

Calibration of nonlocal models for quasi-brittle materials failure prediction

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Outline

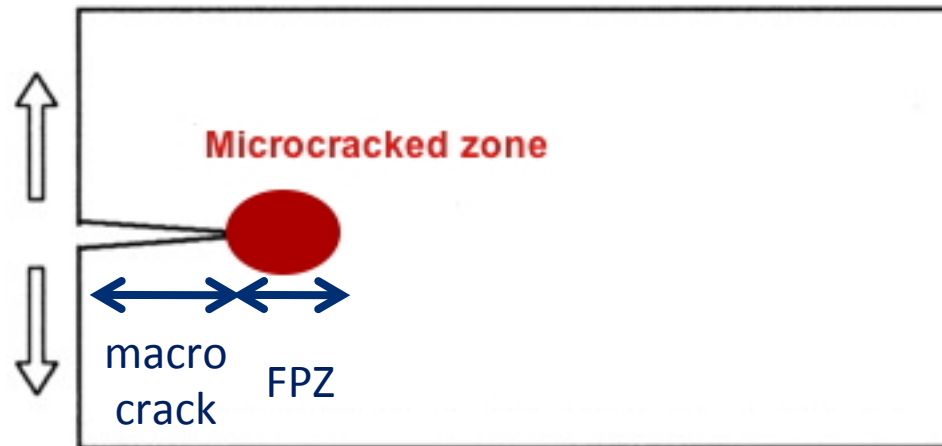
- **Introduction and context: non-local models for quasi-brittle failure**
- **Indirect calibration methods**
- **Direct calibration method ?**
- **Toward the calibration of an evolving characteristic length**
- **Conclusion and perspectives**

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- **Introduction and context: non-local models for quasi-brittle failure**
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Non-local models for quasi-brittle failure

- Propagation of macrocracks in quasi-brittle materials implies the presence of a Fracture Process Zone (FPZ)



- This FPZ leads to typical phenomena such as size effects, boundary effects, strain softening etc.
- Non-local interactions appear within the FPZ and the material points cannot be seen as independent
- It may change transport properties and the FPZ may be the location of complex multi-physics couplings (adsorption, crystallisation, etc.)

Non-local models for quasi-brittle failure

➤ Strain softening materials



• Softening \implies Strain localization \implies Mesh dependencies



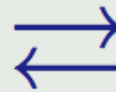
• Regularization techniques : (Non-local models)

Integral-type non-local models

$$\begin{cases} \bar{\epsilon}(\mathbf{x}) = \frac{1}{\Omega_r(\mathbf{x})} \int_{\Omega} \psi_0(\mathbf{x}, \xi) \tilde{\epsilon}(\xi) d\xi \\ \Omega_r(\mathbf{x}) = \int_{\Omega} \psi_0(\mathbf{x}, \xi) d\xi \\ \psi_0(\mathbf{x}, \xi) = \exp\left(-\left(\frac{2\|\mathbf{x} - \xi\|}{l_c}\right)^2\right) \end{cases}$$

(l_c internal length related to the Fracture Process Zone)

☞ Pijaudier-Cabot and Bažant *et al.* (1987)



☞ Peerlings *et al.* (2001)

Gradient damage formulations

$$\begin{cases} \bar{\epsilon}(\mathbf{x}) = \tilde{\epsilon}(\mathbf{x}) + \frac{l_c^2}{2} \nabla^2 \tilde{\epsilon}(\mathbf{x}) + \dots \quad (\text{explicit}) \\ \text{or} \\ \bar{\epsilon}(\mathbf{x}) - \frac{l_c^2}{2} \nabla^2 \bar{\epsilon}(\mathbf{x}) - \dots = \tilde{\epsilon}(\mathbf{x}) \quad (\text{implicit}) \end{cases}$$

(l_c internal length related to the Fracture Process Zone)

☞ Peerlings *et al.* (1996)

The characteristic length l_c needs to be calibrated

and a constitutive law needs to be chosen !

Non-local models for quasi-brittle concrete failure

➤ Simple concrete model

Isotropic damage: $\sigma = (1 - D)\mathbb{C} : \varepsilon$

Equivalent strain: $\varepsilon_{eq} = \sqrt{\sum_{i \in \llbracket 1,3 \rrbracket} \langle \varepsilon_k \rangle_+^2}$ (Mazars, 1986)

Nonlocal averaging: $\bar{\varepsilon}_{eq}(x) = \frac{1}{\Omega_r} \int_{\Omega} \psi(x, \xi) \varepsilon_{eq}(\xi) d\xi$

$\psi(x, \xi)$ with a characteristic length

Damage evolution: $D(h, x) = \left[1 - (1 - A_t) \frac{\varepsilon_{D_0}}{h(x)} - A_t e^{(-B_t(h(x) - \varepsilon_{D_0}))} \right]$
 (Mazars, 1986)

Kuhn-Tucker condition:

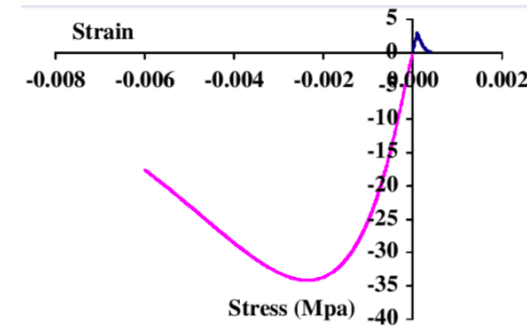
$$\Gamma(\varepsilon, h) = \bar{\varepsilon}_{eq}(\varepsilon) - h, \quad \Gamma(\varepsilon, h) \leq 0,$$

$$\dot{h} \geq 0, \quad \dot{h} \Gamma(\varepsilon, h) = 0$$

$$h = \max(\varepsilon_{D_0}, \max(\bar{\varepsilon}_{eq}))$$

Remark: all the parameters have to be calibrated !

Classical values of the model parameters are provided in Mazars (1986) and Pijaudier-Cabot et al. (1991)

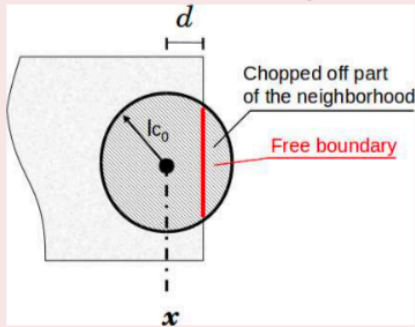


Non-local models for quasi-brittle failure

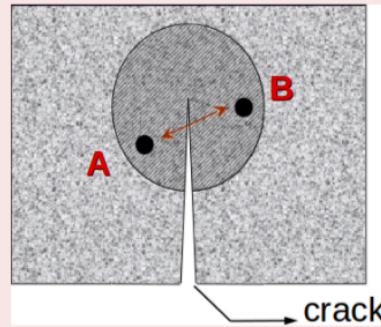
➤ The characteristic length must vary upon failure



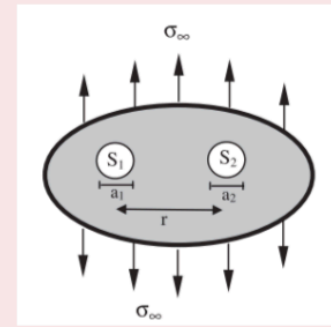
• Why? \implies Effects which are not taken into account :



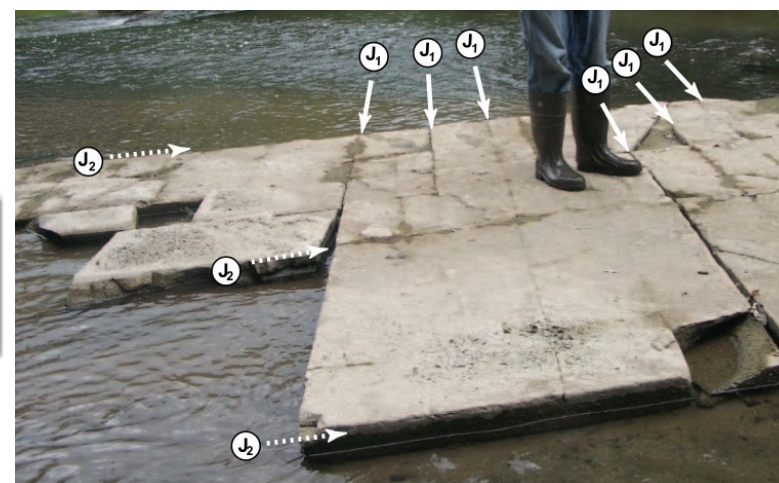
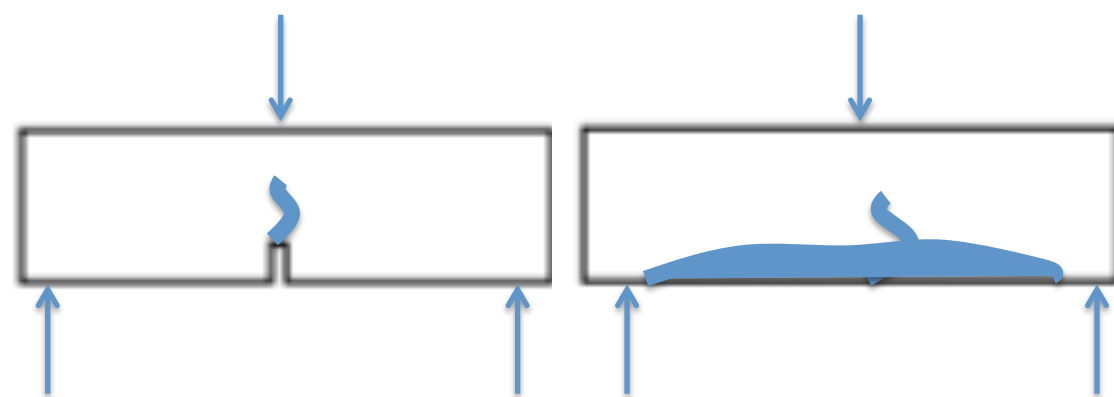
a) Boundary effects ($l_c \searrow$)



b) Shielding effect ($l_c \searrow$)



c) Voids growth effect ($l_c \nearrow$)

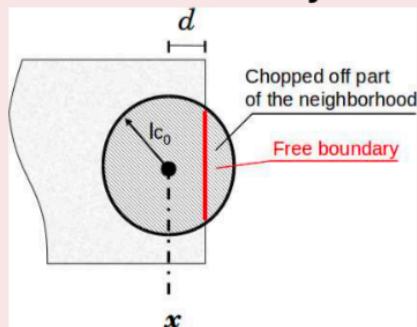


Non-local models for quasi-brittle failure

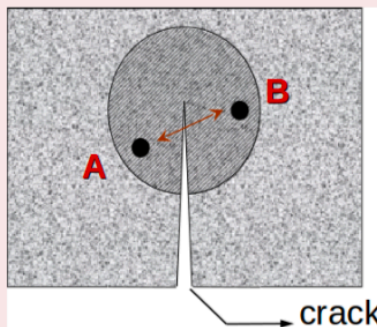
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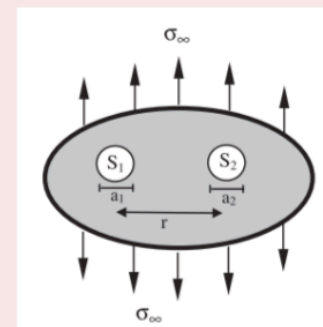
• Why? \implies Effects which are not taken into account :



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Empirical solutions:

Enhanced non-local formulations :

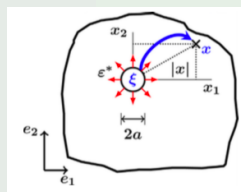
$$l_c = f(d; D) \begin{cases} D : \text{damage, } D \in [0, 1] \\ d : \text{boundary distance} \end{cases}$$

- Krayani et al. *et al.* (2009)
- Pijaudier-Cabot and Dufour *et al.* (2010)
- Grégoire et al. *et al.* (2012)

Less empirical solutions:

Other formulations :

- Stress-based non local damage model
Giry et al. *et al.* (2011)
- Interaction-based non local model
Rojas-Solano et al. (2013)
Pijaudier-Cabot and Grégoire (2014)



Non-local models for quasi-brittle failure

➤ The characteristic length must vary upon failure



• Why? \implies Effects which are not taken into account :

Global motivation for this talk:

Whatever is the non-local model chosen, a characteristic length has to be identified, even as a constant parameter.

a) Boundary effects ($l_c \downarrow$) b) Shielding effect ($l_c \downarrow$) c) Voids growth effect ($l_c \uparrow$)

Additionally, this characteristic length must vary

upon failure...

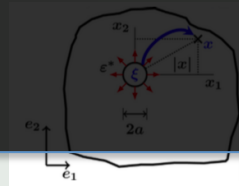
Enhanced non-local formulations :

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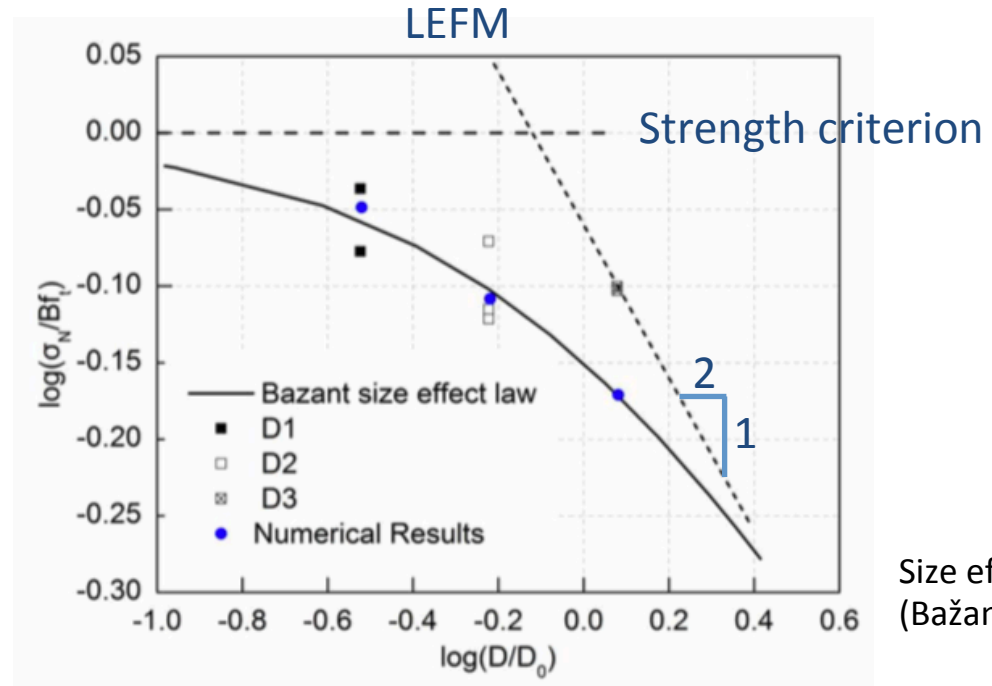
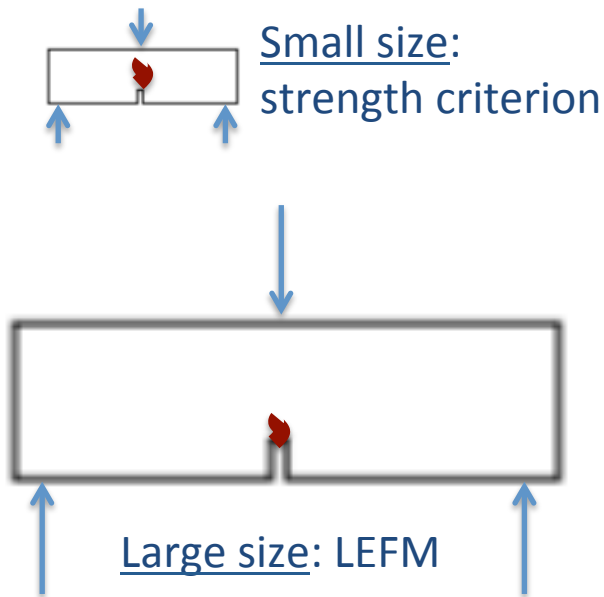


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- Introduction and context: non-local models for quasi-brittle failure
- **Indirect calibration methods**
 - Pick load size effect laws
 - Softening curves of different specimen sizes (1, 3 or 4)
 - Example of calibration failure or success
- **Direct calibration method ?**
- **Toward the calibration of an evolving characteristic length**
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• Indirect calibration methods

