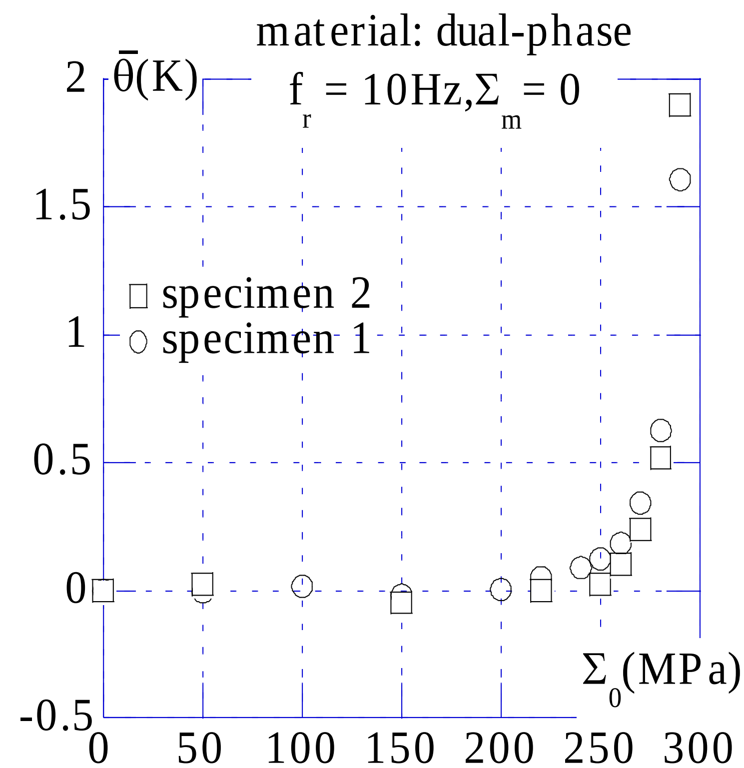
Self-Heating Experiments



5 minutes @ 10 Hz

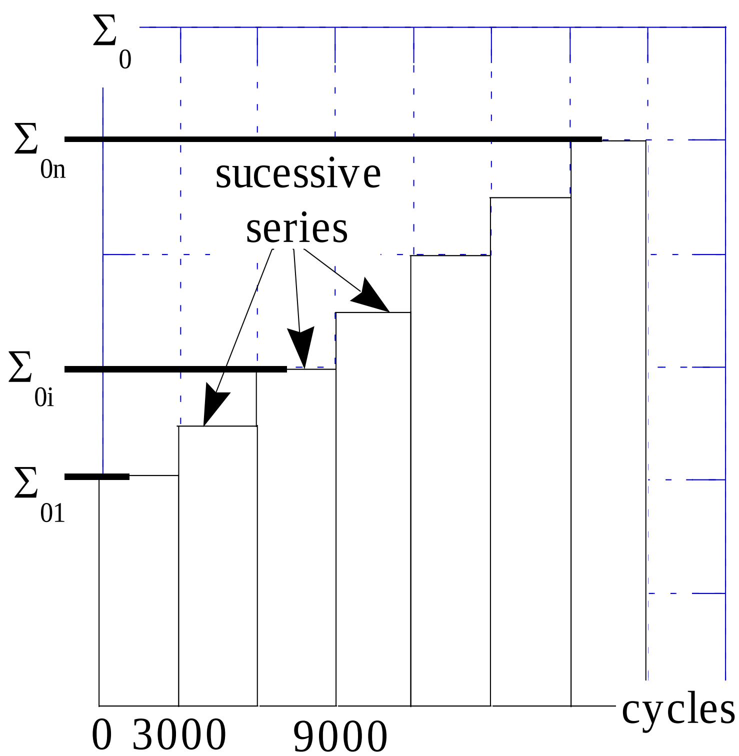
Self-Heating Experiments

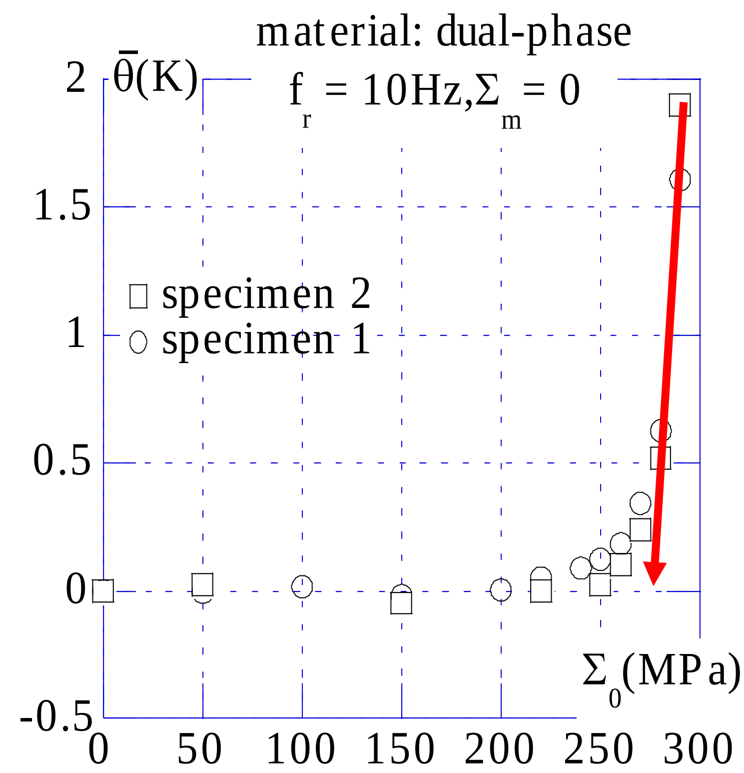




≈ 35 minutes @ 10 Hz

Self-Heating Experiments

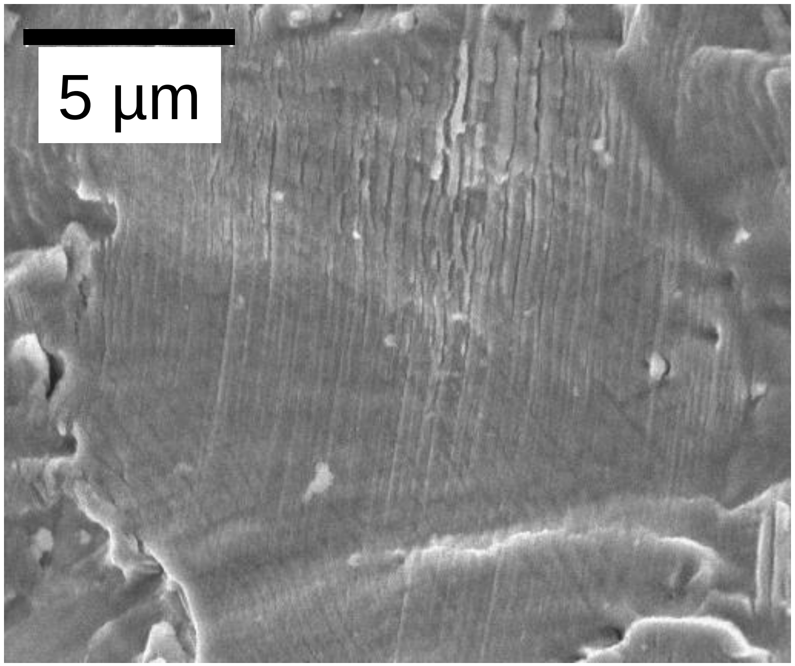


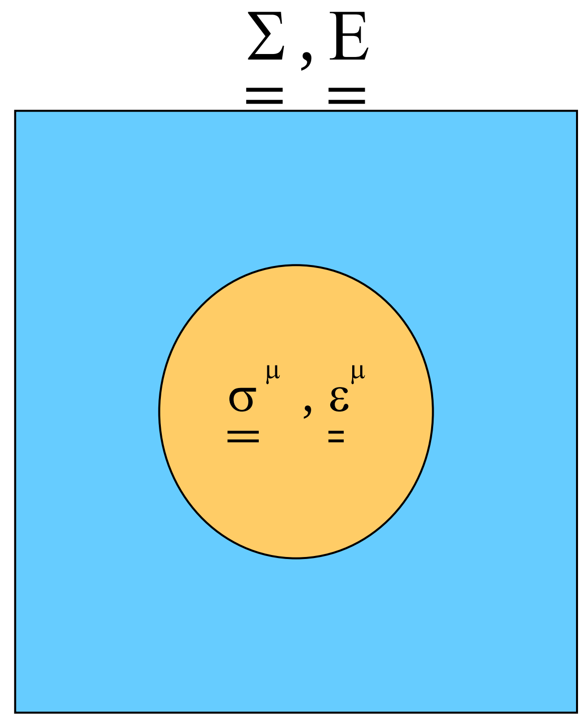


≈ 35 minutes @ 10 Hz

[Galtier, 1993, *PhD dissertation*, ENSAM; Luong, 1995, *Nucl. Eng. Design* 158 pp. 363-376; Bérard et al., 1998, *Mat. Techn.* 1-2 55-57; Krapez *et al.*, 1999, *Proc. 5th AITA* pp. 379-385; La Roza and Risitano, 2000, *Int. J. Fat*. 22 pp. 65-73]

