MODEL Reduction strategy applied to a problem of crack propagation in anisotropic materials

W. Tezeghdanti[a], S. Pommier[b], Y. Guilhem[c]

a. *LMT-Cachan/ENS Cachan/CNRS/Université Paris-Saclay*, tezeghdanti@lmt.ens-cachan.fr
b. *LMT-Cachan/ENS Cachan/CNRS/Université Paris-Saclay*, sylvie.pommier@universite-paris-saclay.fr
c. *LMT-Cachan/ENS Cachan/CNRS/Université Paris-Saclay*, yoann.guilhem@ens-cachan.fr

Abstract

In order to predict the life of high pressure turbine blades, a damage tolerance approach must be set. This approach requires a study of crack propagation. Furthermore, these components can be made as single crystals which creates a challenge in studying crack propagation in anisotropic materials. Instead of running expensive elastic-plastic computations, an elastic analysis is applied over the whole structure while a confined elastic-plastic one, fed by the previous, is processed on the K-dominance zone around the crack tip. Kinematic fields from these analyses are projected on a reduced basis in order to obtain a non-local condensed model that serves as an input for an incremental model. This condensed model gives a variation of blunting intensity factors of different modes as a function of the loading given through stress intensity factors.

Keywords: Incremental model; Model reduction; POD; Crack propagation; Anisotropy …

1. Introduction

Based on the work done by Laird and Pelloux [1], Pommier and Risbet [2] proposed an incremental model based on the plastic blunting of the sharp crack tip. This concept introduces the blunting intensity factor ρ as an additional variable that describes the plastic flow in the plastic zone. To build the incremental model for fatigue crack growth, the kinematic displacement field in the crack tip region is decomposed into an elastic field and a plastic one, where the plastic part is the origin of the crack growth and auto-stresses during unloading [3]. Each part of this decomposition is assumed to be a combination of a reference field that depends on spatial coordinates (r,θ) and an intensity factor depending on the external forces and the crack geometry (K_i, ρ_i). The model was previously set for isotropic materials and we aim to make it closer to real behaviour of single crystals by applying it on anisotropic materials.
The method used here is the Proper Orthogonal Decomposition (POD) which is based on the orthogonal projection of the kinematic displacement field into a well-chosen basis to get proper orthogonal modes [4]. This decomposition is powered by the hypothesis that elasticity and plasticity are kinematically independent:

$$\mathbf{u}(\boldsymbol{x},t) = \sum_i g_i(t).f_i(\boldsymbol{x}) \tag{1}$$

It was shown that the first two modes of the POD are sufficient to describe the physics around the crack tip where the first mode corresponds to the elastic kinematic field and the second mode refers to the plastic field [5]:

$$\partial\mathbf{u}(\boldsymbol{x},t) = \partial\widetilde{K}(t).\boldsymbol{\varphi}^{el}(\boldsymbol{x}) + \partial\rho(t).\boldsymbol{\varphi}^{pl}(\boldsymbol{x}) \quad (2)$$

where $\widetilde{K}$ is the stress intensity factor (including the nominal stress intensity factor and the one containing shielding effect) and ρ is the blunting intensity factor. In our case, we limited our study to a 2D model with plane deformation and a mode I/II crack. Hence, we proceeded to a second decomposition of the kinematic field with respect to the crack plane into a symmetric part that describes mode I and a skew-symmetric part that describes mode II. We obtain finally:

$$\partial\mathbf{u}(\boldsymbol{x},t) = \partial\widetilde{K}_I(t).\boldsymbol{\varphi}^{el}_{I\,ref}(\boldsymbol{x}) + \partial\widetilde{K}_{II}(t).\boldsymbol{\varphi}^{el}_{II\,ref}(\boldsymbol{x}) + \partial\rho_I(t).\boldsymbol{\varphi}^{pl}_{I\,ref}(\boldsymbol{x}) + \partial\rho_{II}(t).\boldsymbol{\varphi}^{pl}_{II\,ref}(\boldsymbol{x}) \quad (3)$$

2. Kinematic basis

2.1 Reference fields

Based on the hypothesis of self-similarity of the geometry and the displacement field around the crack tip, reference fields are decomposed using the POD into a radial dependent function giving the scale and an angular dependent function giving the shape.

$$\varphi^{k}_{i\,ref}(r,\theta) = f^{k}_{i}(r).g^{k}_{i}(\theta) \qquad i \in [I, II], k \in [el, pl] \quad (4)$$

Angular functions are given in figure [1]. Elastic functions are compared with the solution proposed by Sih, Paris and Irwin in the case of anisotropic material [6]. We chose the cubic anisotropy and we assumed that the direction (001) is normal to the model and α is the angle between the (100) direction and the crack plane. Linear plots show the components of reference fields as function of the angular coordinate θ. Circular plots show the deformation of a unit circle submitted to the corresponding displacement. Elastic fields are sized to obtain a CTOD and CTSD equal to 1 in mode I and mode II respectively.