1) Pick load size effect laws

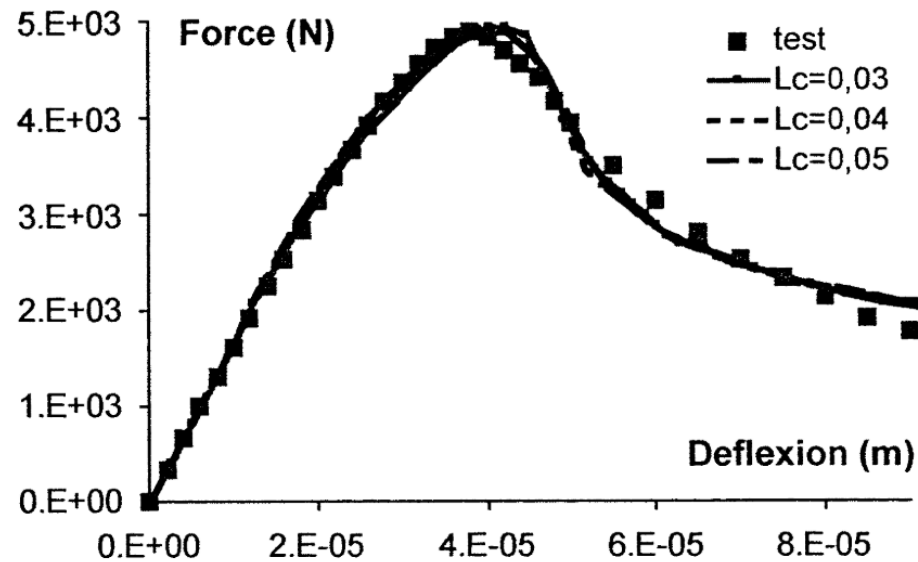


Commonly performed: only notched specimen, only 3 sizes

Pick load size effect laws should not be used alone to calibrated model parameters

- Indirect calibration methods

- 2) Softening curve (1 size)

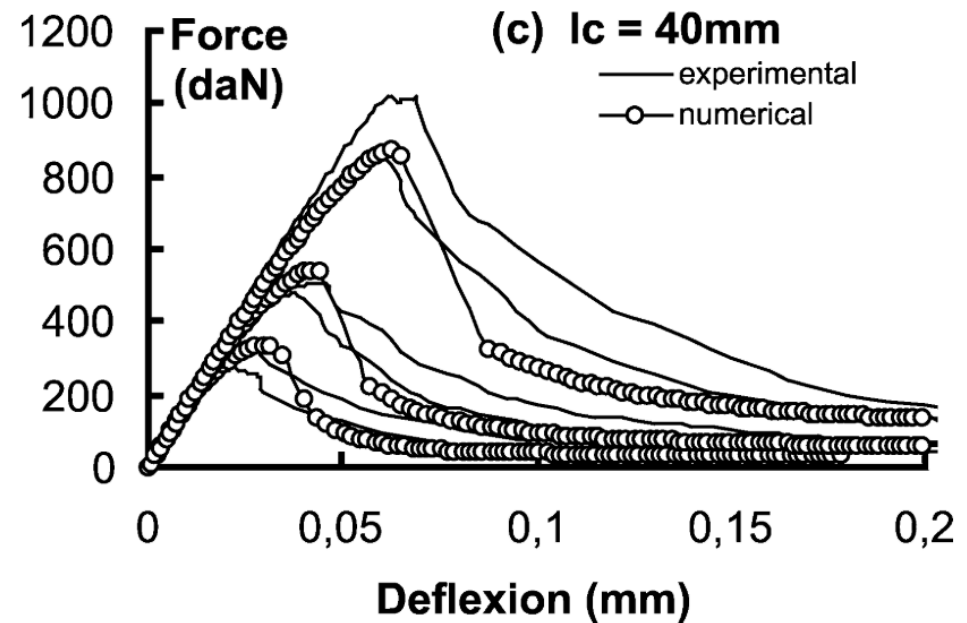
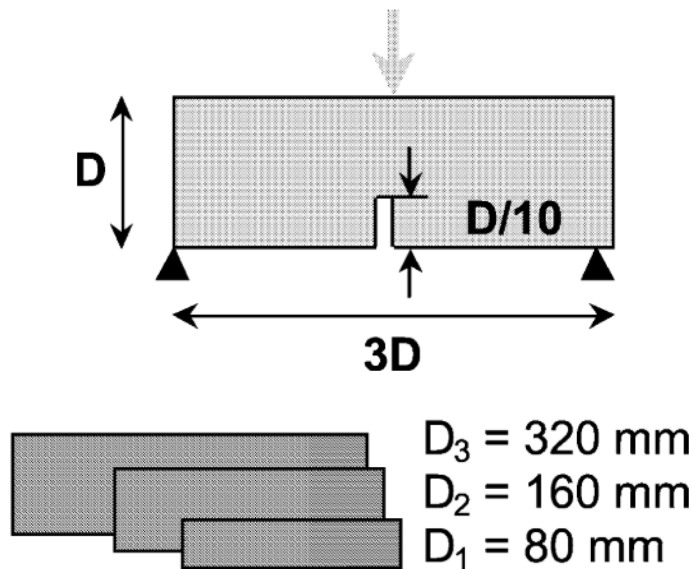


(Le Bellégo et al., 2003)

A model cannot be calibrated from inverse analysis of a single load deflexion curve.

• Indirect calibration methods

2) Softening curves (3 sizes) (most studies)



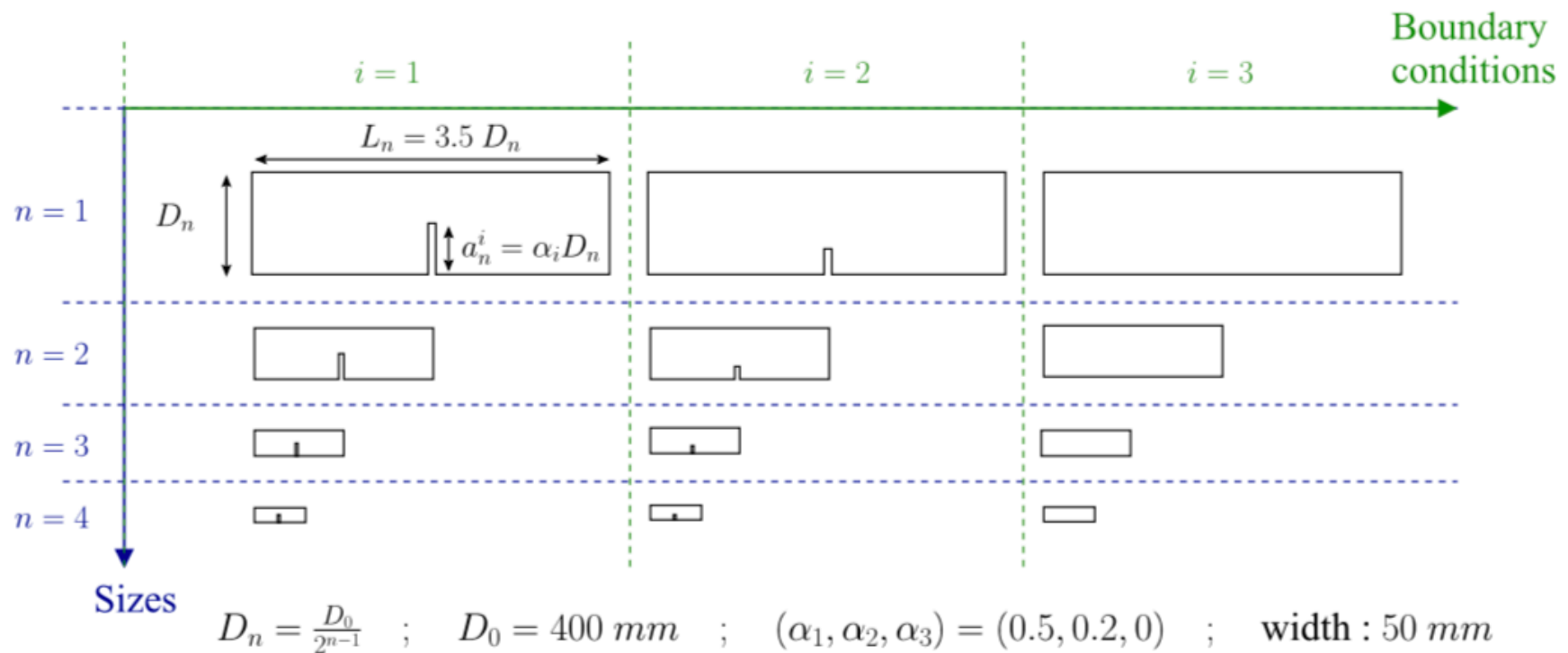
(Le Bellégo et al., 2003)

Most of the time, no problem for a good calibration on three sizes...

... but things get more difficult for four sizes.

• Indirect calibration methods

2) Softening curves (4 sizes)



34 three bending tests

+ 51 characterisation tests

(Compressive strength, splitting tensile strength, Young's modulus, Poisson ratio)

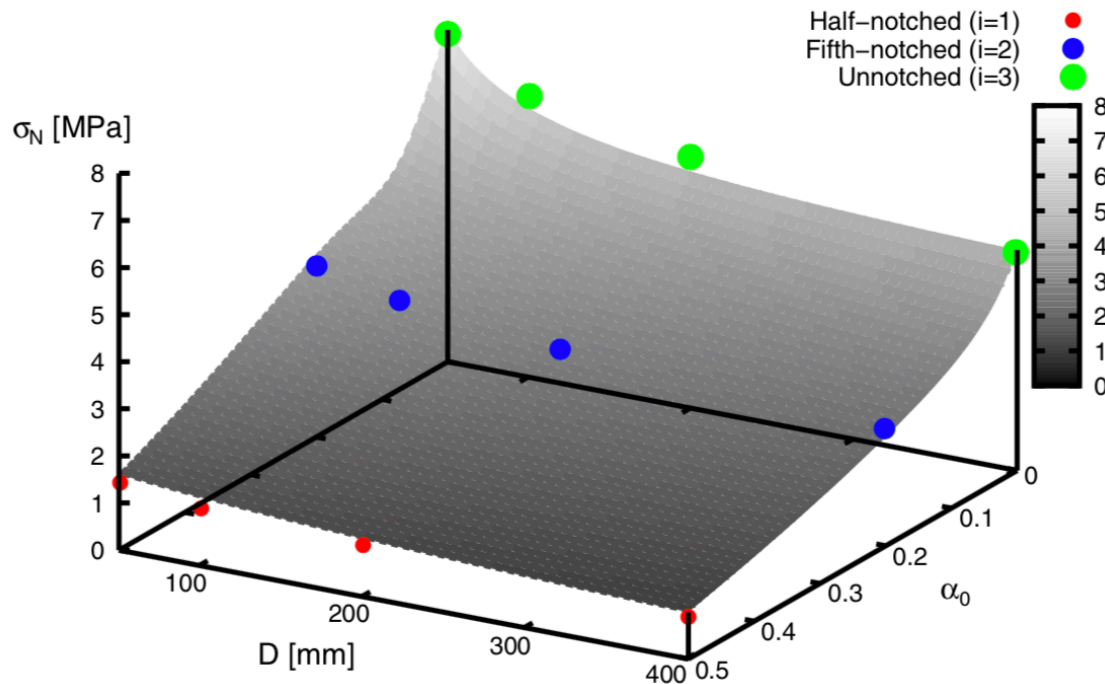
(Grégoire et al., 2013)

see also (Hoover et al., 2013)

• Indirect calibration methods

2) Softening curves (4 sizes)

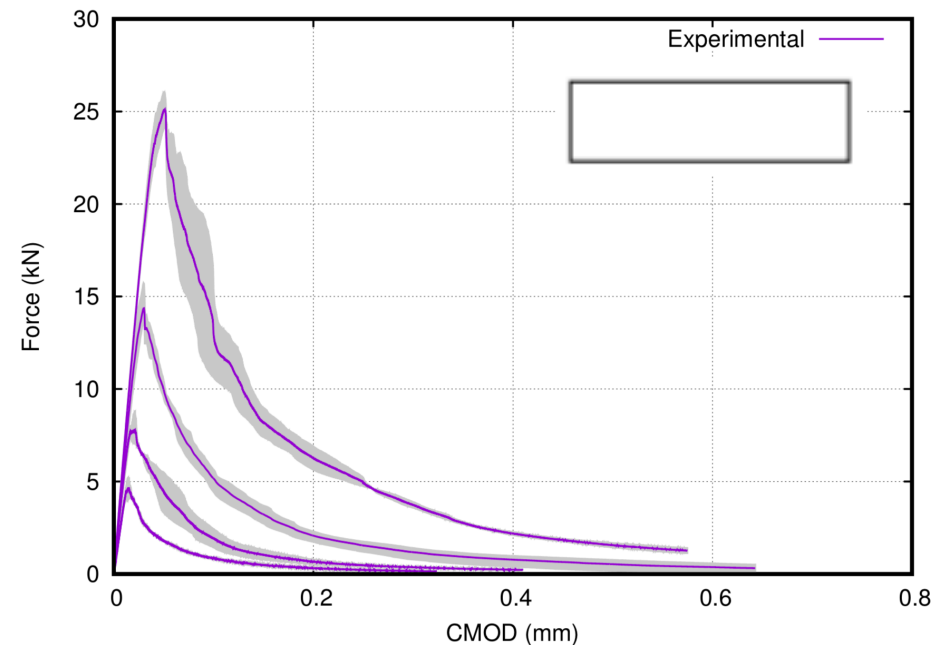
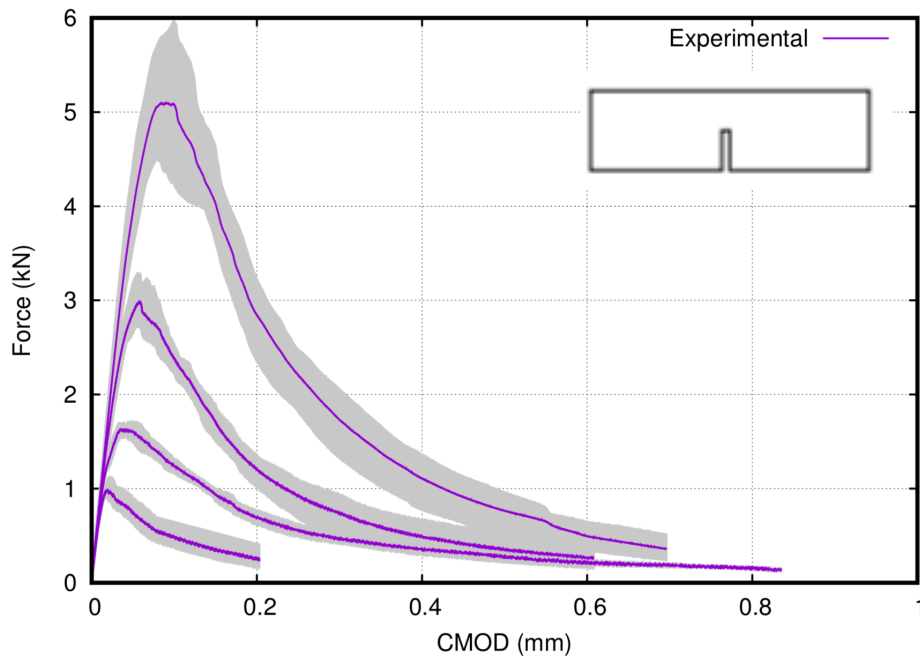
Remark: if you really want to use a size effect law, use a universal one with notched and unnotched specimens but still not the best for failure model calibration...



Size effect law from:
(Bažant and Yu, 2009)

(Grégoire et al., 2013)

- Indirect calibration methods
 - 2) Softening curves (4 sizes)

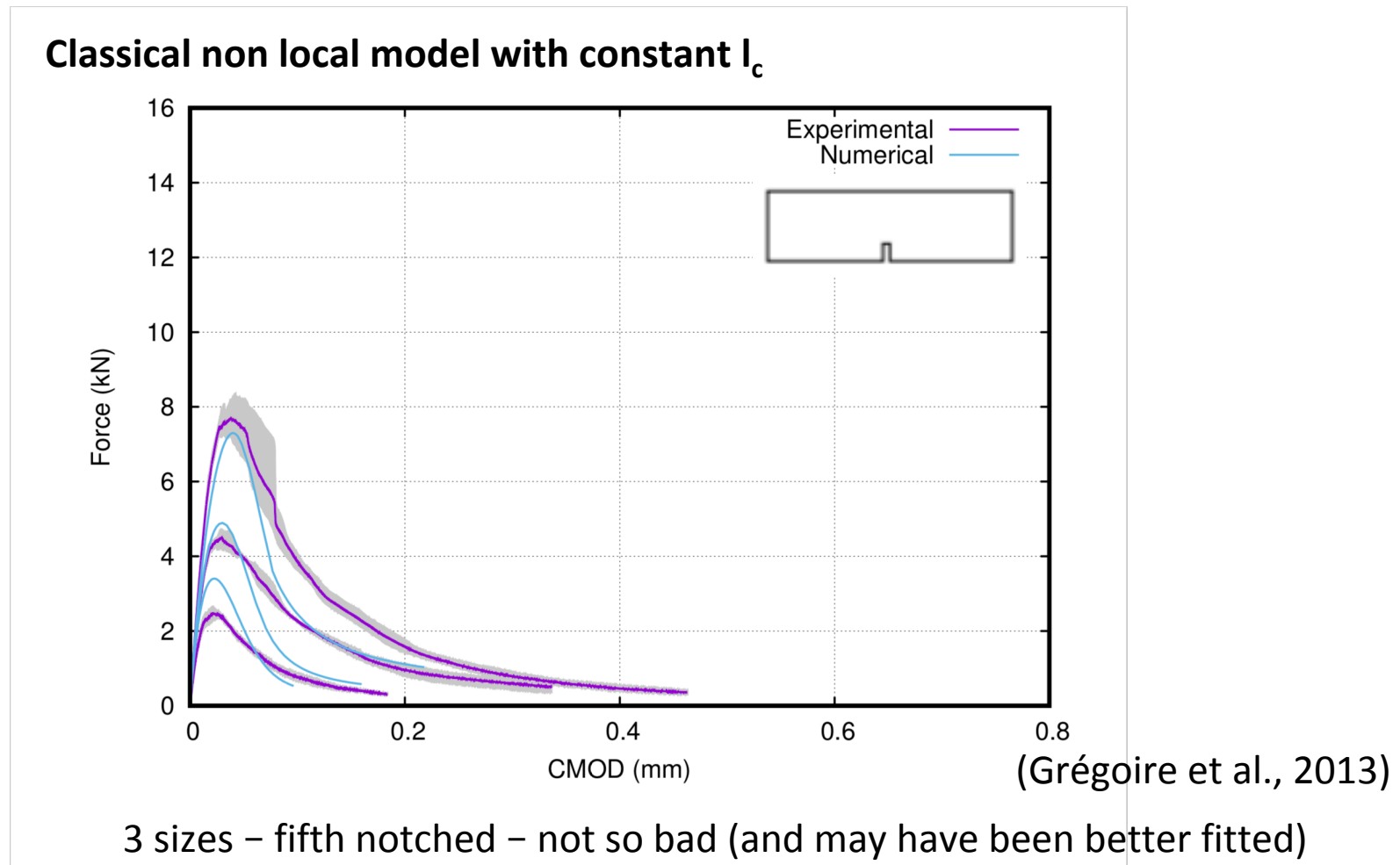


(Grégoire et al., 2013)

Perfect for model calibration but...