Microplasticity



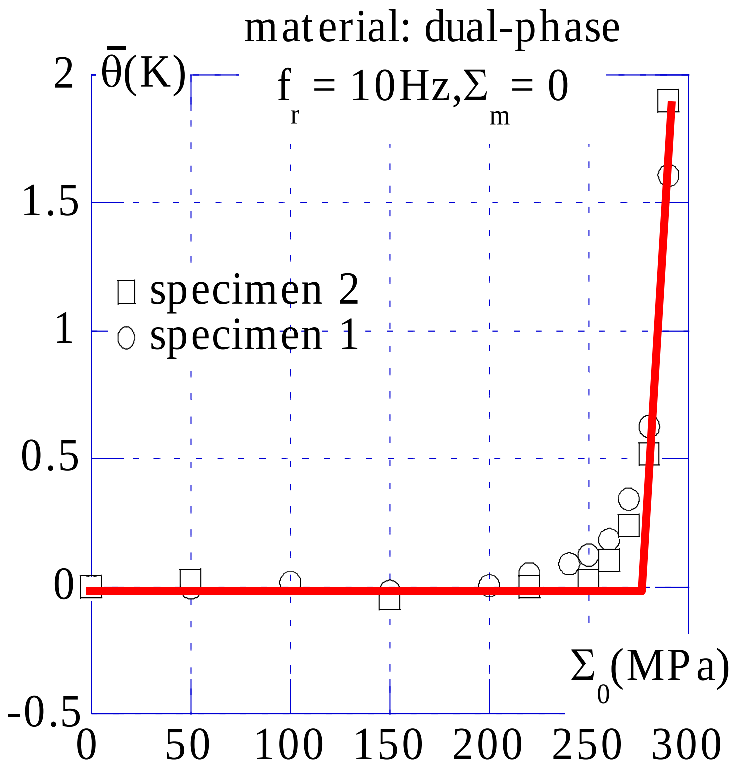


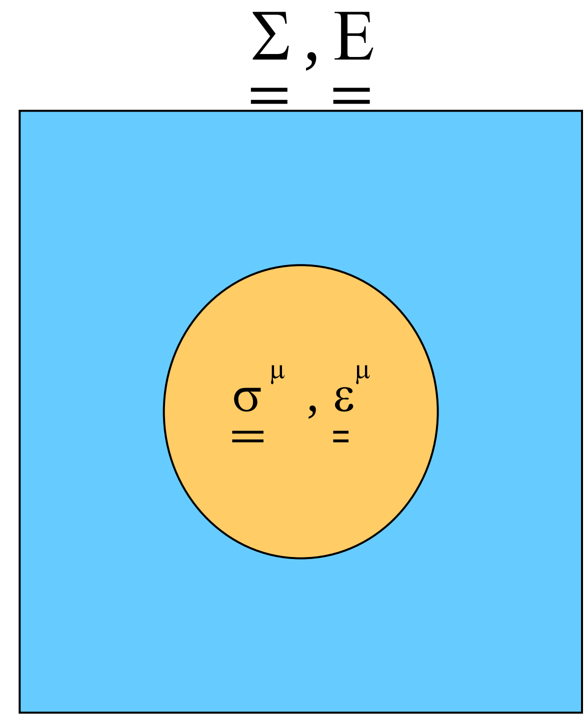
Mean dissipated power per cycle

$$\bar{S} = \frac{4 f_v f_r \sigma_y^{\mu} \left\langle \Sigma_0 - \sigma_y^{\mu} \right\rangle}{C + 3\mu(1-\beta)} = f_r \times D$$

[Doudard *et al.*, 2005, *FFEMS* 28 pp. 279-288]

Microplasticity

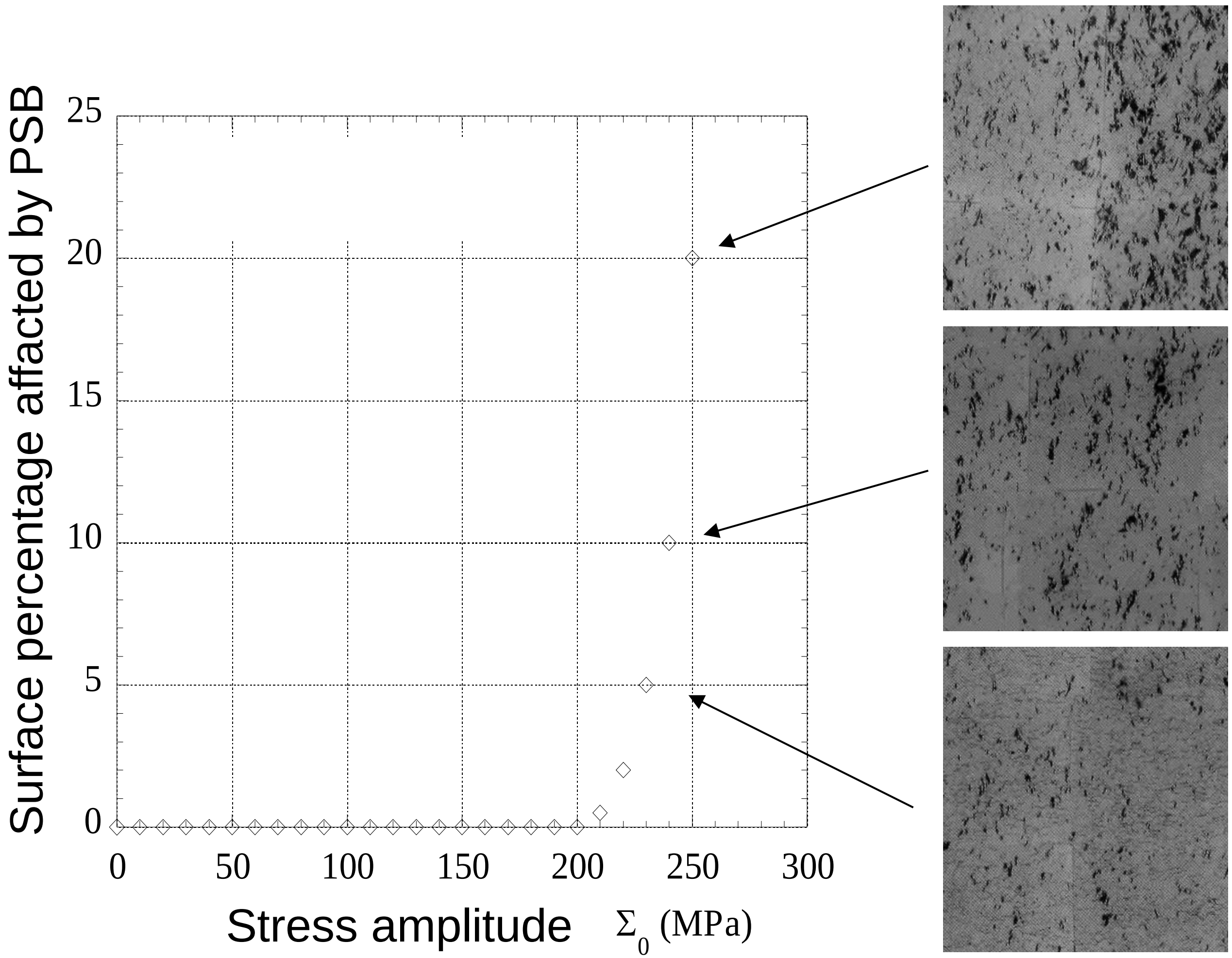




$$\bar{S} = \frac{4 f_v f_r \sigma_y^\mu \left\langle \Sigma_0 - \sigma_y^\mu \right\rangle}{C + 3\mu(1-\beta)} = f_r \times D$$

[Doudard *et al.*, 2005, *FFEMS* 28 pp. 279-288]

Gradual Development of Microplasticity



[Cugy and Galtier, 2002, *IRSID* internal report]

Poisson Point Process

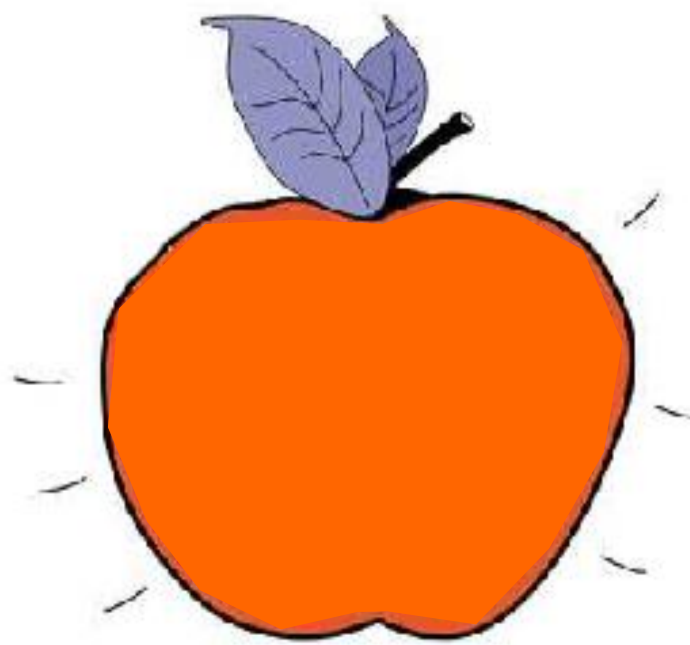
Probability of finding *k* defects in Ω:

$$P_k(\Omega) = \frac{[N(\Omega)]^k}{k!} \exp[-N(\Omega)]$$

Average number of defects in Ω: $N(\Omega) = V\lambda$

Process intensity:

$$\lambda_t(\sigma) = \frac{1}{V_0}\left(\frac{\Sigma_0}{\sigma_0}\right)^m$$



Poisson Point Process

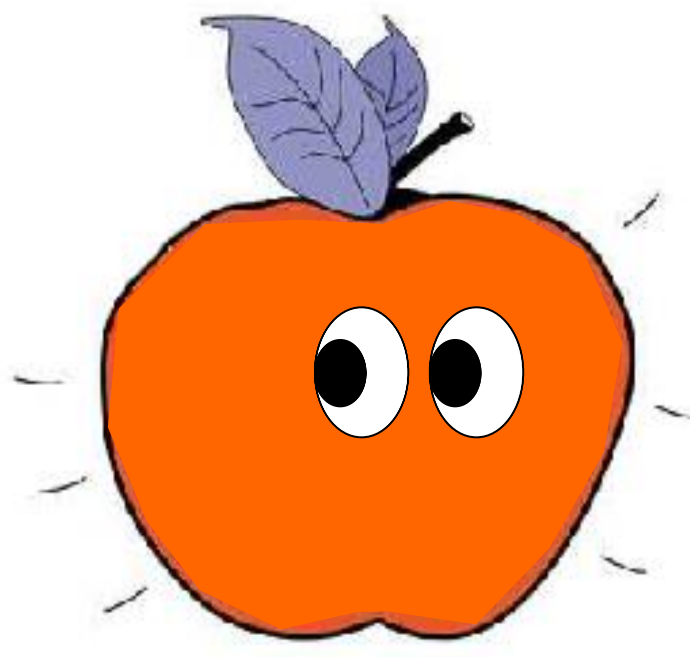
Probability of finding *k* defects in Ω:

$$P_k(\Omega) = \frac{[N(\Omega)]^k}{k!} \exp[-N(\Omega)]$$

Average number of defects in Ω: $N(\Omega) = V\lambda$

Process intensity:

$$\lambda_t(\sigma) = \frac{1}{V_0}\left(\frac{\Sigma_0}{\sigma_0}\right)^m$$



Poisson Point Process

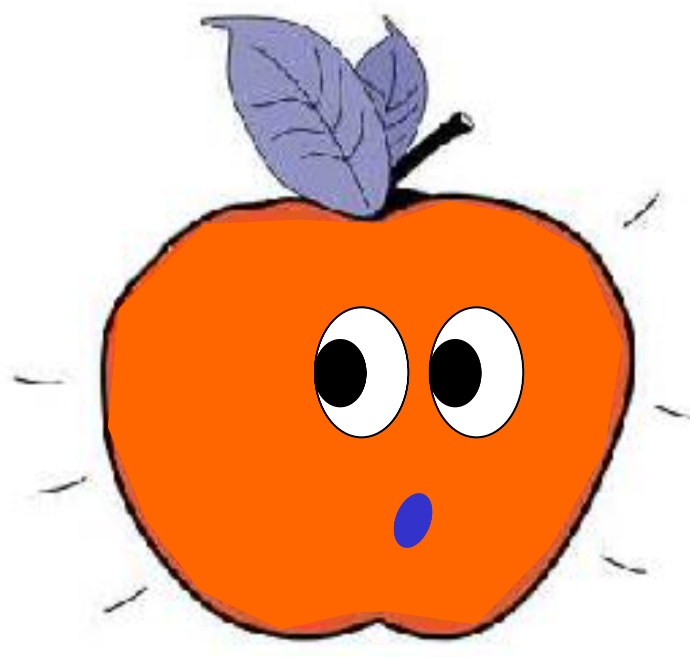
Probability of finding *k* defects in Ω:

$$P_k(\Omega) = \frac{[N(\Omega)]^k}{k!} \exp[-N(\Omega)]$$

Average number of defects in Ω: $N(\Omega) = V\lambda$

Process intensity:

$$\lambda_t(\sigma) = \frac{1}{V_0}\left(\frac{\Sigma_0}{\sigma_0}\right)^m$$



Poisson Point Process

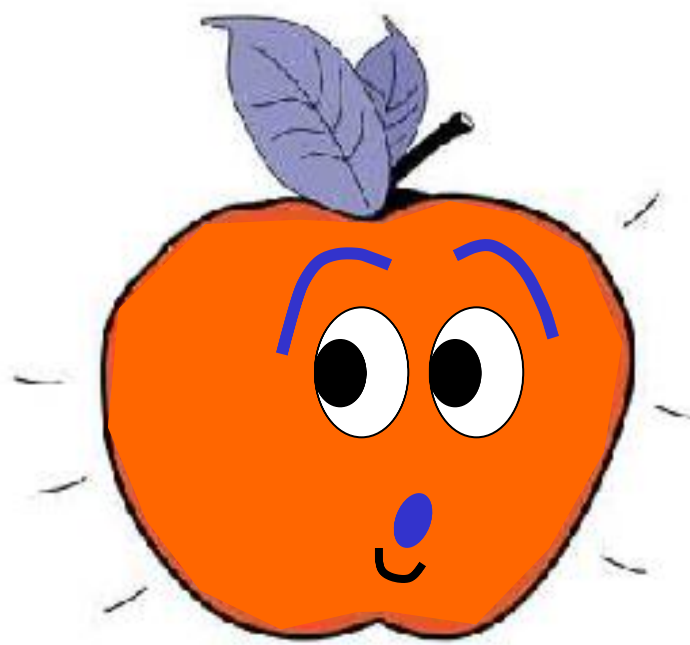
Probability of finding *k* defects in Ω:

$$P_k(\Omega) = \frac{\left[N(\Omega)\right]^k}{k!} \exp\left[-N(\Omega)\right]$$

Average number of defects in Ω: $N(\Omega) = V\lambda$

Process intensity:

$$\lambda_t(\sigma) = \frac{1}{V_0}\left(\frac{\Sigma_0}{\sigma_0}\right)^m$$



[Palin-Luc, 2017, *MECAMAT*]

Heat Equation

$$\dot{\theta} + \frac{\theta}{\tau_{eq}} = \frac{\overline{S}}{c\rho}$$

$$\overline{S} = f_r \times D$$

D: Mean dissipated power per cycle

For 1 site: $$\delta\left(\sigma_y^\mu, \Sigma_0\right) = \frac{4V_0}{h}\sigma_y^\mu \left\langle \Sigma_0 - \sigma_y^\mu \right\rangle$$

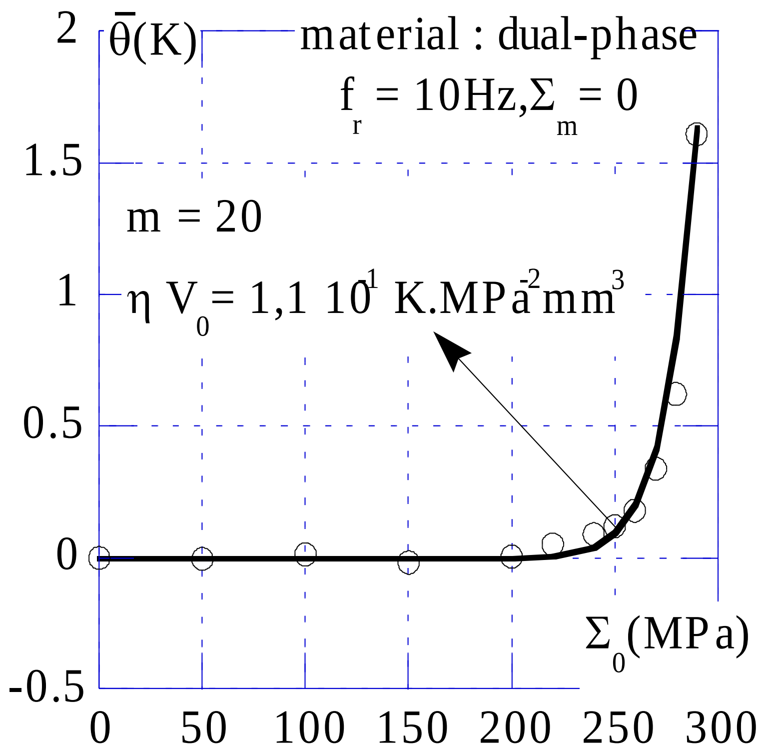
$$D = \frac{1}{V}\int_0^{\Sigma_0} \underbrace{\delta\left(\Sigma, \Sigma_0\right)}\underbrace{\frac{d\lambda}{d\Sigma} V d\Sigma} = \frac{4V_0 m}{h(m+1)(m+2)} \frac{{\Sigma_0}^{m+2}}{\left(V_0^{1/m} S_0\right)^m}$$

Mean dissipation for a site with yield stress Σ

Number of sites with yield stress between Σ and Σ + *d*Σ

[Charkaluk, 2017, *MECAMAT*]

Self-Heating Experiments



Link with Endurance?

Weakest link hypothesis: 1 active site ≡ failure of sample

$$P_F = P_{k>0}(\Omega) = 1 - P_{k=0}(\Omega) = 1 - \exp(-\lambda V)$$

$$P_F = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\Sigma_0}{S_0}\right)^m \right]$$

[Weibull, 1951, *J. Appl. Mech.* 18 pp. 293-297]
[Gulino and Phoenix, 1991, *J. Mater. Sci.* 26 pp. 3107-3118]
[Jeulin, 1991, *thèse d'État*, University of Caen]

Link with Endurance?