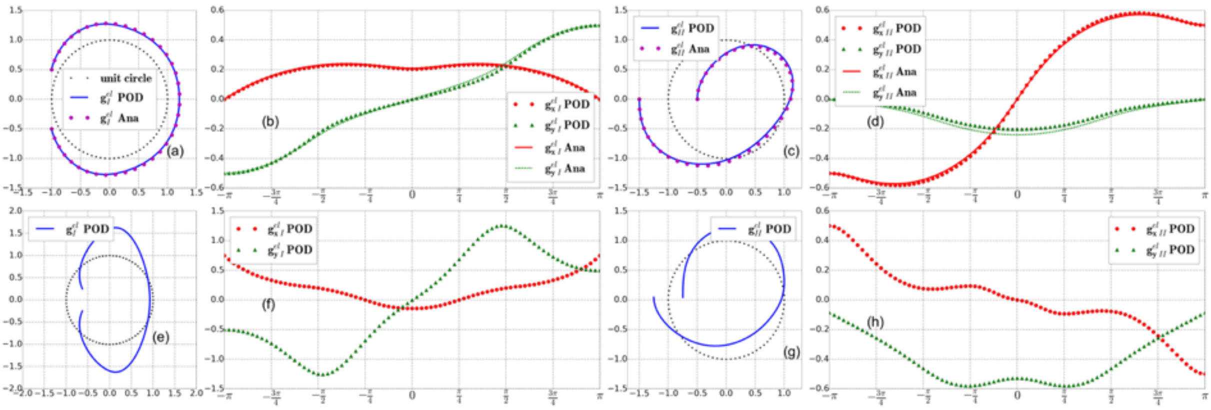

Figure 1. *Angular functions of elastic reference fields of (a-b) mode I and (c-d) mode II and plastic reference fields of (e-f) mode I and (g-h) mode II for a cubic elastic material oriented at α=45° with respect to the crack.*

It seems that the elastic basis is consistent with results found in literature. Results from cyclic elastic-plastic analysis are projected on these bases. This projection will give a nonlocal model that relates loading, described by stress intensity factors $\widetilde{K}_i$, to the crack tip opening and sliding, described by blunting intensity factors ρ_i.

2.2 Material model

Once this kinematic approximation is obtained and validated for different aspects of anisotropy, a model yielding the response of the material subjected to this kinematics field can be defined. This model characterises the yield surface criterion, the evolution law of internal variables and the plastic flow direction as function of the anisotropy of the material. The proposed approach makes it possible to observe the response of the material within the crack tip region and hence to identify the corresponding elements in the model.

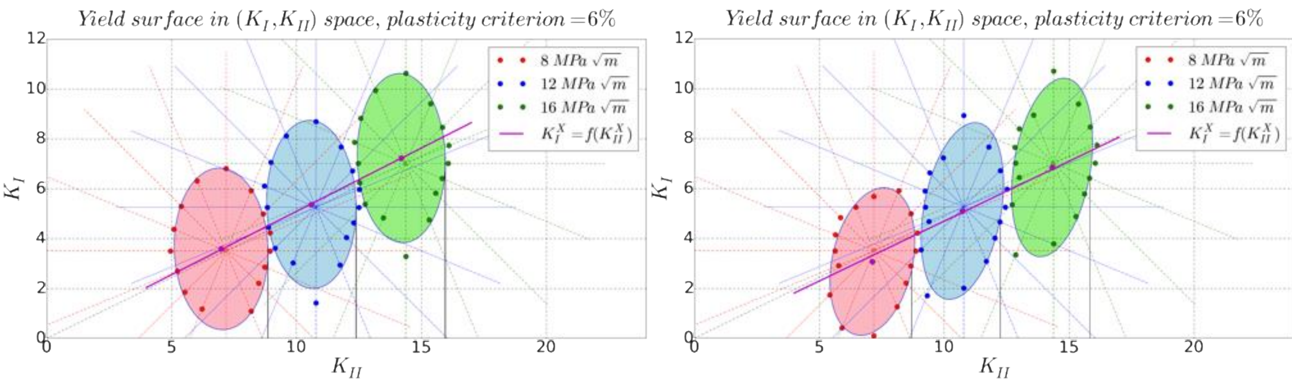

Figure 2. *Yield surfaces for different loading levels and for (a) isotropic materials and for (b) a cubic elastic material oriented at α=30° with respect to the crack.*

3. Conclusion

The kinematic basis found for both isotropic and cubic materials seem to describe well the associated material model. This basis will serve to project more sophisticated fields and extract a non-local condensed model that relates blunting intensity factors ρ_i directly to the loading described by stress intensity factors K_i and some internal variables. This method is able to identify the shape, the size and the evolution of plastic yield surfaces as function of the material crystallographic orientation and the applied loading. In a further work, a material model can be created according to this kinematic basis.

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