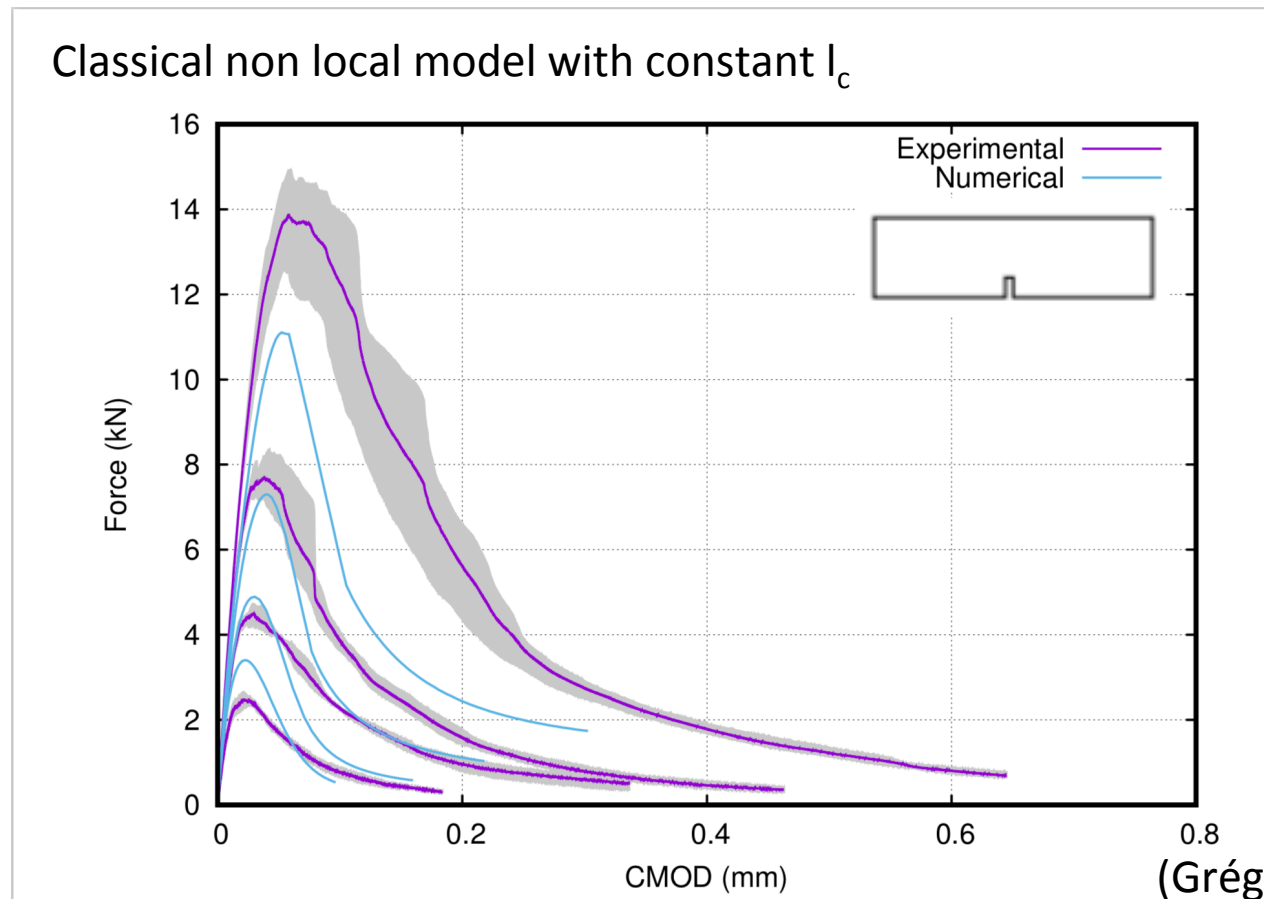
- Indirect calibration methods

- 3) Example of calibration failure – NL model with constant length



- Indirect calibration methods

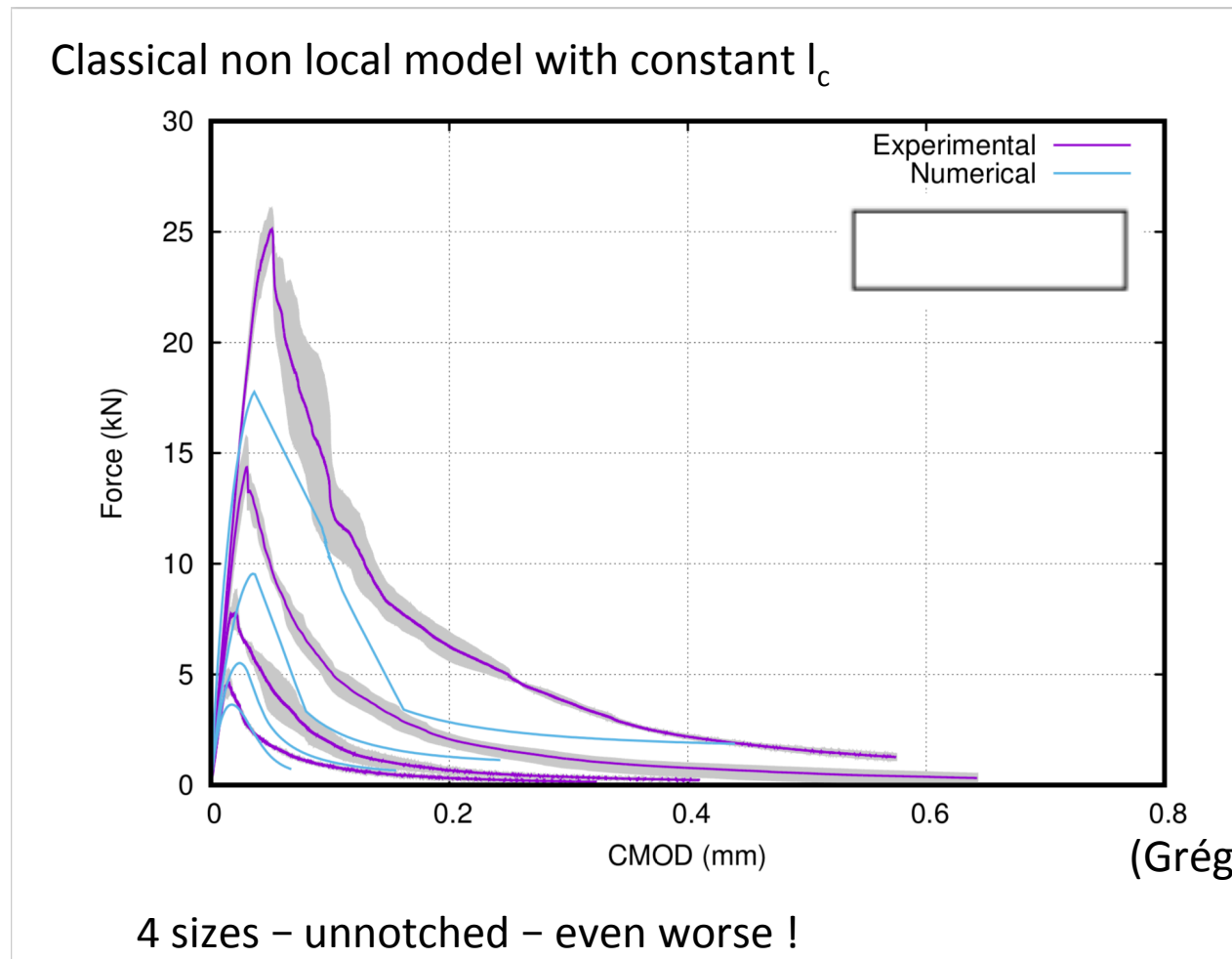
- 3) Example of calibration failure – NL model with constant length



4 sizes – fifth notched – completely lost

- Indirect calibration methods

- 3) Example of calibration failure – NL model with constant length

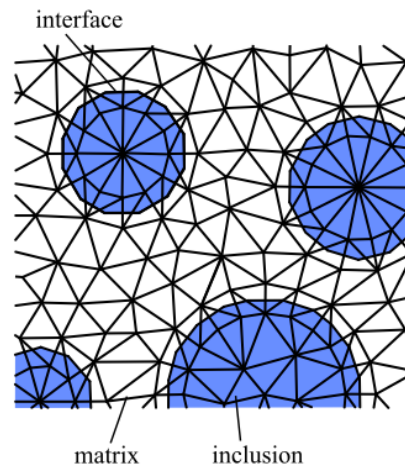


• Indirect calibration methods

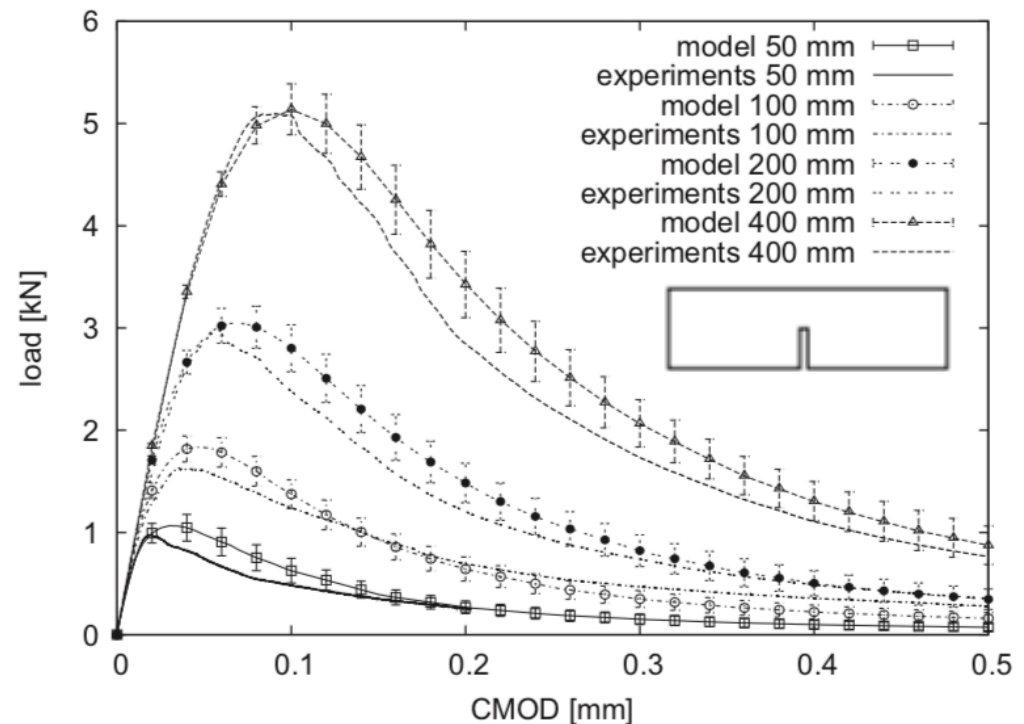
3) Example of calibration success – Mesoscale lattice model

But some calibration may work !!!

Mesoscale lattice model



(Grassl et al., 2012)

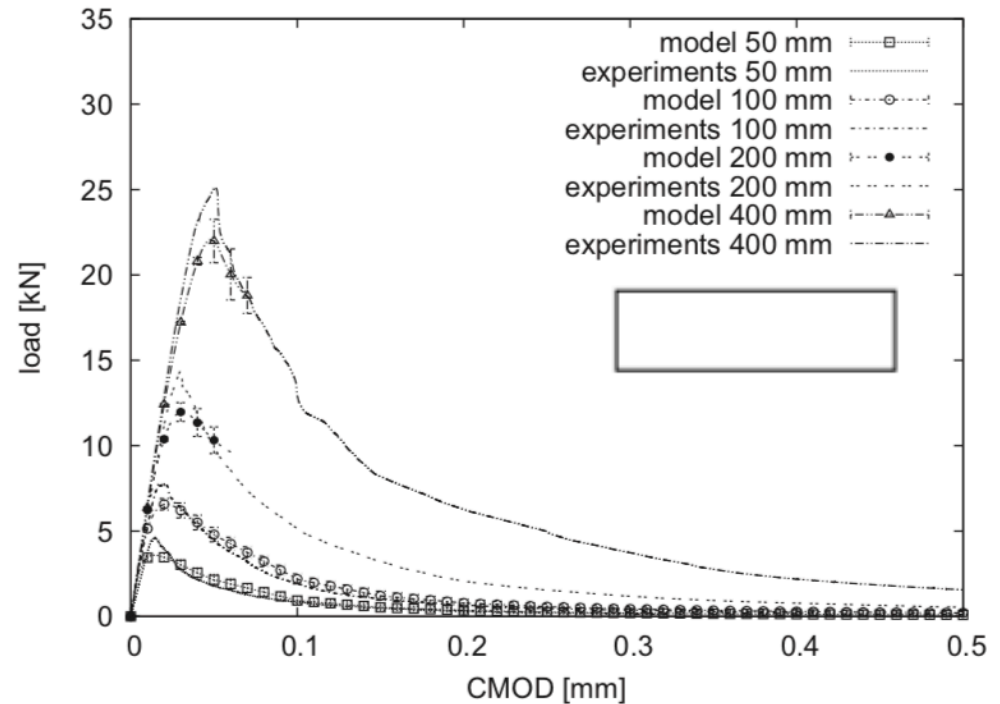
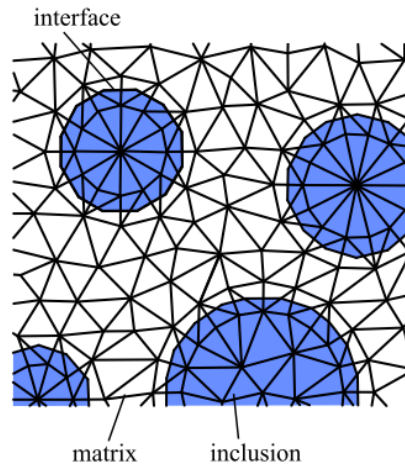


• Indirect calibration methods

3) Example of calibration success – Mesoscale lattice model

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Mesoscale lattice model



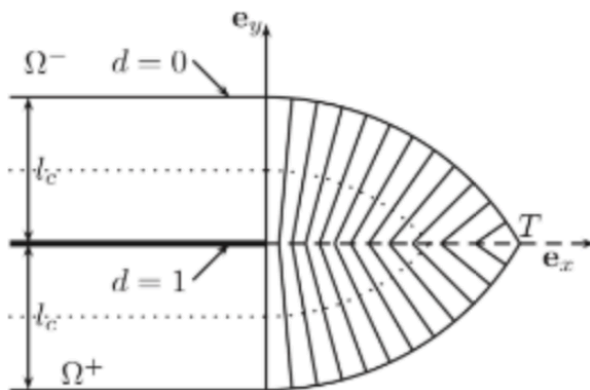
(Grassl et al., 2012)

• Indirect calibration methods

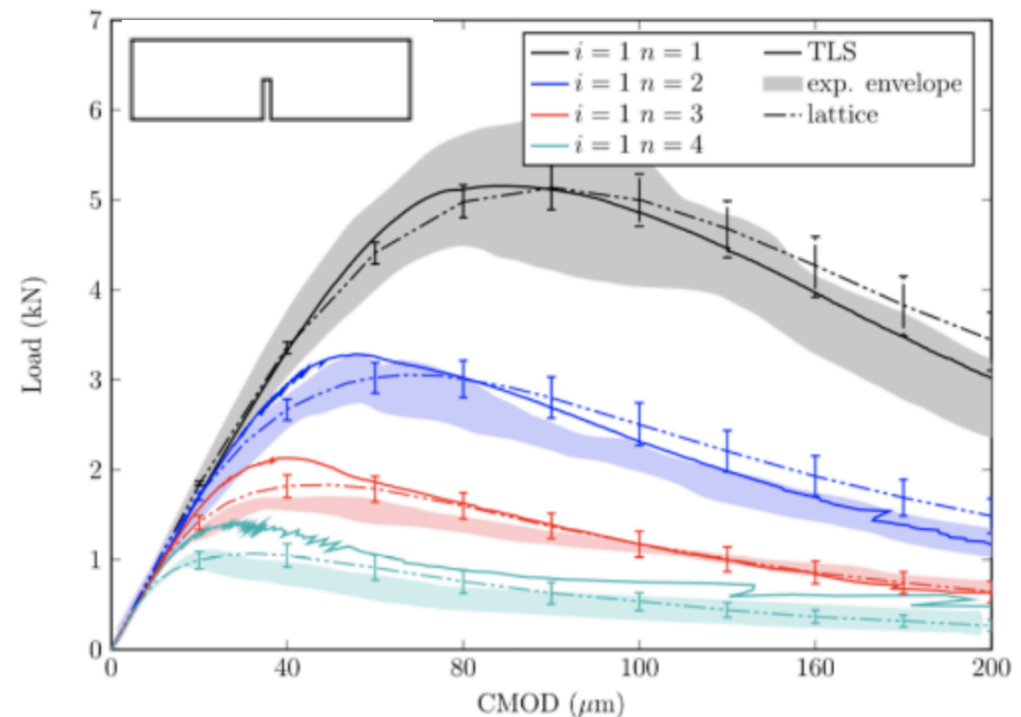
3) Example of calibration success – Thick level set model

But some calibration may work !!!