Case of non-uniform stress field

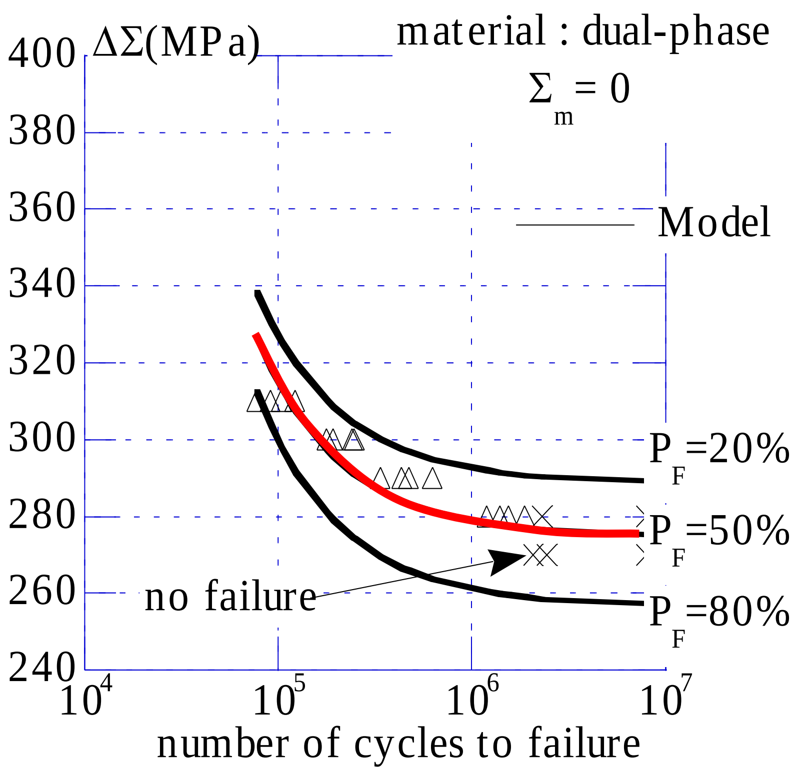
$$P_F = 1 - \exp\left[-\frac{V H_m}{V_0} \left(\frac{\Sigma_F}{S_0} \right)^m \right]$$

$$H_m = \frac{1}{V} \int_\Omega \left(\frac{\Sigma_0}{\Sigma_F} \right)^m dV \leq 1$$

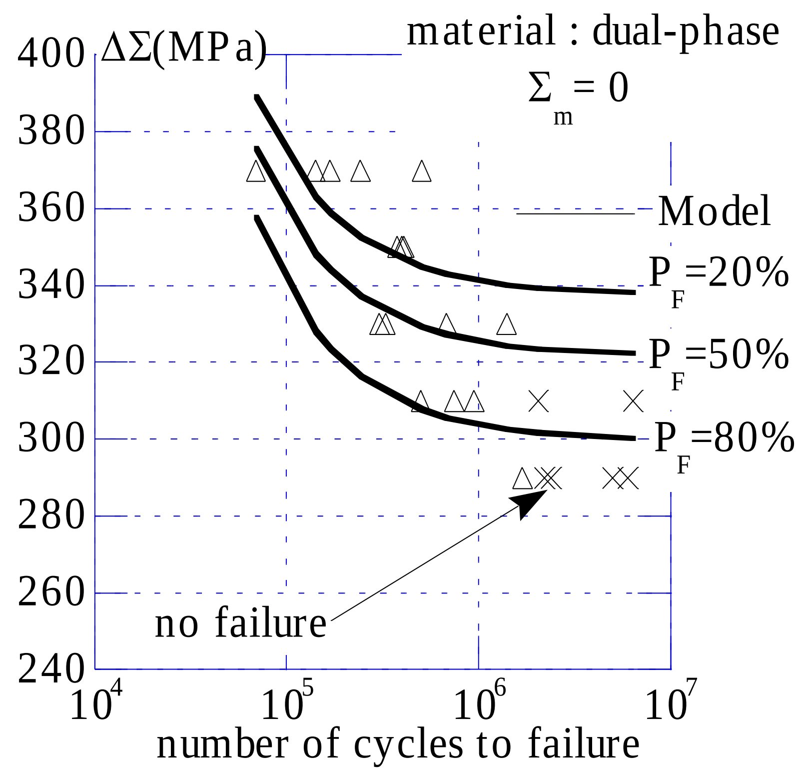
⇒ Accounts for volume and stress heterogeneity effects

[Weibull, 1952, *Appl. Mech. Rev.* 5 pp. 449-451]
[Davies, 1973, *Proc. Brit. Ceram. Soc.* 22 pp. 429-452]
[HF *et al.*, 1992, *CRAS* 315 (II) pp. 1293-1298]

Validation



Tension/compression



Flexure

[Doudard *et al.*, 2005, *FFEMS* 28 pp. 279-288]

Outline

Damage mechanisms / examples

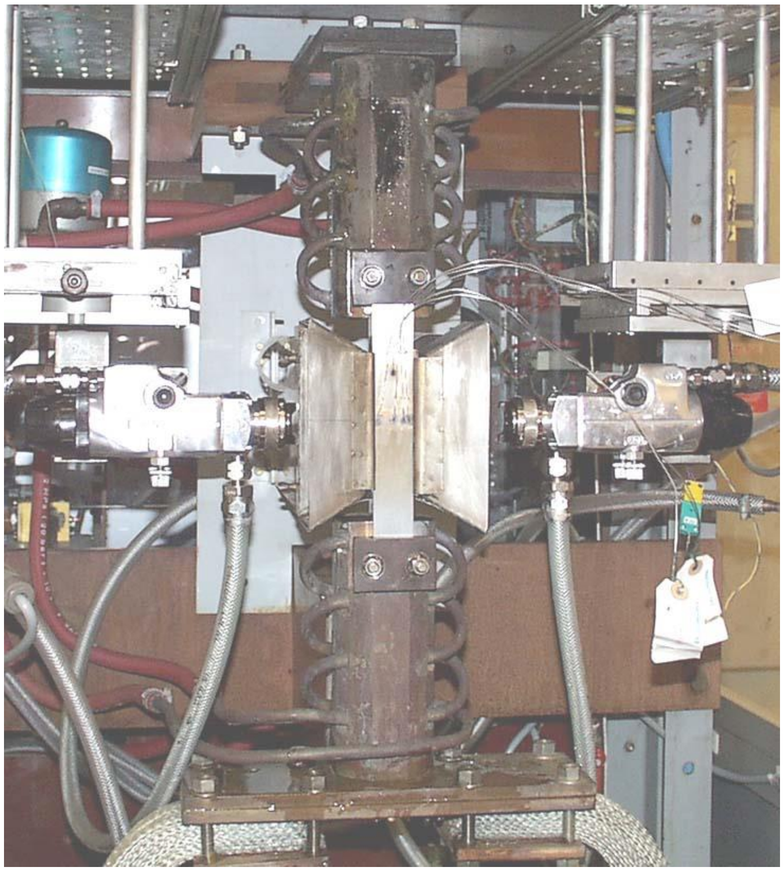
Weakest link hypothesis

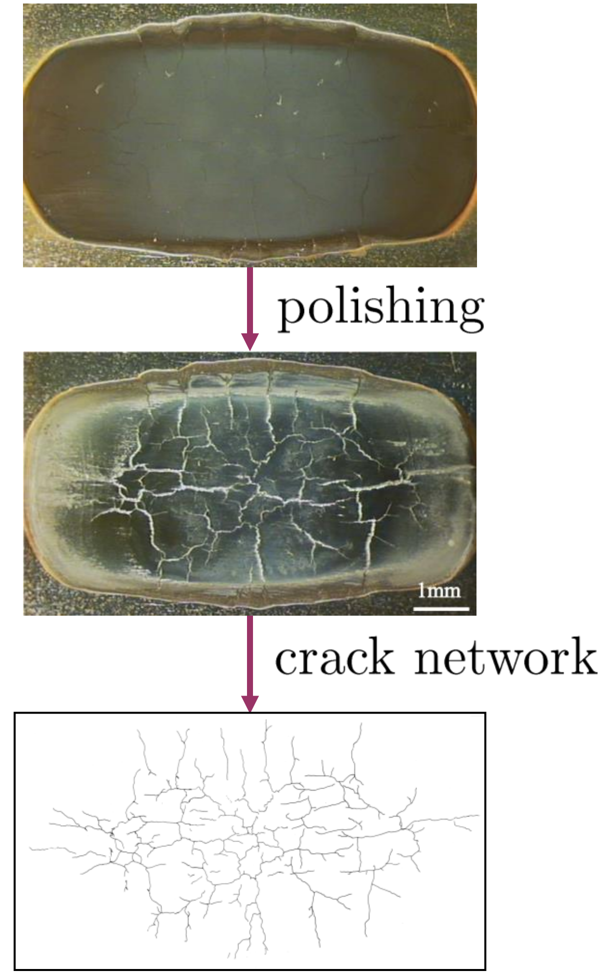
Self-heating experiments / endurance limits

Crack networks: thermal striping

Summary and perspectives

Crack Networks in Stainless Steel





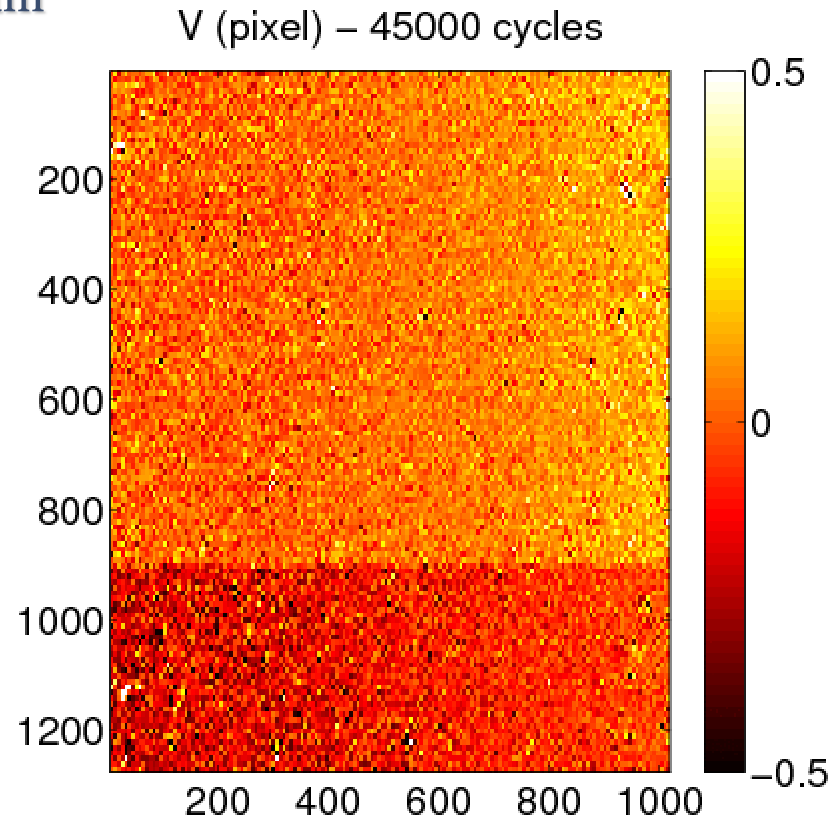
after 300 000 cycles
with $\Delta T = 150°C$