Thick level set (TLS)



(Parrilla Gómez, 2017)

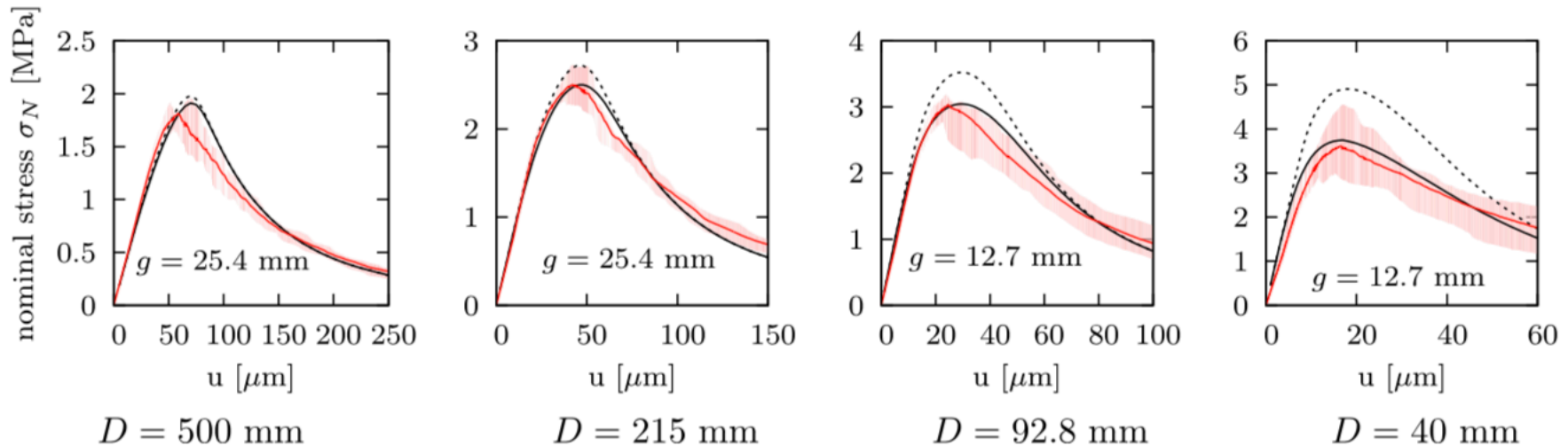


• Indirect calibration methods

3) Example of calibration success – NL model with varying length

But some calibration may work !!!

Integral non-local model with varying L_c



(Havlásek et al., 2016)

exp. from (Hoover et al., 2013)

Outline

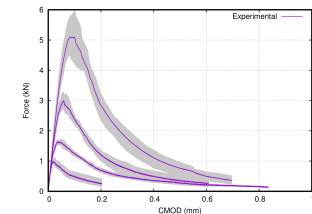
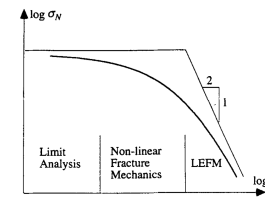
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Softening curves of different specimen sizes (1, 3 or 4)

Example of calibration failure or success



– Direct calibration method ?

Digital image correlation

X-ray tomography

Fracture surface roughness

– Toward the calibration of an evolving characteristic length

Acoustic emission

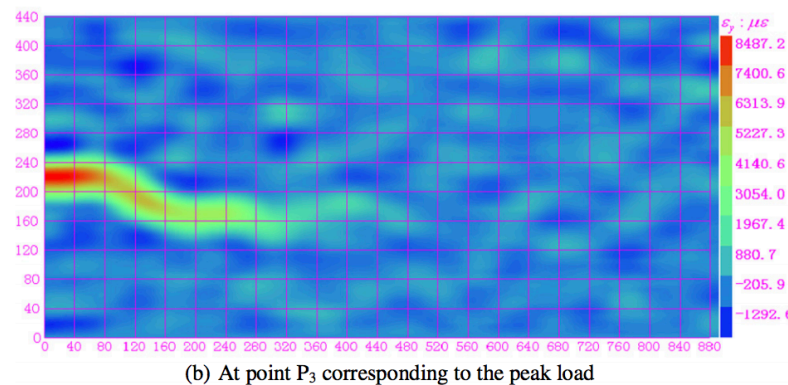
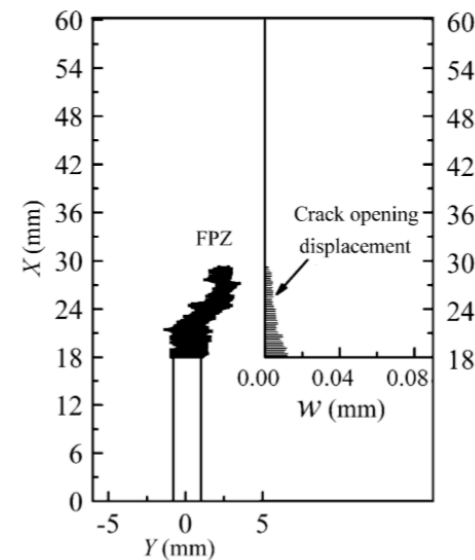
Mesoscale modelling

Spatial ecology and Ripley's functions

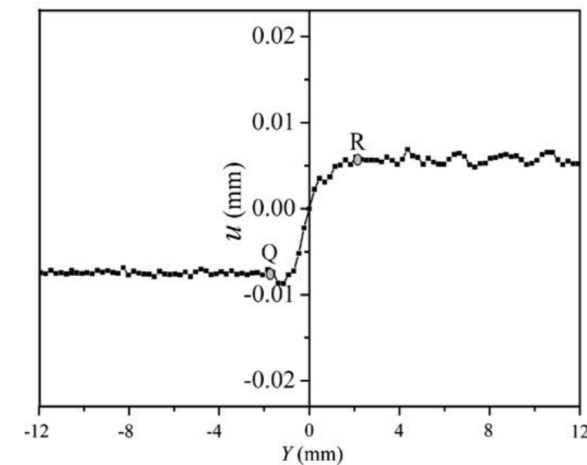
– Conclusion and perspectives

• Direct calibration method ?

1) Digital image correlation e.g. (Wu et al., 2011) (Alam et al., 2012)
(Ł. Skarżyński and J. Tejchman, 2016)



(Wu et al., 2011)

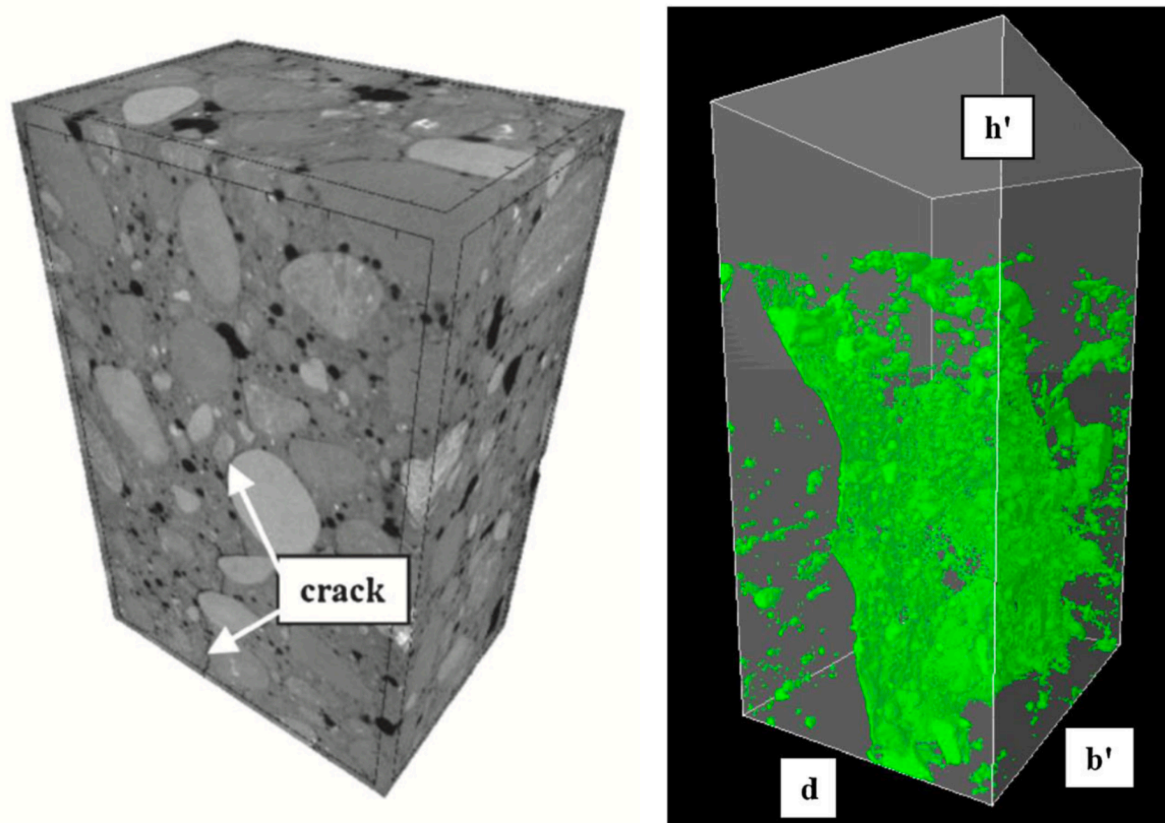


Problems:

- Continuous DIC generally but discontinuous DIC formulation exists
e.g. (Réthoré et al., 2007), (Grégoire et al., 2009), (Grégoire et al., 2011)
- No information about non local interactions
- Difficult to use it for direct model calibration

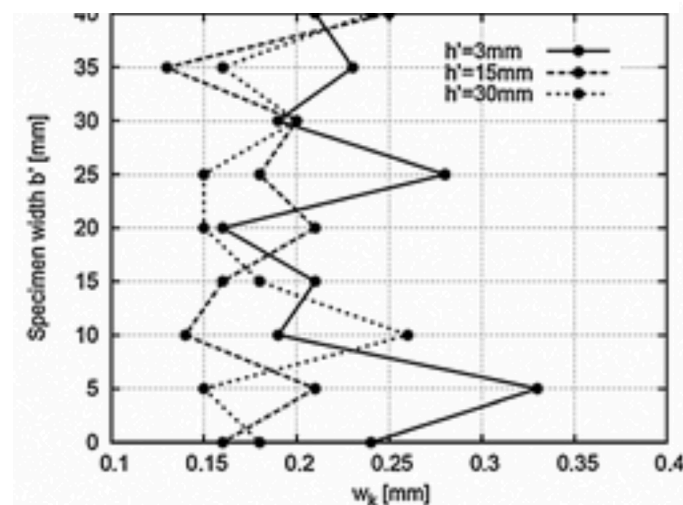
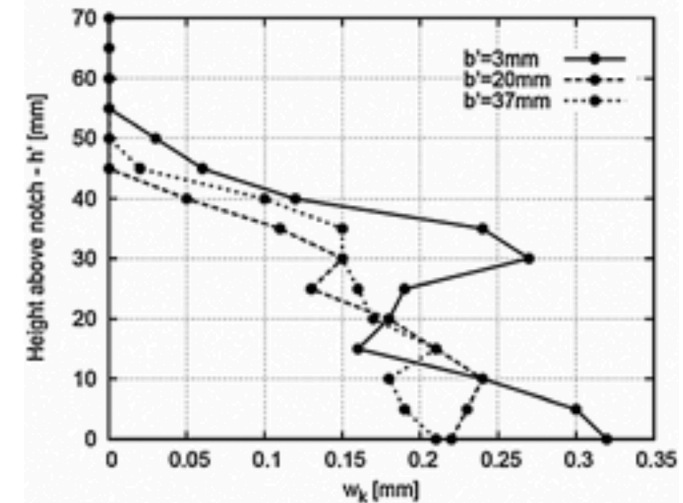
• Direct calibration method ?

2) X-ray tomography e.g. (Ł. Skarżyński and J. Tejchman, 2016)



Problem:

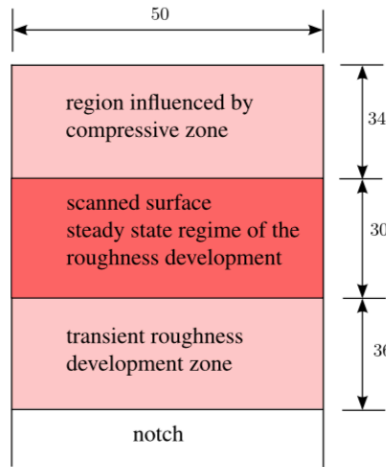
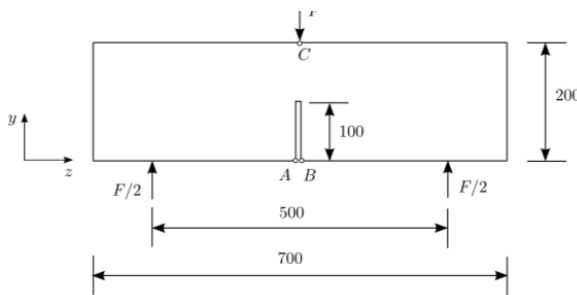
- Postmortem analysis (no evolution)
- Still difficult to use it for direct model calibration



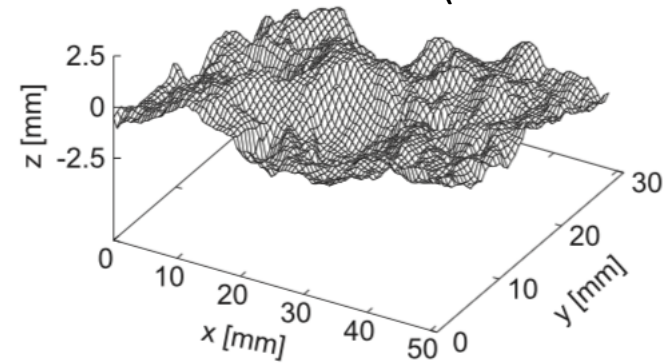
• **Direct calibration method ?**

3) Fracture surface roughness

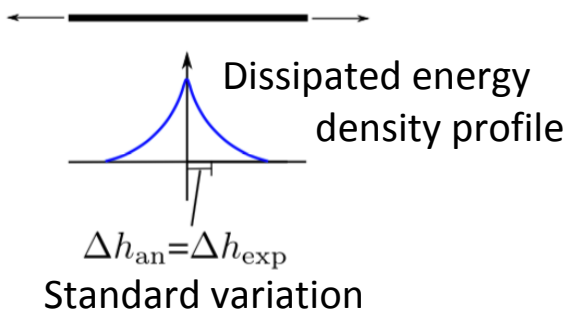
1) Experimental fracture test



(Xenos et al., 2015)



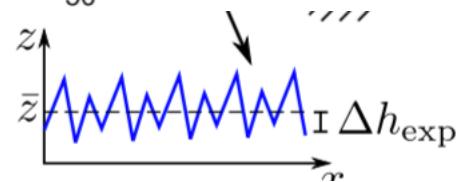
2) 1D non local model failure test



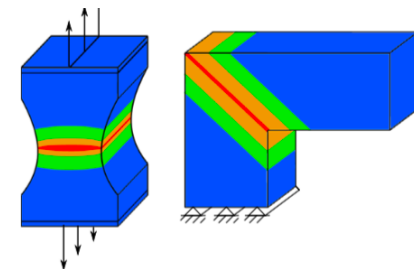
3) Direct calibration

$$\bar{f}(\mathbf{x}) = \int_V \alpha(\mathbf{x}, \xi, R) f(\xi) d\xi$$

Standard variation:



4) Structure failure analyses



- Based on the assumption that the large majority of energy is dissipated in a rough crack
- **Straight forward for constant characteristic length calibration**
- **Quite unique and very promising but needs to be further validated**

Outline

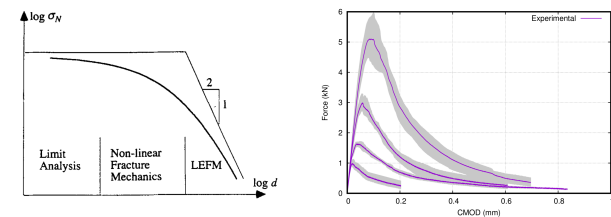
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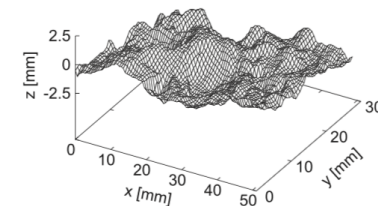
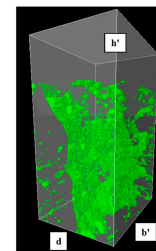
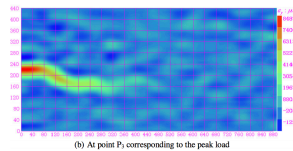


– Direct calibration method ?

Digital image correlation

X-ray tomography

Fracture surface roughness



– Toward the calibration of an evolving characteristic length

Acoustic emission

Mesoscale modelling

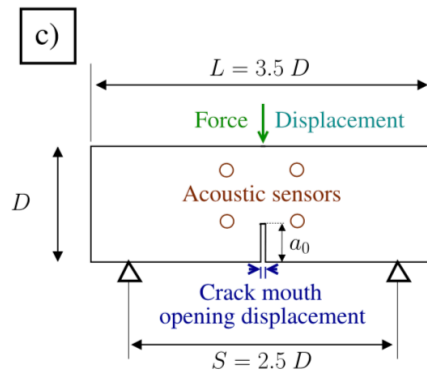
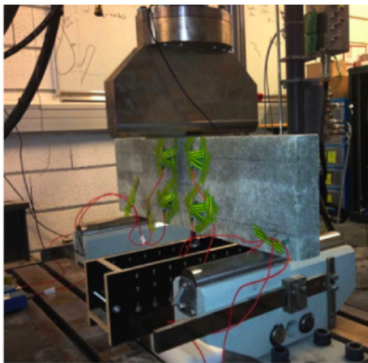
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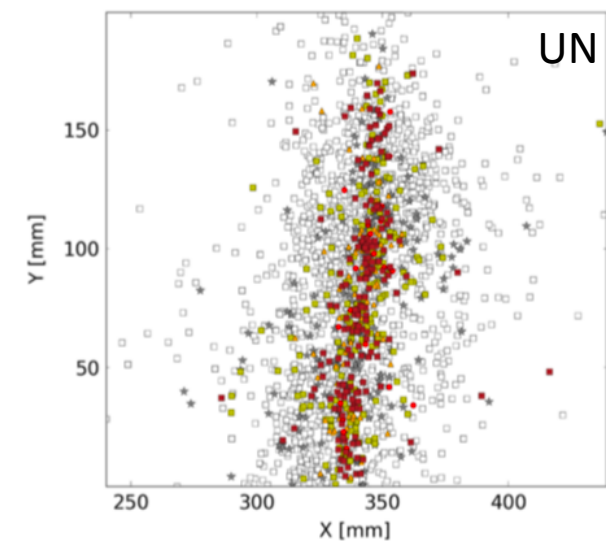
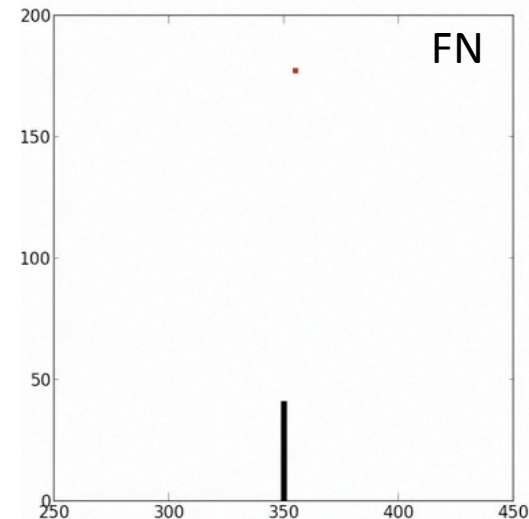
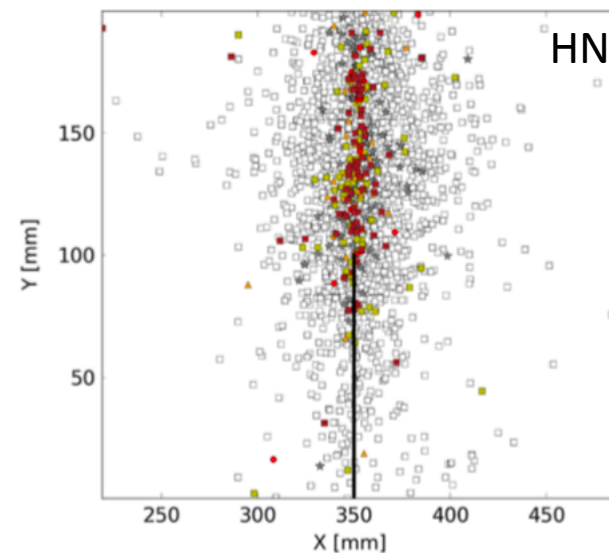
1) Acoustic emission

e.g. (Landis, 1999) (Granger et al., 2007)
(Grégoire et al., 2015) (Saliba et al., 2016)



Unnotched and notched specimen

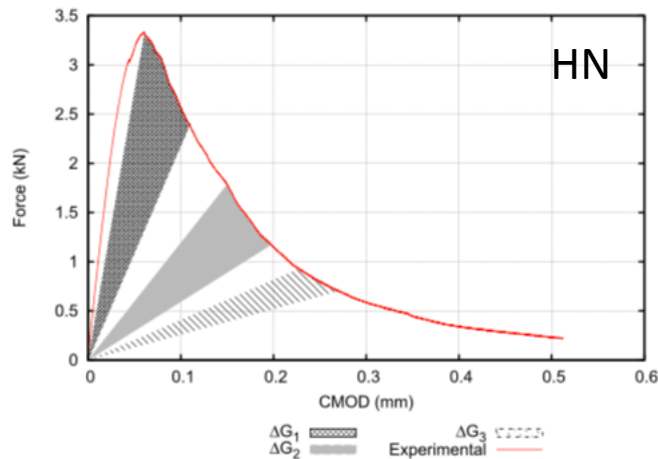
(Grégoire et al., 2015)



Calibration of an evolving characteristic length

2) Acoustic emission

e.g. (Landis, 1999) (Granger et al., 2007)
(Grégoire et al., 2015) (Saliba et al., 2016)



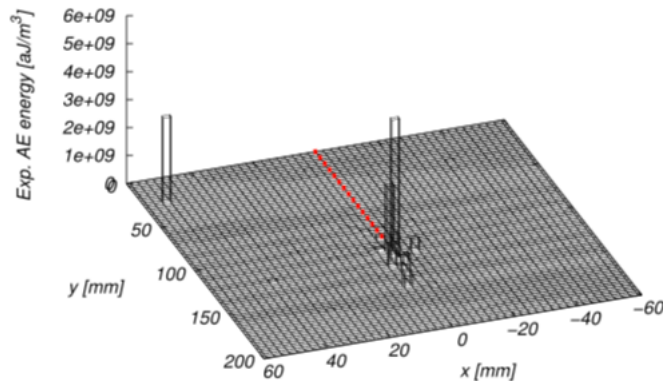
(a) HN200: Half-notched beams, $D = 200$ mm

Unnotched and notched specimen

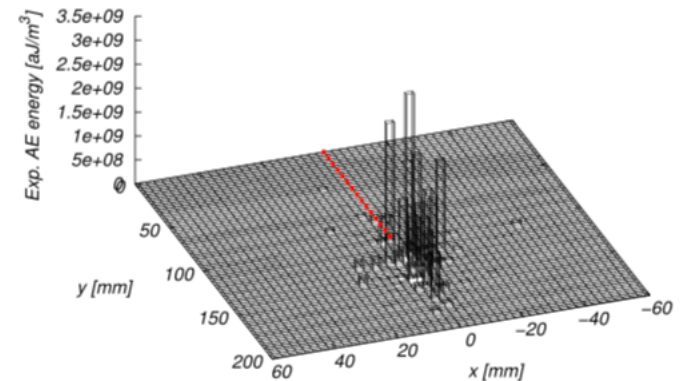
(Grégoire et al., 2015)

Problem:

Not so much points !
(almost nothing prepick)



(a) Δ_{G1} - Experimental

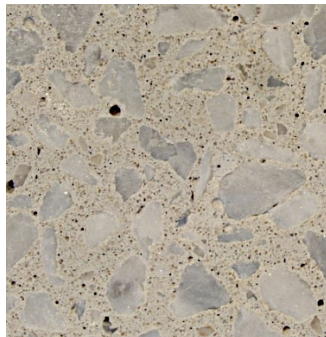


(c) Δ_{G2} - Experimental

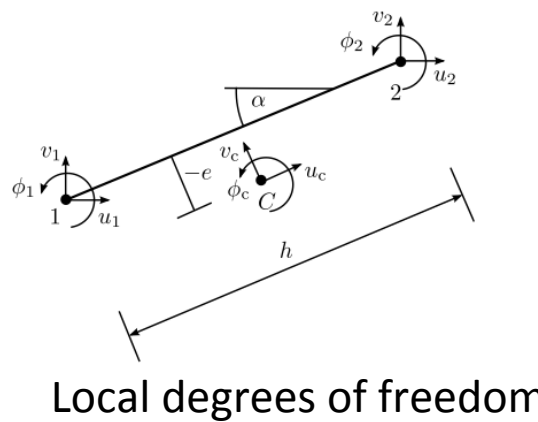
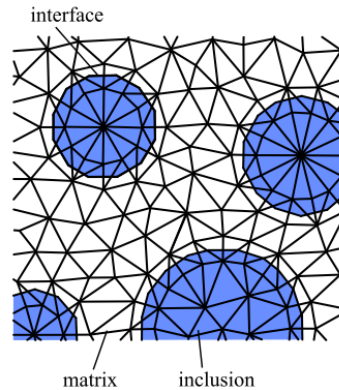
• Calibration of an evolving characteristic length

2) Mesoscale modelling e.g. (Schlangen et Van Mier, 1992) (Delaplace et al., 1996) (Grassl and Jirásek, 2010) (Grassl et al., 2012)

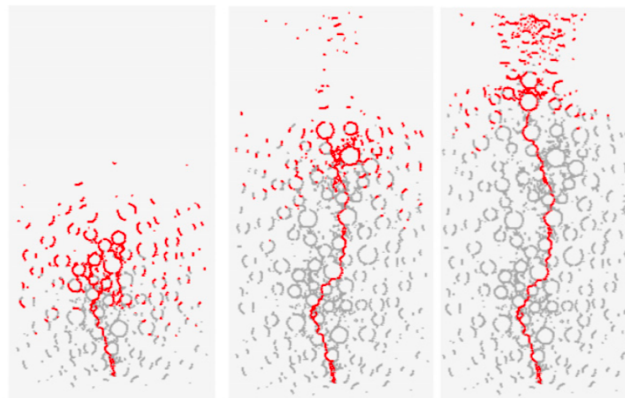
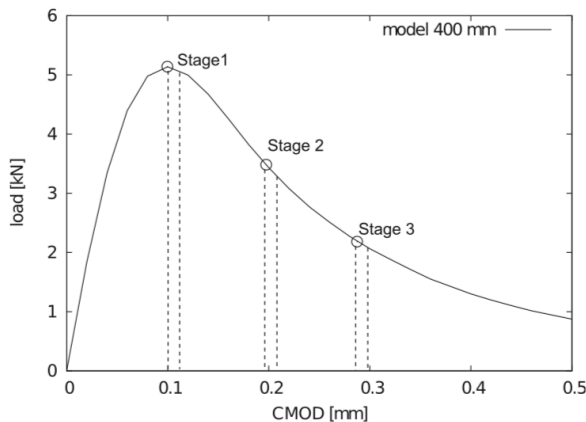
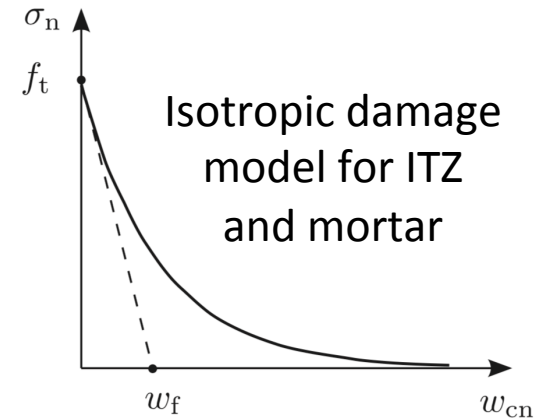
Heterogeneities are explicitly meshed



Concrete



Local degrees of freedom



A lot of damage point may be identified !

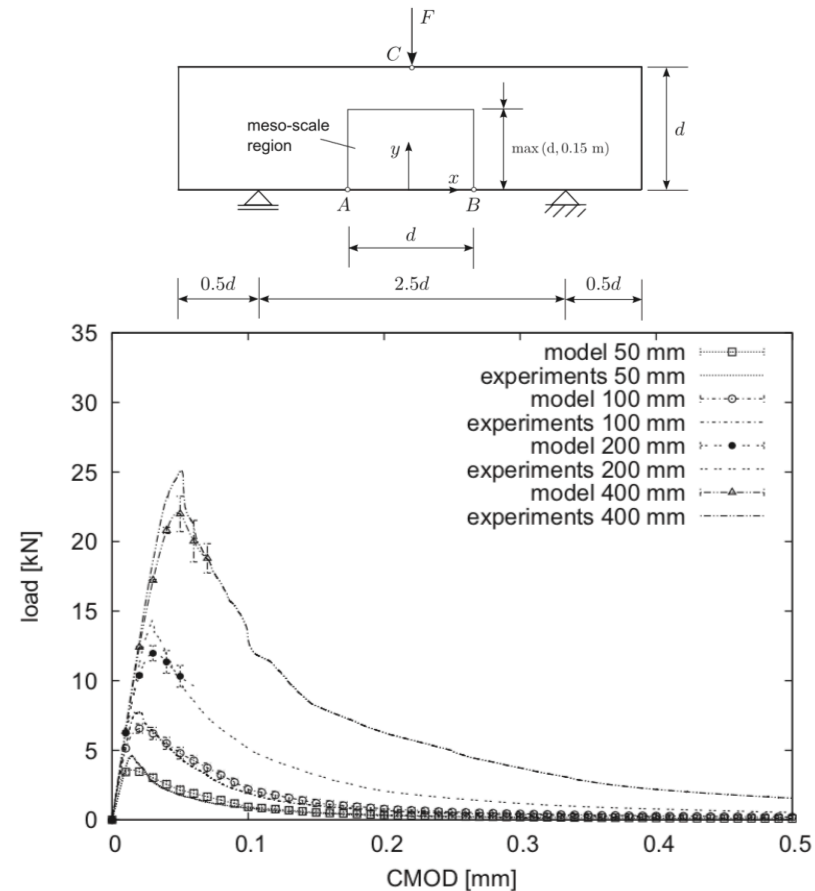
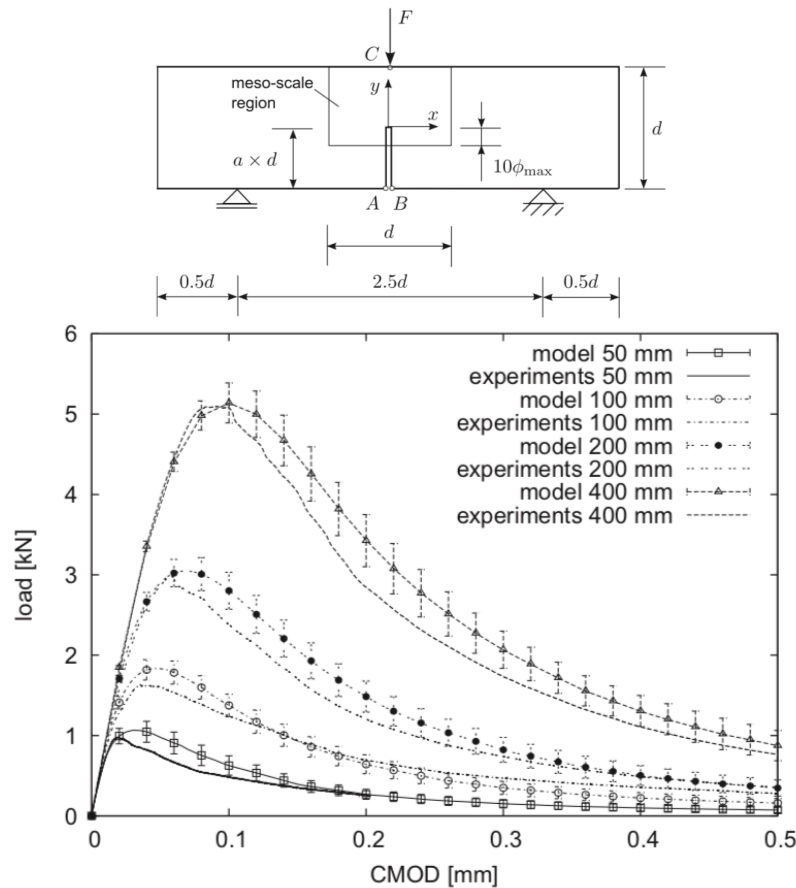
(Grassl et al., 2012)

- Calibration of an evolving characteristic length

- 2) Mesoscale modelling

Consistent and predictive in term of global response !