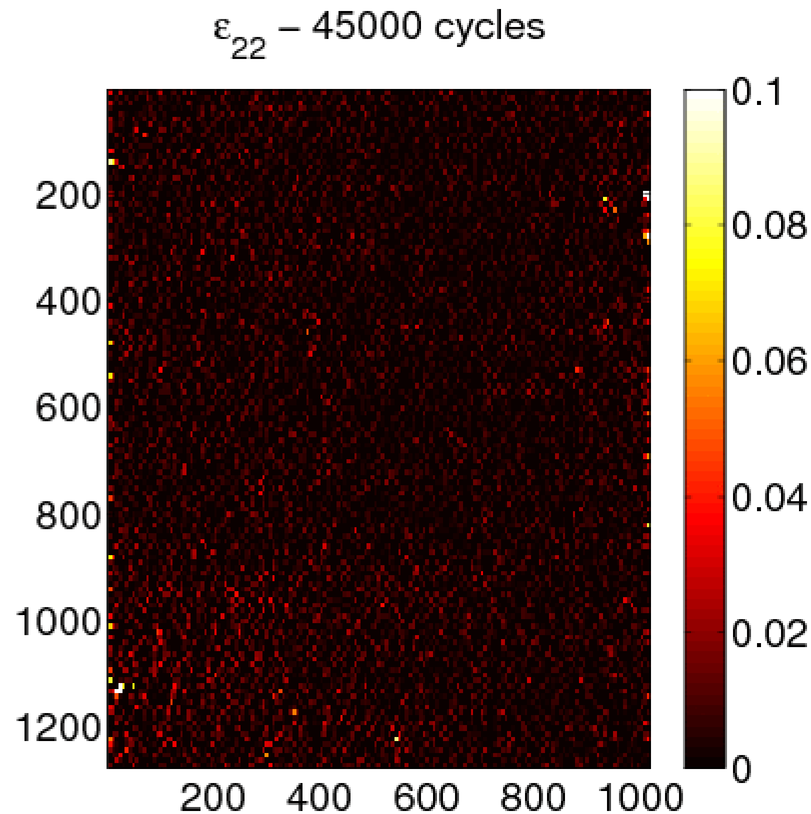
[Maillot, 2003, *PhD dissertation* UST Lille]

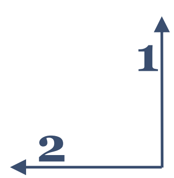
Mechanical Test

- Displacement field U_2
- Strain field ε_{22}

1px ≈ 2μm



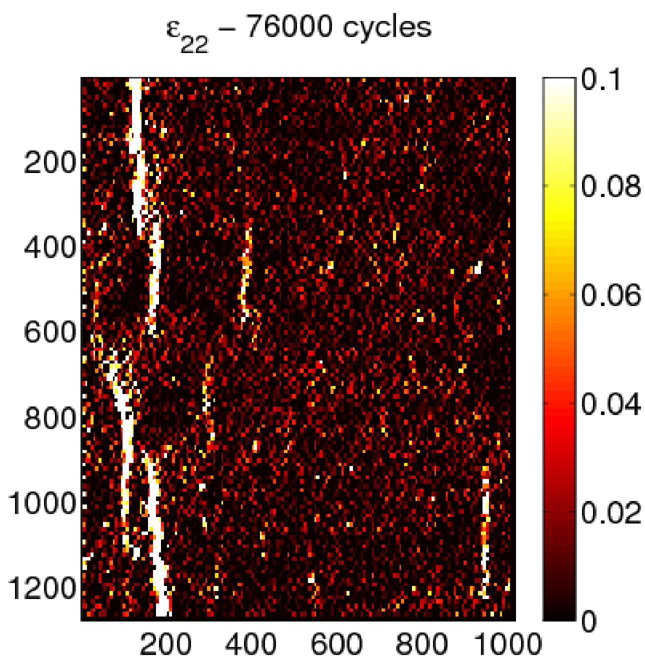


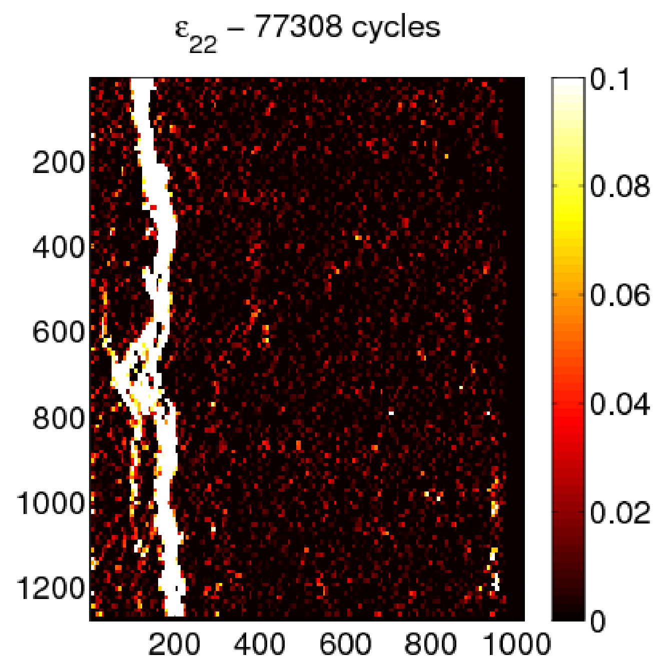


[Malésys, 2007, *PhD dissertation* ENS Cachan]
[Rupil *et al.*, 2009, JP SF2M]

Observations

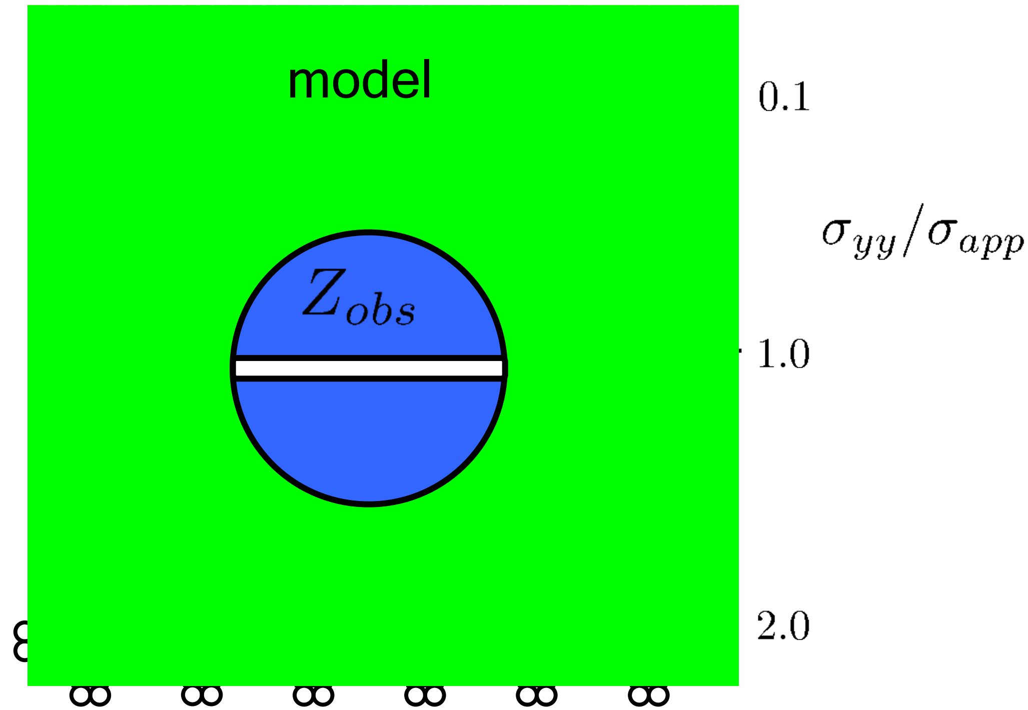
- Multiple crack initiation
- Crack arrest
- Crack coalescence





Model Hypotheses

- Shielding zone Z_{obs}: relaxed stress field around crack

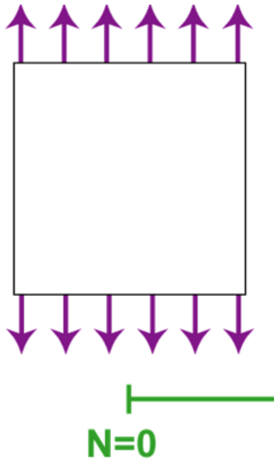


- Continuous initiation of cracks:
  - each site may initiate a crack after an incubation time (random variable)

[Malésys *et al.*, 2006, *CRAS* 334 pp. 419-424]