(Grassl et al., 2012)

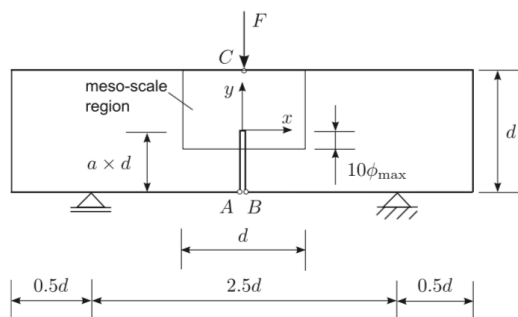


- Calibration of an evolving characteristic length

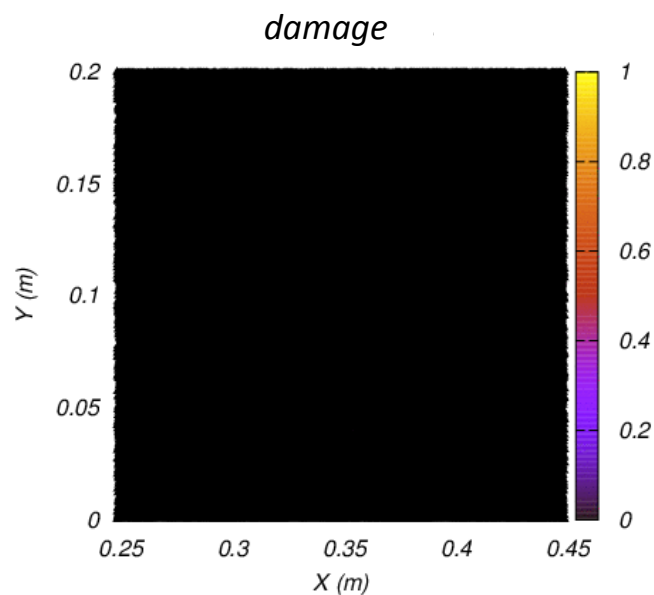
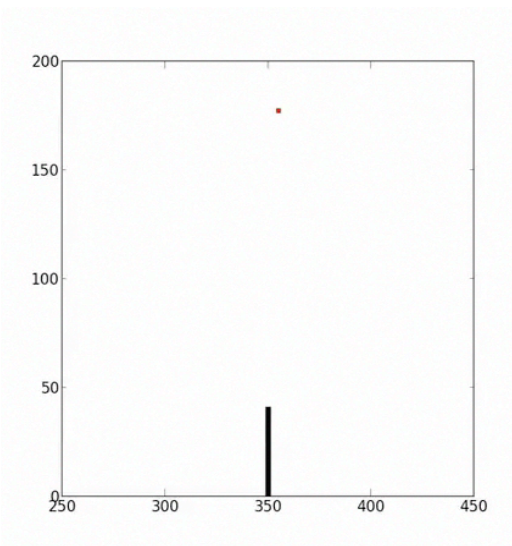
- 2) Mesoscale modelling

Consistent and predictive in term of local response !

(Grégoire et al., 2015)



Acoustic emission vs lattice model

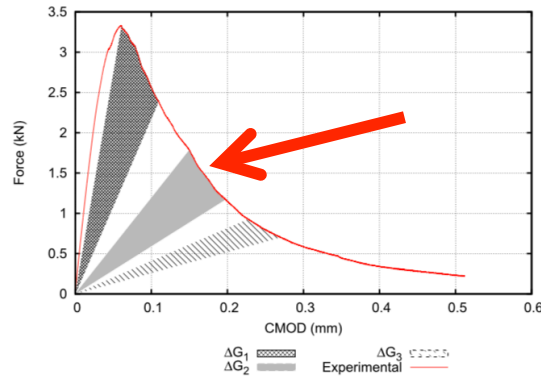


• Calibration of an evolving characteristic length

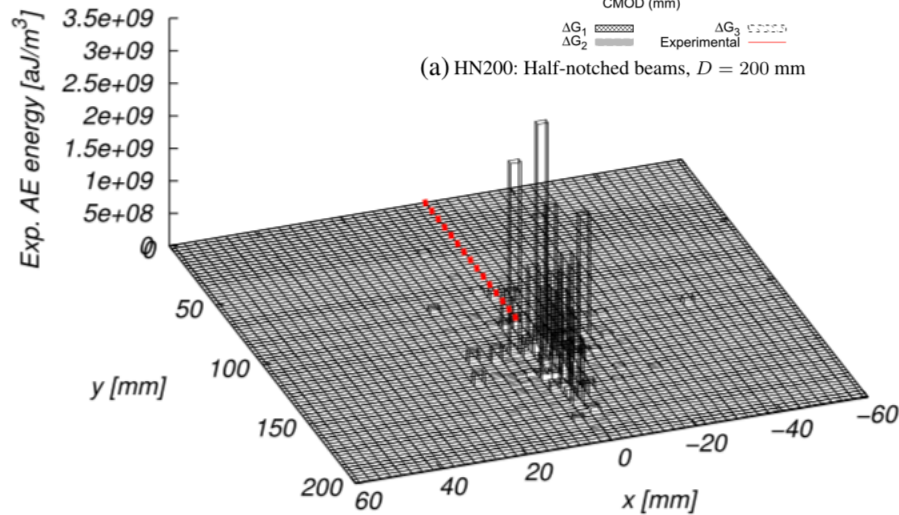
2) Mesoscale modelling

Consistent and predictive in term of local response !

(Grégoire et al., 2015)

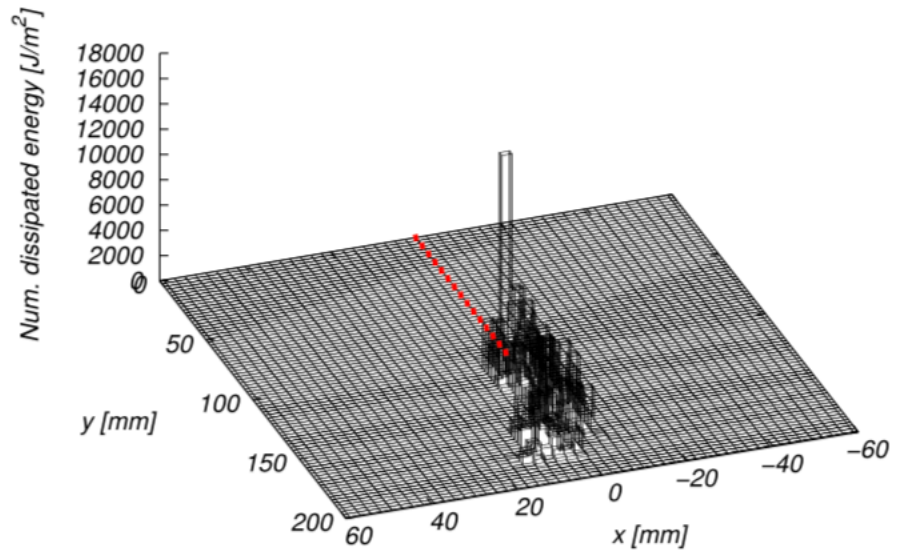


(a) HN200: Half-notched beams, $D = 200$ mm



(c) Δ_{G2} - Experimental

Acoustic emission vs lattice model



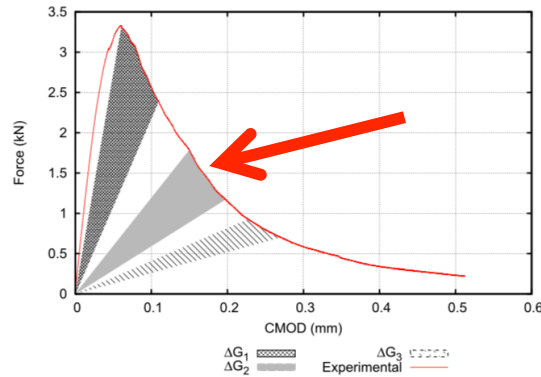
(d) Δ_{G2} - Numerical

• Calibration of an evolving characteristic length

2) Mesoscale modelling

Consistent and predictive in term of local response !

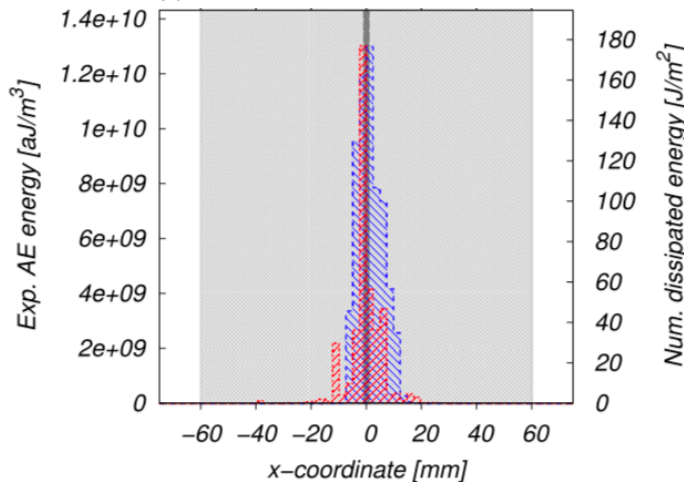
(Grégoire et al., 2015)



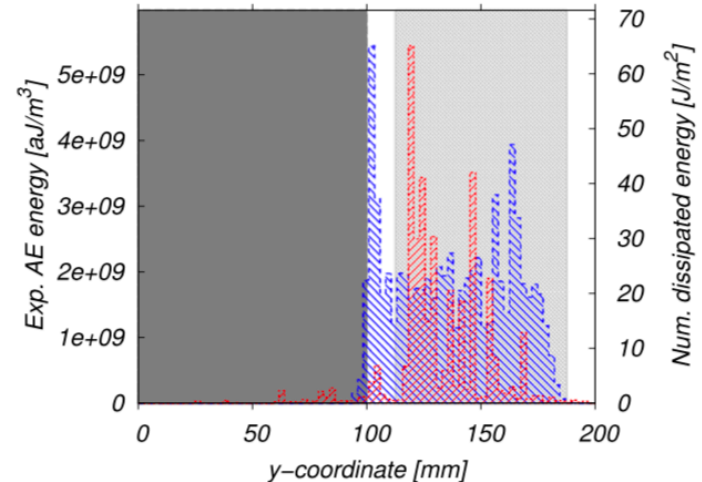
(a) HN200: Half-notched beams, $D = 200$ mm

Acoustic emission vs lattice model

Problem: How interpreting these data in a way that we can identify the characteristic length evolution ?



(c) Δ_{G2} – Horizontal projection



(d) Δ_{G2} – Vertical projection

- **Calibration of an evolving characteristic length**

- 3) Ripley's function: a spatial ecology tool

We want to characterize how microcracks interact from damage patterns that start randomly and then localise

This has been performed for years in spatial ecology:

(Ripley, 1977) – Cell migration

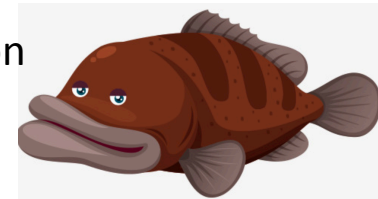
(Stamp, 1990) – Plant spreading

(Diggle, 1991) – Disease spreading

(Duncan, 1993) – Tree spreading

(Dixon, 2002) – Review on Ripley's function

(Tentelier and Piou, 2011) – Anadromous fish migration

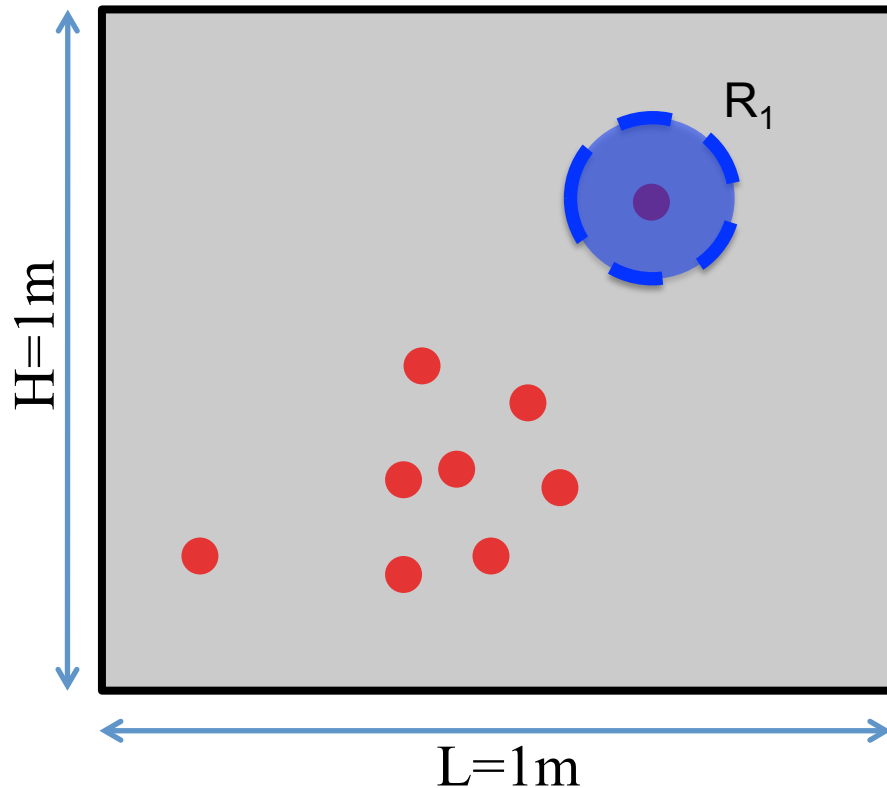


and more recently in mechanics:

(Tordesillas et al., 2012) – Diffuse granular failure

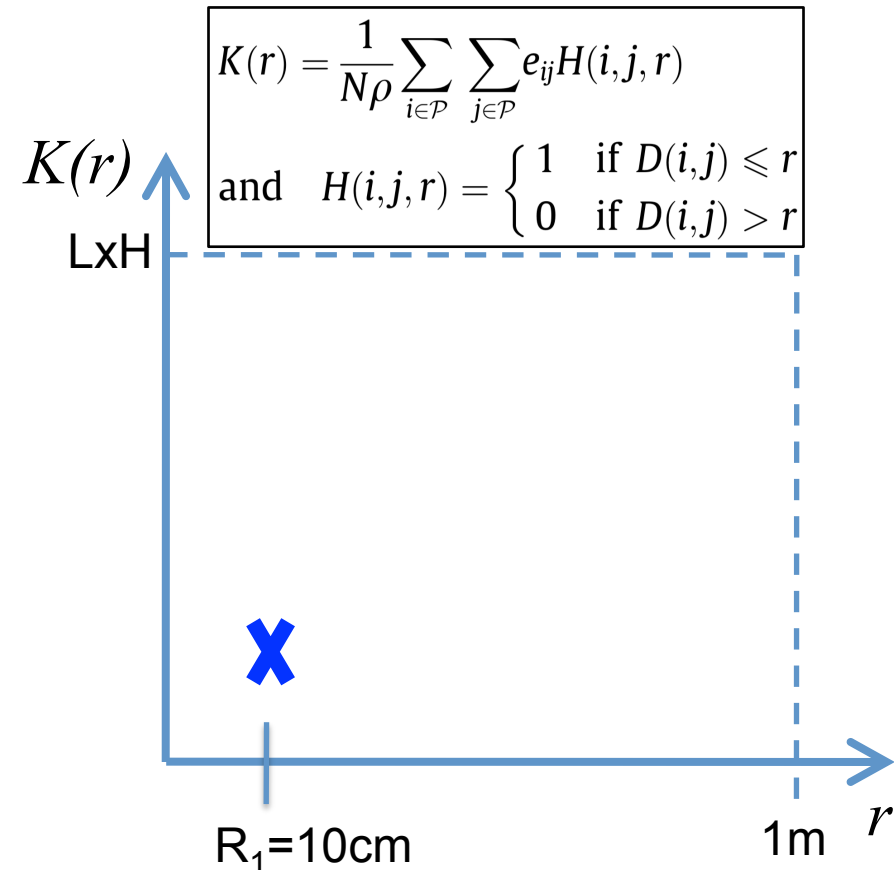
(Lefort et al., Eng Fract Mech, 2015) – Concrete failure (this work)

Principle of Ripley's functions



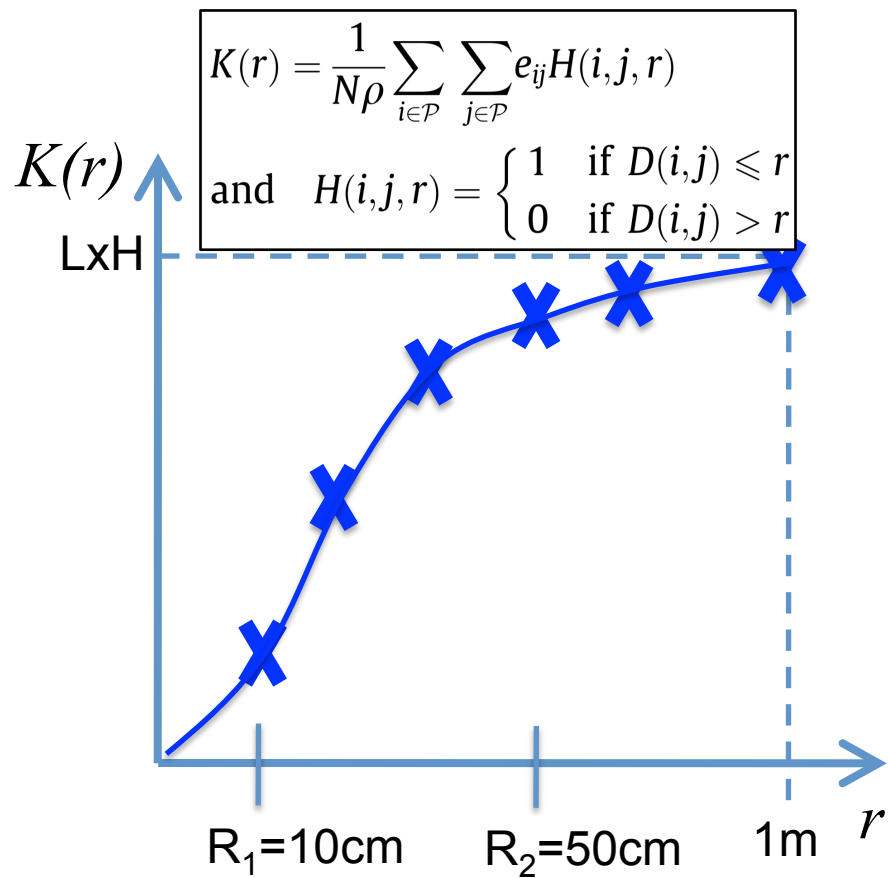
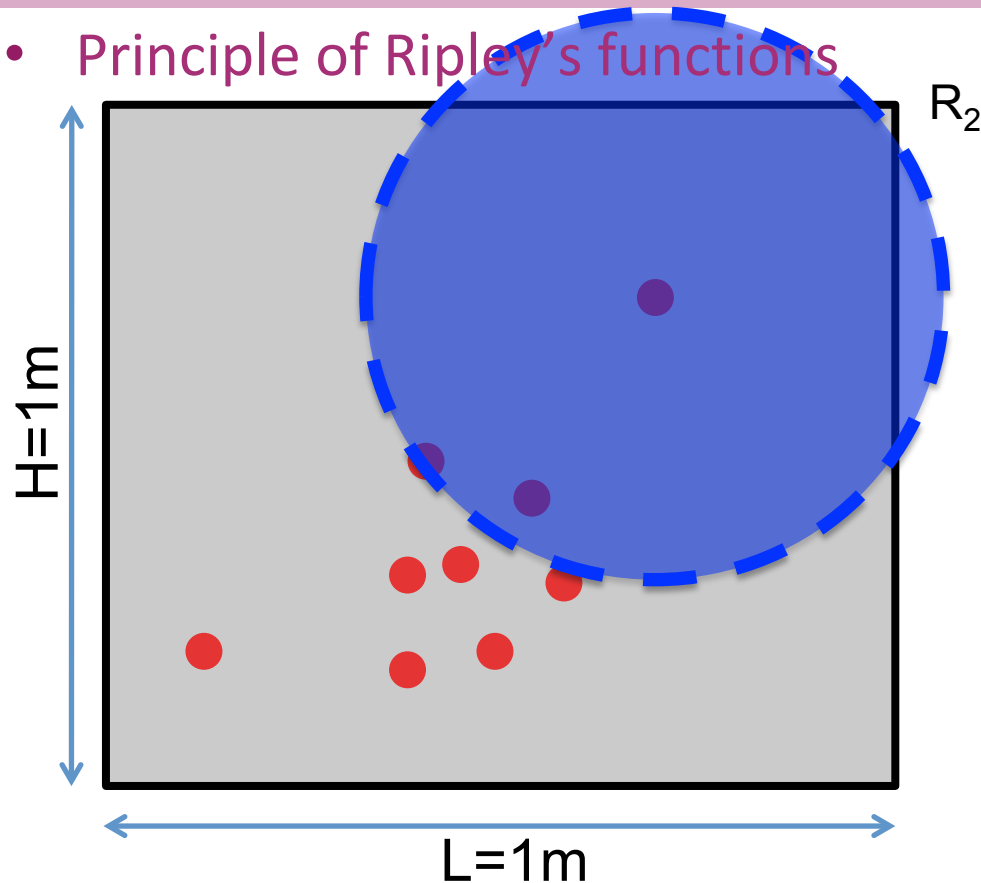
Average density of points : $\rho=9/(L \times H)=9\text{points}/\text{m}^2$

Average number of neighbours counted in the disk of radius R_1 : $N_{\text{moy},R_1} = 0,50$



$$\Rightarrow K(R_1) = N_{\text{moy},R_1} / \rho = 0,06$$

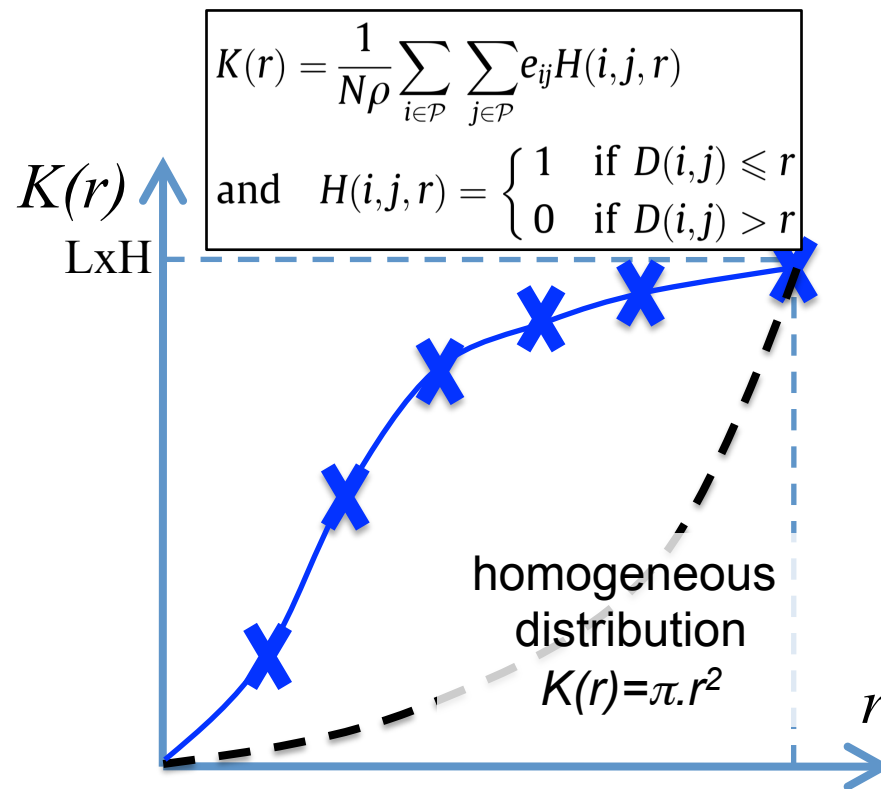
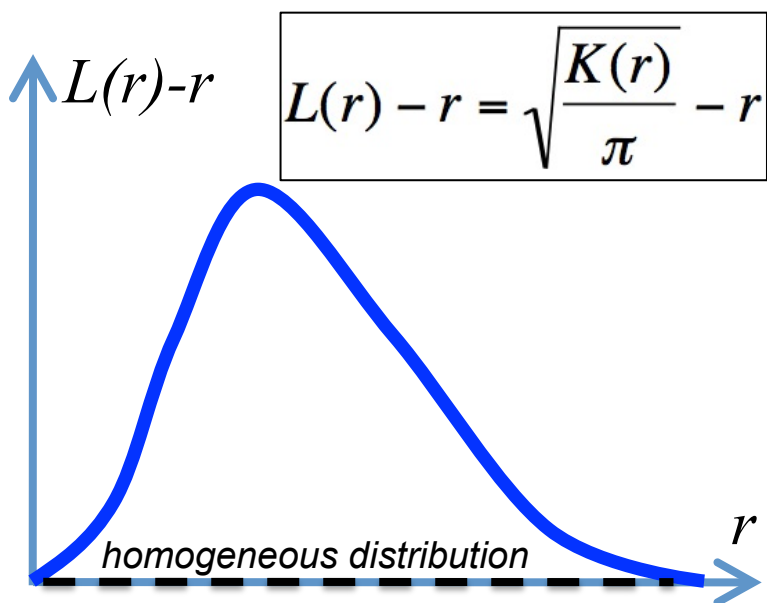
- Principle of Ripley's functions



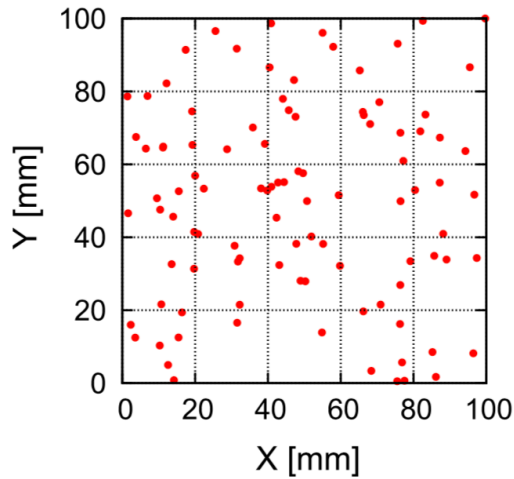
Average number of neighbours counted in the disk of radius R_2 : $N_{\text{moy},R_2} = 7$

$$\Rightarrow K(R_2) = N_{\text{moy},R_2} / \rho = 0,78$$

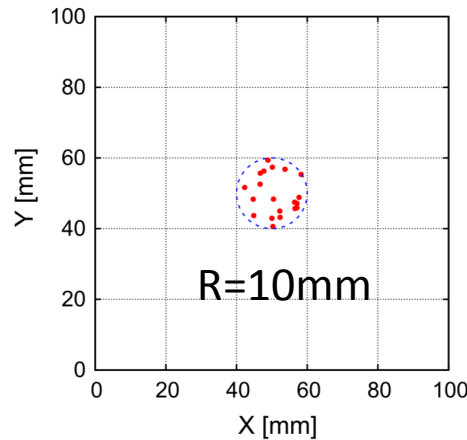
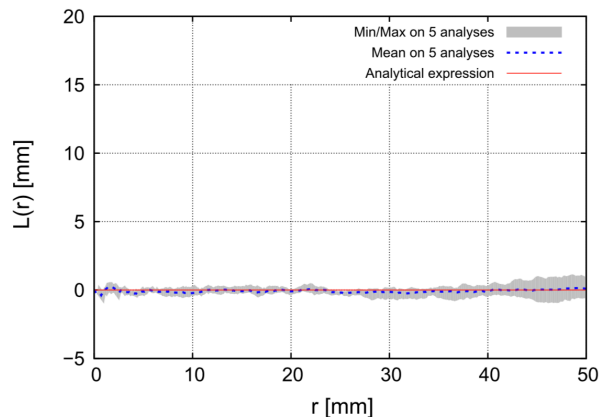
• Principle of Ripley's functions



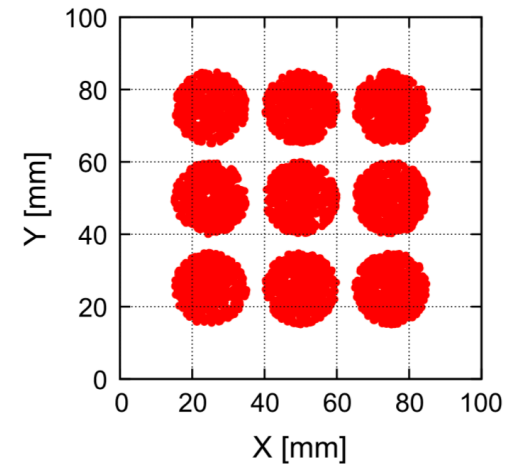
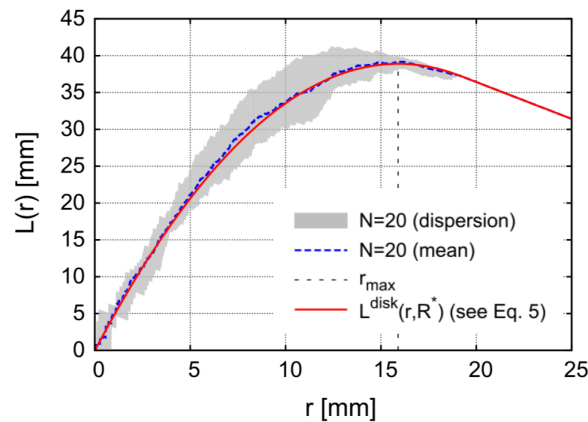
• Typical shapes of $L(r)$ functions



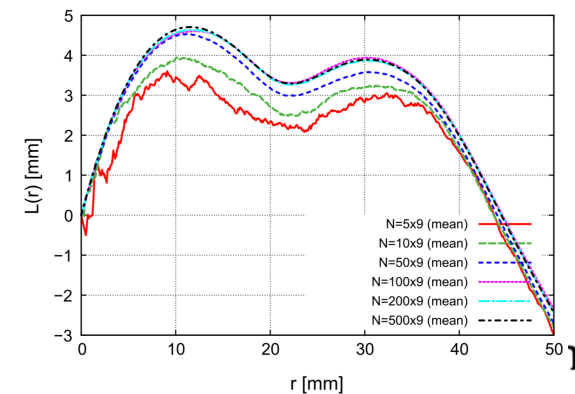
Random points pattern



Points concentrated in 1 disc

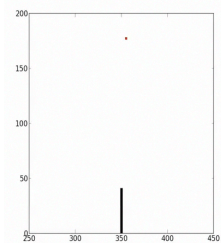


Points concentrated in 9 discs

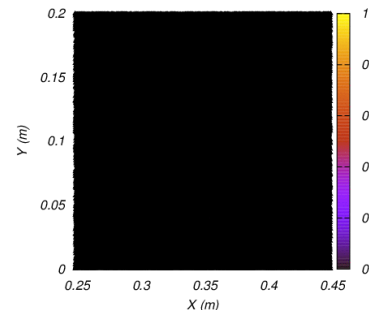
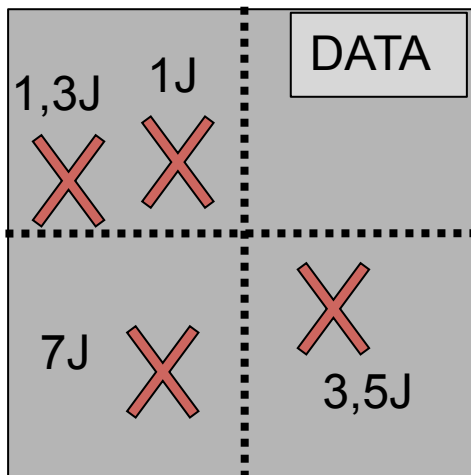
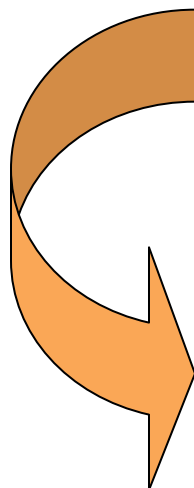