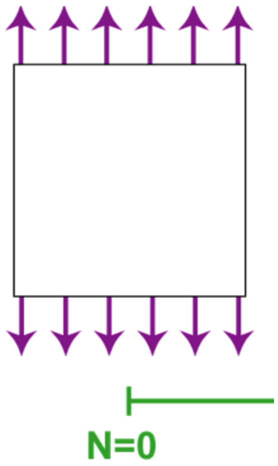
Model Hypotheses

Continuous initiation

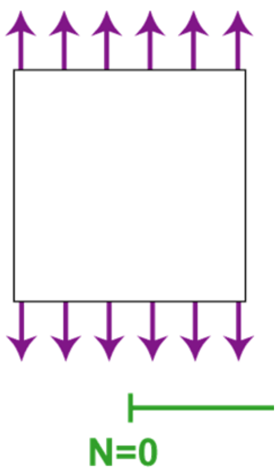


Model Hypotheses

Continuous initiation



Continuous initiation + Shielding process



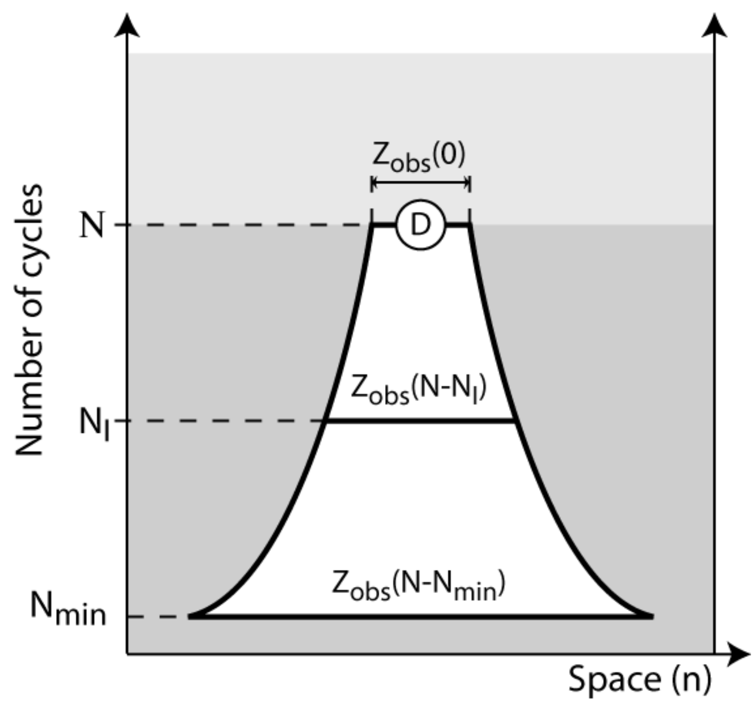
shielded site

propagating crack and its obscuration zone

stopped crack and its obscuration zone

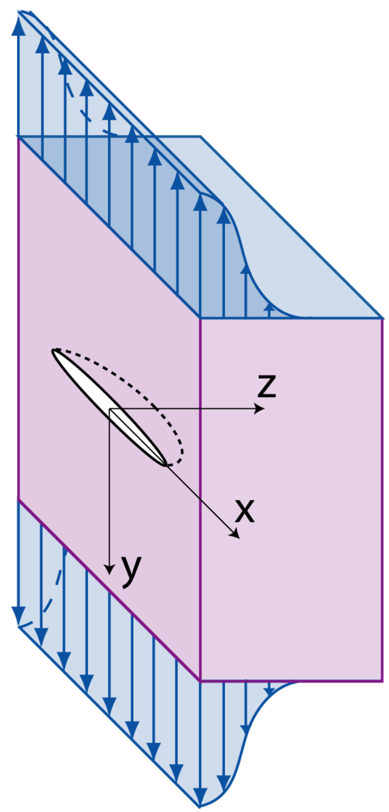
Model Hypotheses

- Horizon for given site D:
  - space-time zone where D at least obscured by another crack (white zone)
- Necessary condition to allow site D to initiate a crack: no crack in horizon of D



[Malésys *et al.*, 2006, *CRAS* 334 pp. 419-424]
[Malésys *et al.*, 2009, *Int. J. Fat.* 31 pp. 565-574]

Model Outputs



a = 1/2 crack size
c = crack depth

- a and c calculated with propagation law
- Size of shielding zone $$Z_{obs} = \pi ac$$
- Obscuration probability

$$P_{obs} = 1 - \exp\left(-\int_0^N Z_{obs}(N - N_i) \frac{d\lambda}{dN_i} dN_i \right)$$

- Initiated crack density

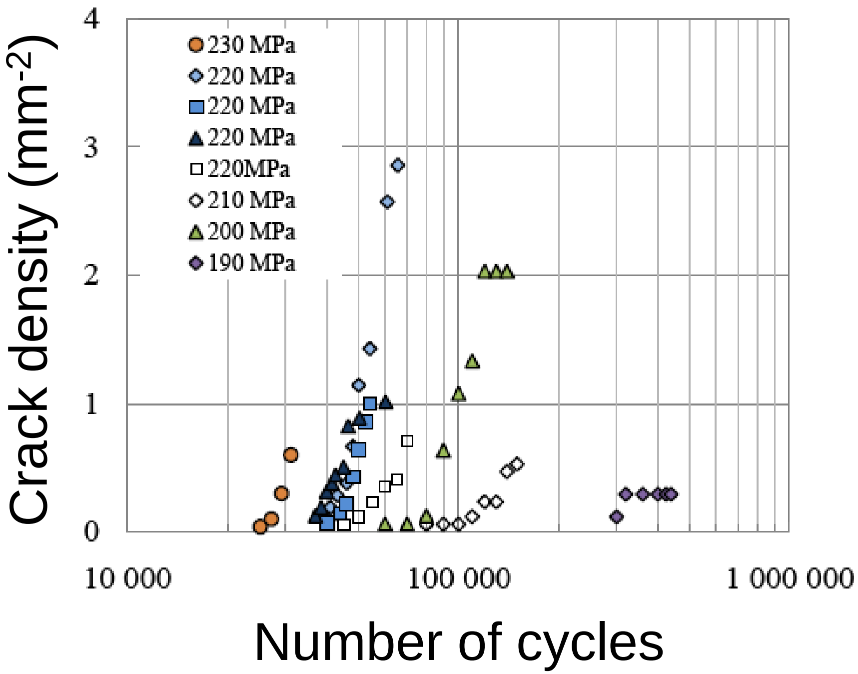
$$\frac{d\lambda_i}{dN} = (1 - P_{obs}) \frac{d\lambda}{dN}$$

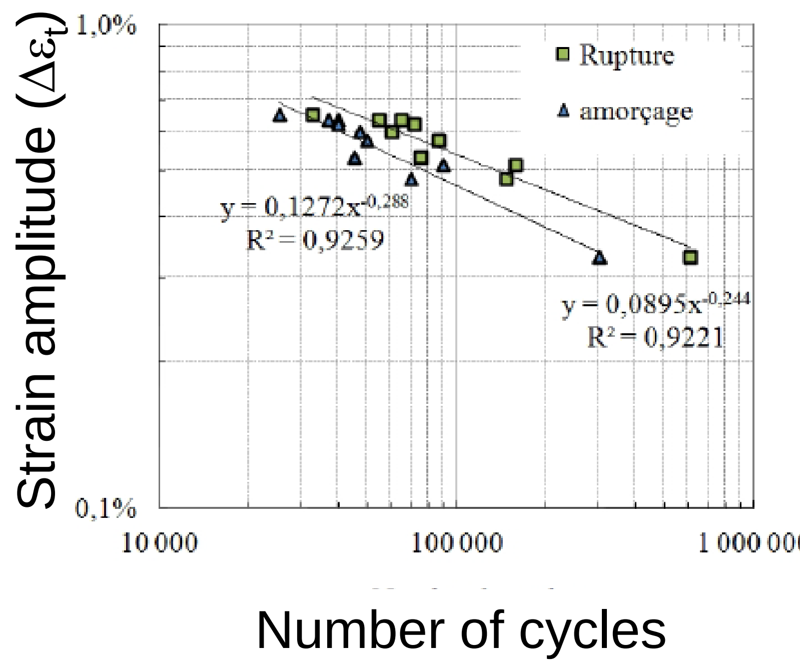
- Propagating crack density

$$\lambda_p = (1 - P_{obs}) \lambda_i$$

[Malésys *et al.*, 2009, *Int. J. Fat.* 31 pp. 565-574]

Uniaxial Tests

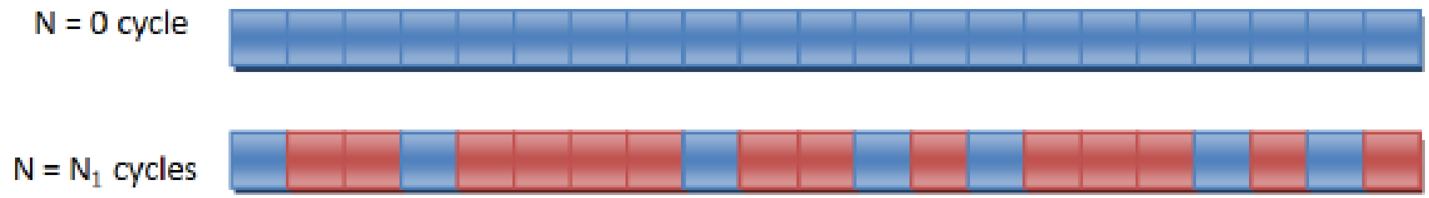




[Rupil *et al.*, 2011, *J. Strain Analysis* 46(6) pp. 492-509]
[Rupil, 2012, *PhD dissertation* ENS Cachan]

Intensity of PPP

$$P(N_i < N) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\ln(K) - (1/2)\ln(N_i) - \mu_\Delta}{2\sqrt{2}\sigma_\Delta}\right)\right]$$



Microcrack coalescence in neighboring grains

$$P(N_i < N_1) = p$$

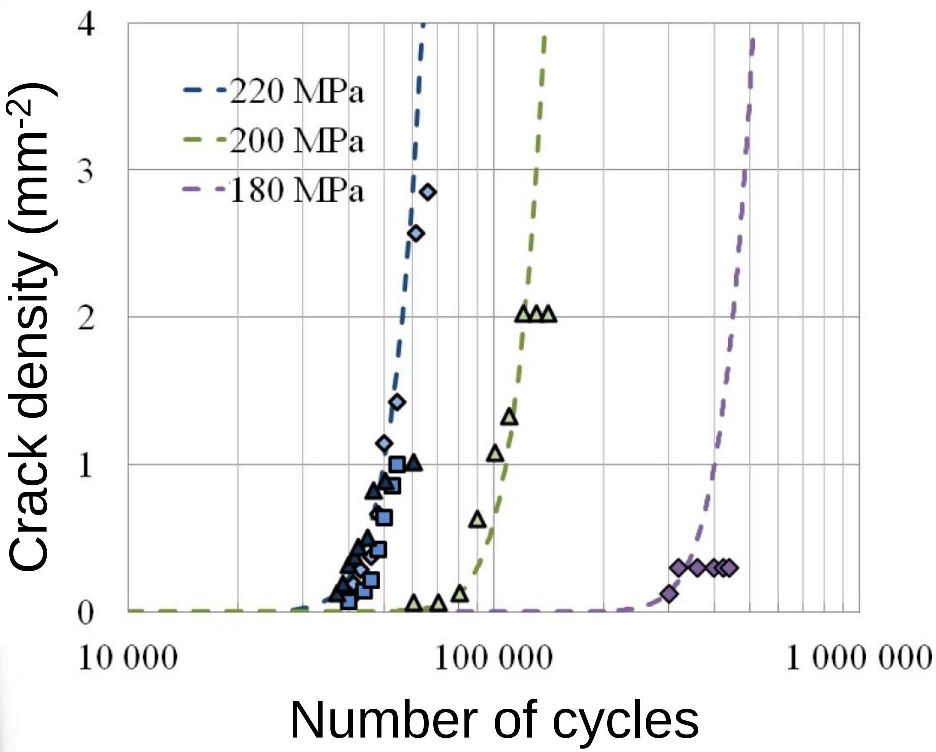
$$q = 1 - p$$

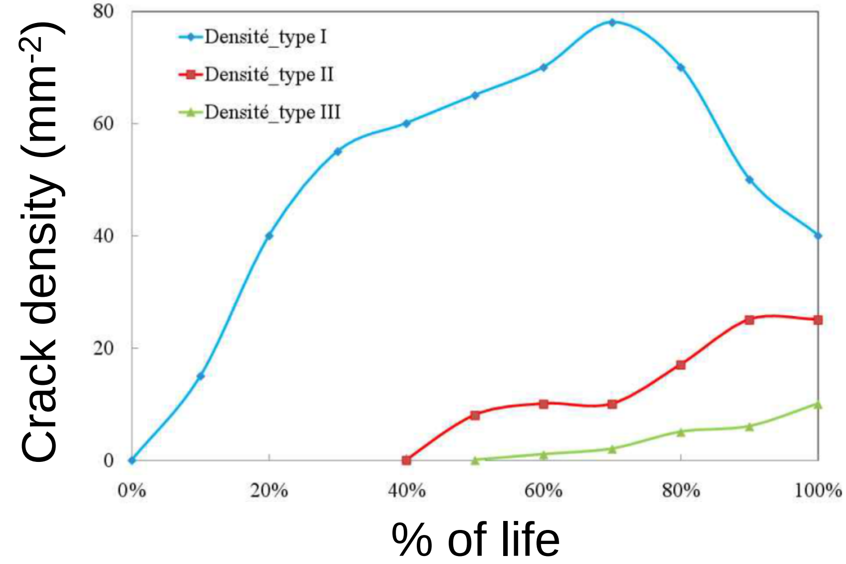
$$\lambda = \frac{(1-p) \times p^m}{{D_g}^2}$$

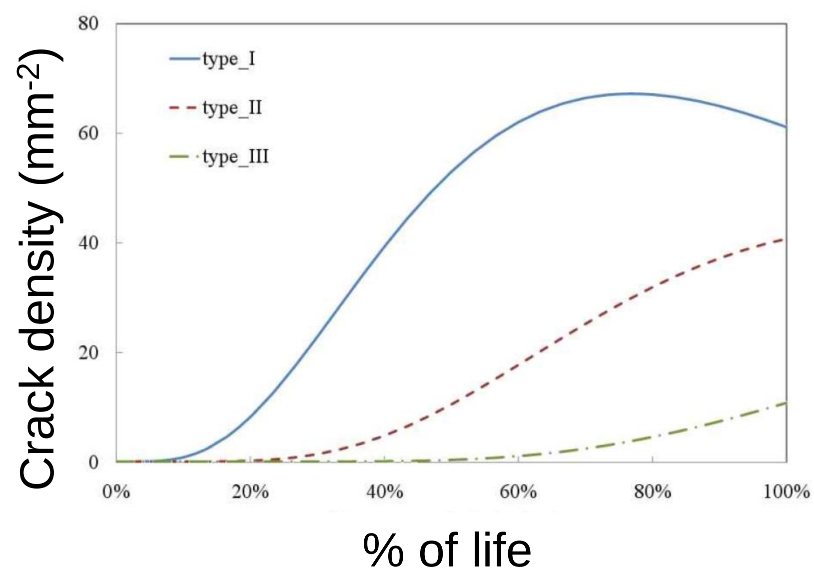
$$\lambda(\Delta\sigma, N) = \lambda_0 \left(\frac{\langle \Delta\sigma - \Delta\sigma_u(N) \rangle}{\sigma_0}\right)^m$$

[Osterstock, 2008, *PhD dissertation*, Ecole Centrale de Lille]
[Rupil *et al.*, 2011, *Int. J. Multiscale Comput. Eng*. 9(4) pp. 445-458]

Experimental Validation





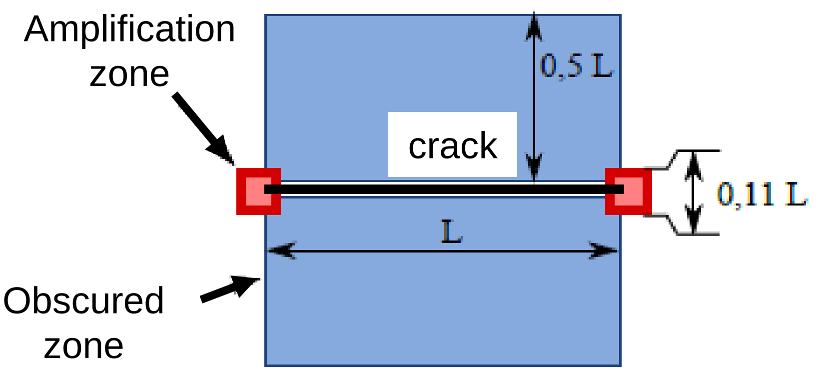


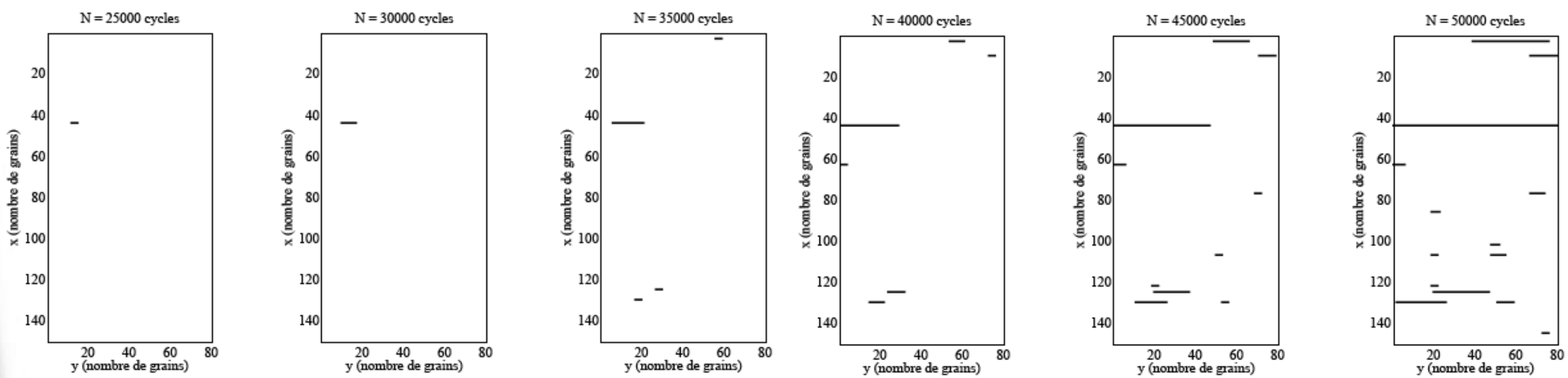
[Magnin *et al.*, 1985, *Scripta Met.* 19(12) pp. 1487–1490]

Accounting for Coalescence

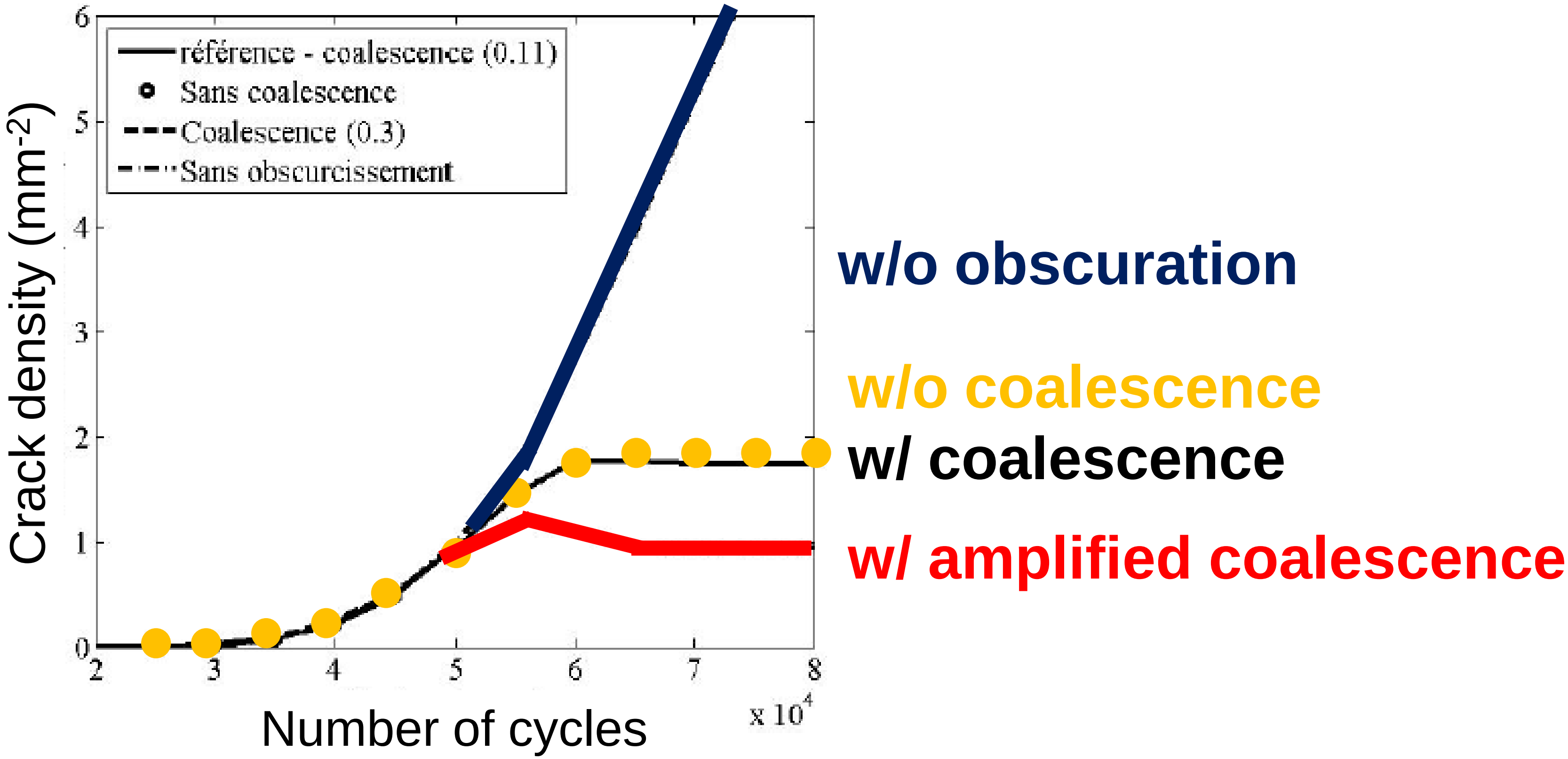
Discrete stochastic model

- **Initiation** (crack > 3 grains)
- Propagation
- **Obscuration**
- **Coalescence**





Effect of Coalescence



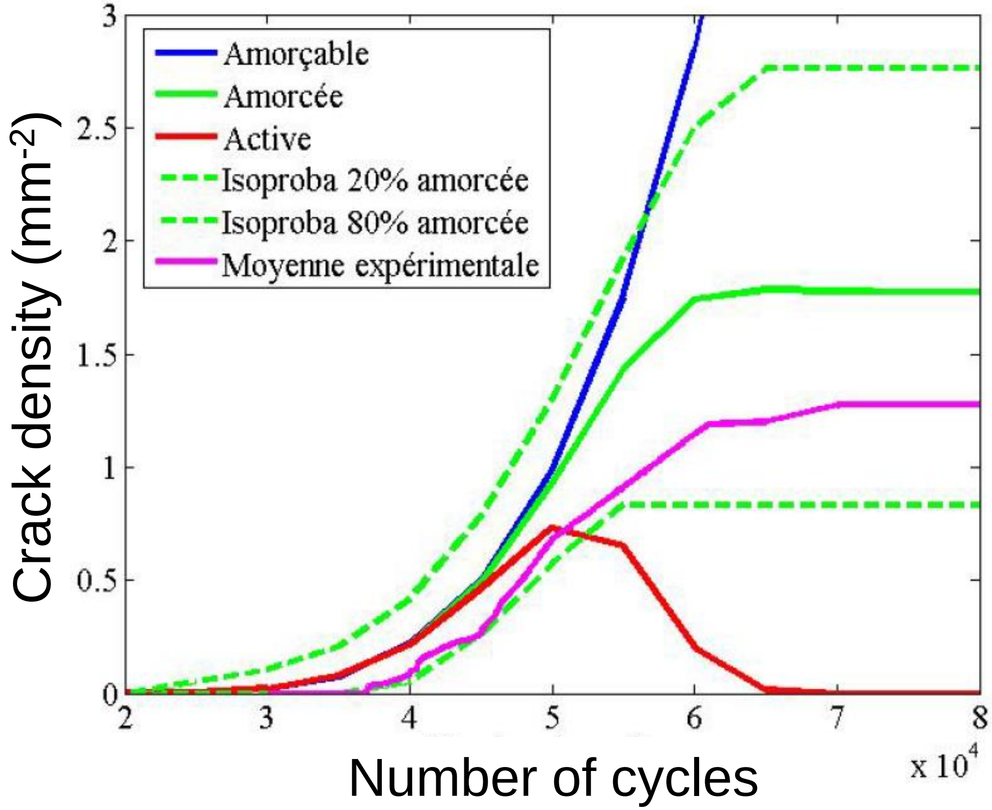
Obscuration

Change of initiated crack density

Coalescence

Comparison with Experiments

Initiated cracks (w/o shileding)



20%

Initiated cracks

Experiments

80%

Active cracks

[Rupil, 2012, *PhD dissertation* ENS Cachan]

Mécamat

Outline

Damage mechanisms / examples

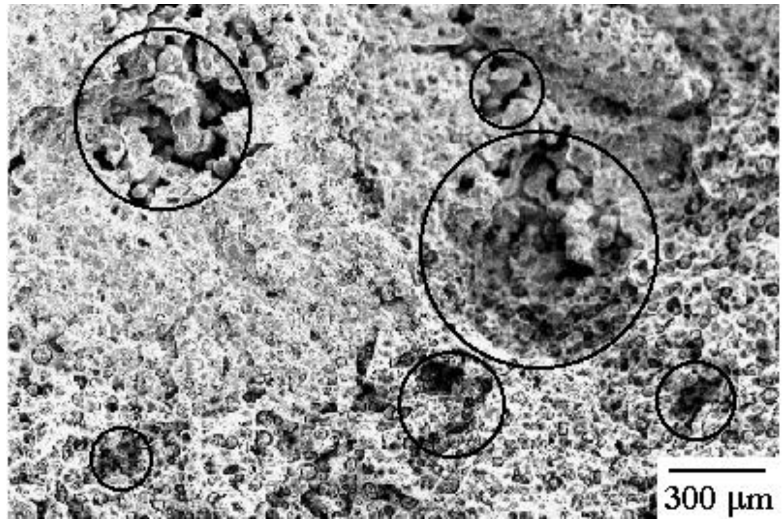
Weakest link hypothesis

Self-heating experiments / endurance limits

Crack networks: thermal striping

Summary and perspectives

Initial defects and HCF



Mechanical description of a defect

Multiaxial (non-proportional) loading

Defect distributions

[Murakami *et al.*, 1980, *Int. J. Fat.* 2 pp. 23-30]
[Murakami *et al.*, 1999, *Extremes* 2 pp. 123-147]
[Flacelière and Morel, 2004, *FFEMS* 27 pp. 1123-1135]
[Doudard *et al.*, 2007, *FFEMS* 30 pp. 107-114]
[Nasr et al., 2009, *FFEMS* 32 pp. 292-309]

Inspection Procedures

Contrôle d'une installation industrielle

Détection d'une fissure :

Incertitude sur sa géométrie

Longueur	Orientation

Estimation de durée de vie

Influence des incertitudes ?	Probabilité de défaillance ?

[Boutet, 2015, *PhD dissertation* Univ. Paris-Saclay]

Self-heating / HCF links

- Modeling:
  - mechanisms*
  - multiaxial loading conditions**
  - other classes of materials
- Source identification*** (IR thermography)
- Structures

***[Boulanger *et al.*, 2004, *Int. J. Fat.* 26 pp. 221-229]
*[Charkaluk and Constantinescu, 2006, *CRAS* 334 pp. 373-379]
**[Doudard *et al.*, 2007, *Int. J. Fat.* 29 pp. 748-757]
*[Mareau *et al.*, 2007, *Proc. CFM 2007*]
**[Poncelet *et al.*, 2007, *CRAS* 335 pp. 81-86]
***[Maquin *et al.*, 2009, *Mech. Mat.* 41 pp. 928-942]

Crack Networks and HCF

- Initiation conditions
- Propagation laws
- Coalescence
- Quantitative predictions wrt. experiments

Remerciements

- I. Chantier-de Lima, C. Doudard, N. Malésys,
  M. Poncelet , J. Rupil, D. Seyedi, H. Yaacoub Agha

- R. Billardon, D. Marquis, S. Roux (LMT)
- S. Calloch, C. Doudard (LBMS)

- A.-S. Béranger, V. Denier, P. Lory, J.-F. Vittori (Renault)
- P. Cugy, A. Galtier, B. Weber (ArcelorMittal)
- J.-M. Stéphan, S. Taheri (EDF)
- L. Vincent (CEA)