• Experimental Campaign vs Numerical Model

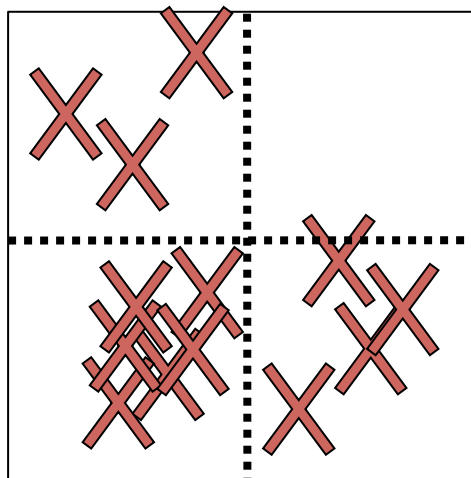
– Taking into account the energy...



intensity coding
(here 1J=1point)

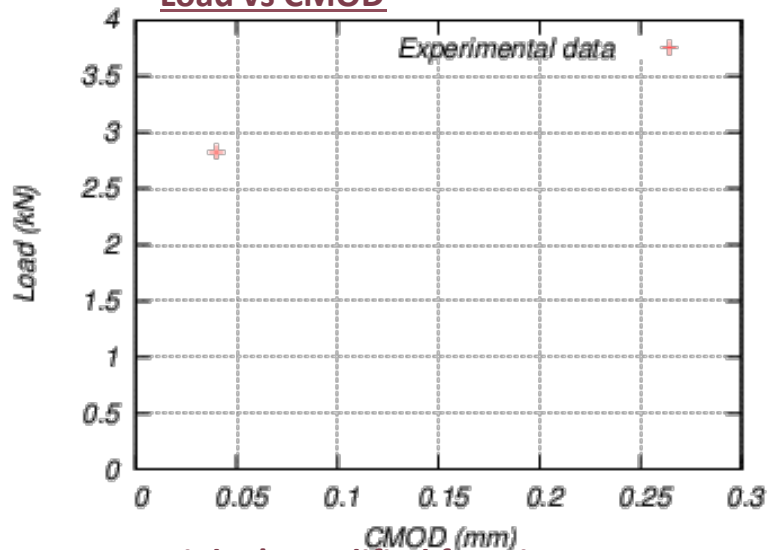


Point pattern analyzed with Ripley's functions

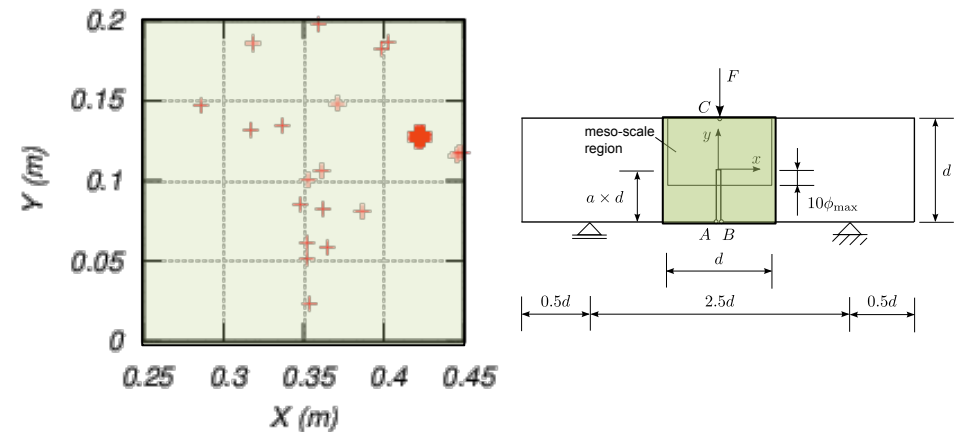


EXPERIMENTAL

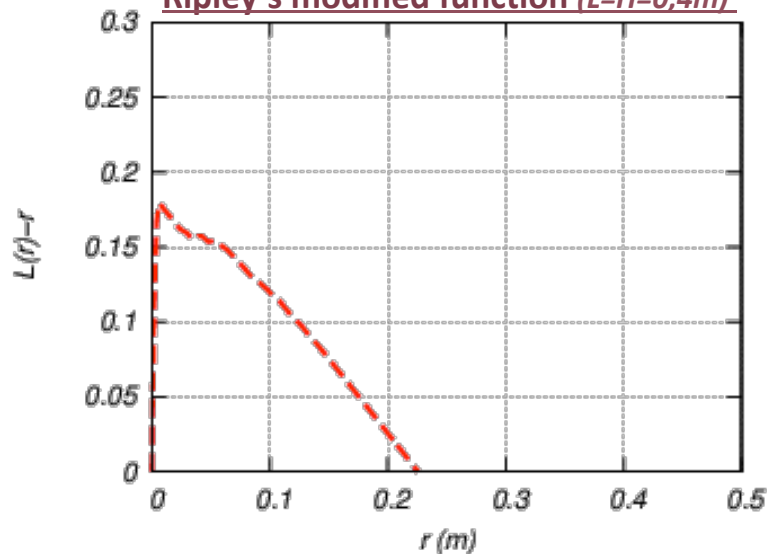
Load vs CMOD



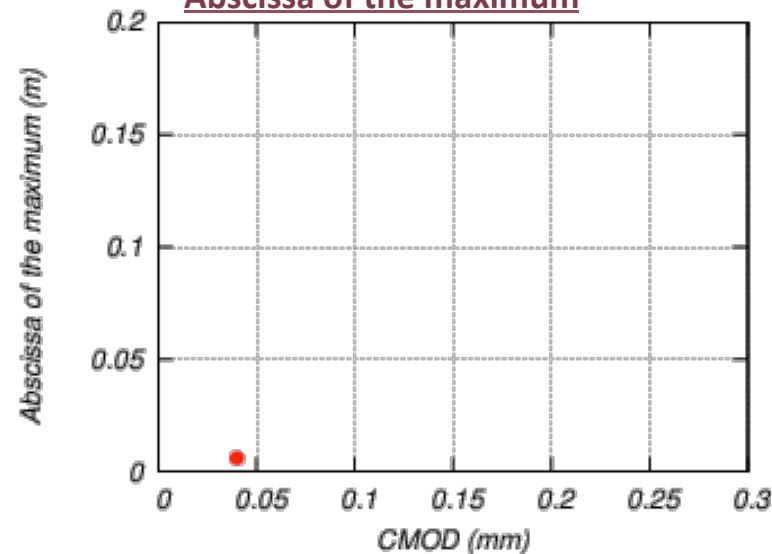
Position of events occurring during the time step



Ripley's modified function (L=H=0.4m)

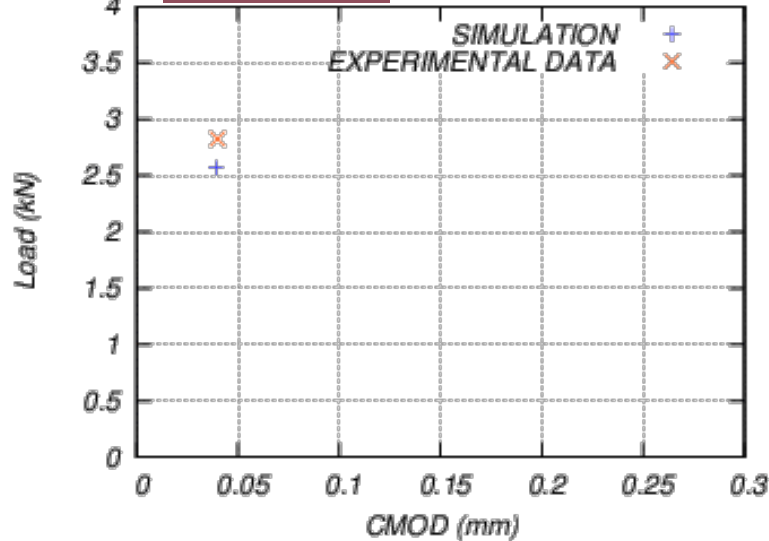


Abscissa of the maximum

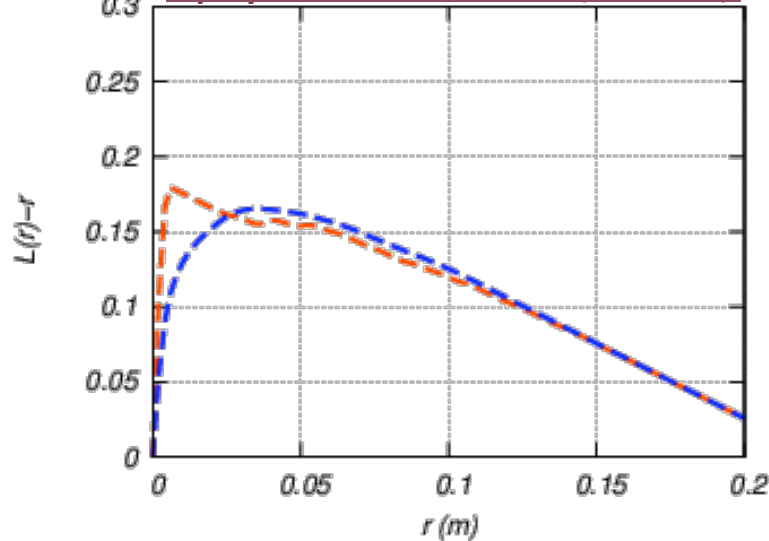


SIMULATION vs EXPERIMENTAL

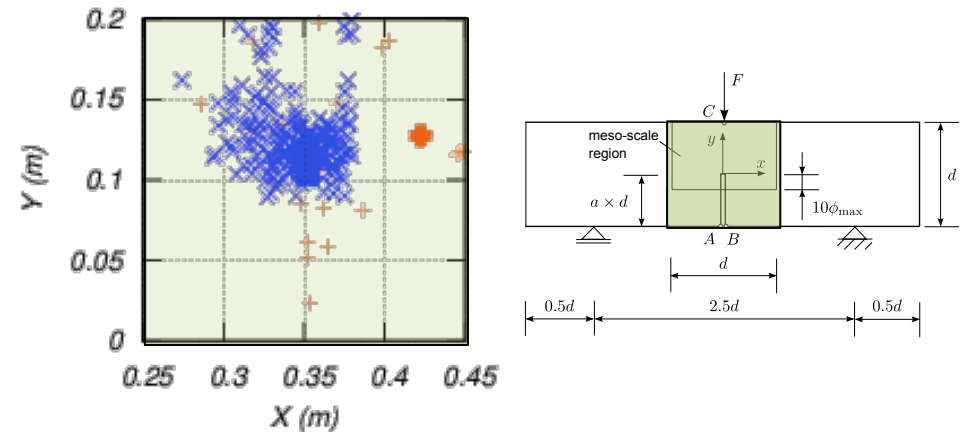
Load vs CMOD



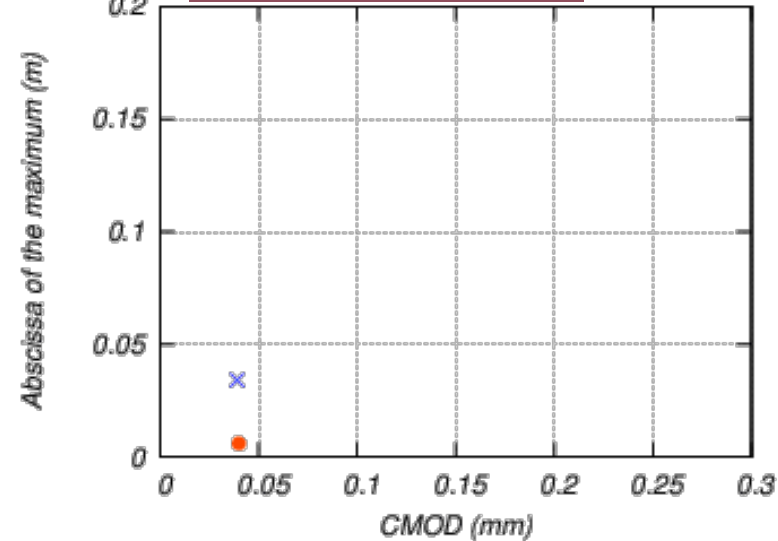
Ripley's modified function ($L=H=0,4m$)



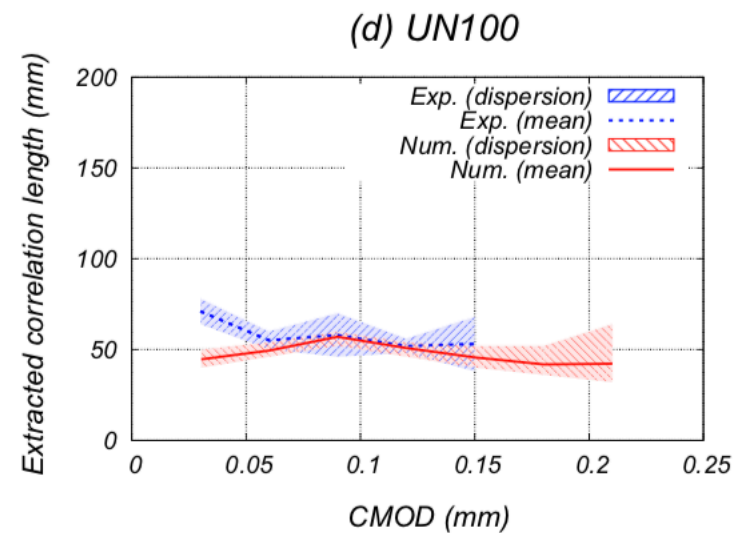
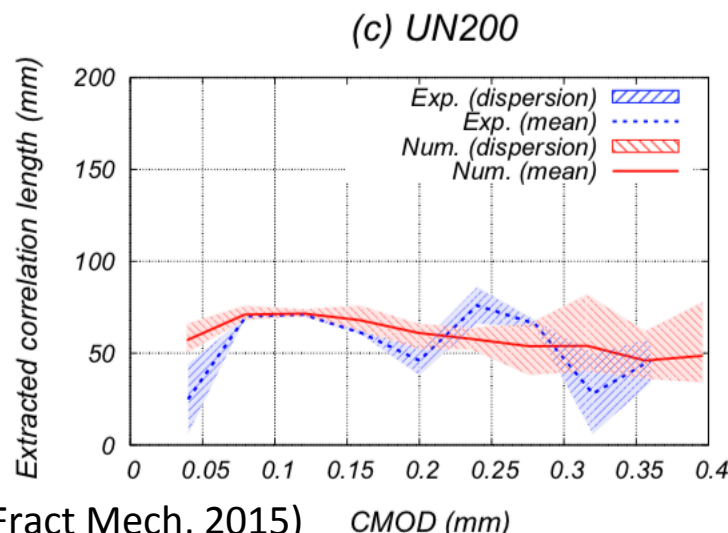
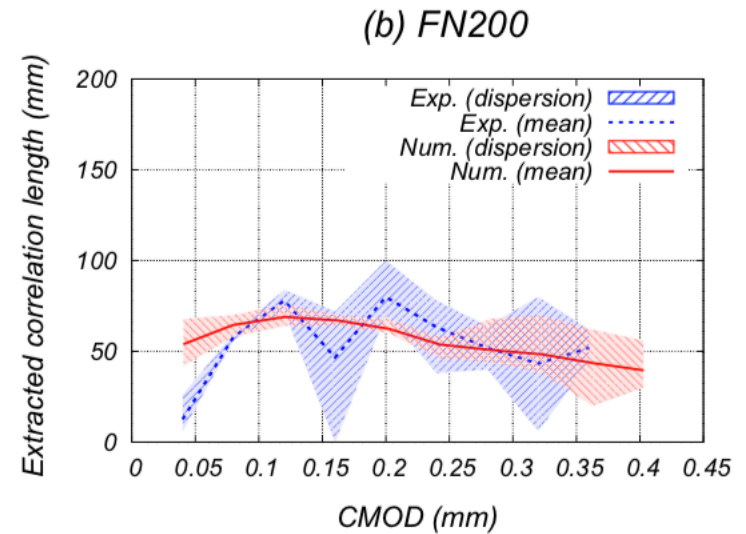
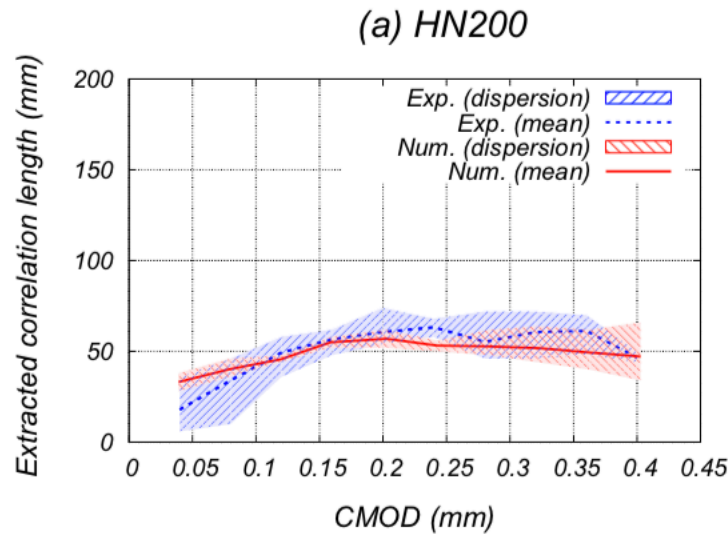
Position of events occurring during the time step



Abscissa of the maximum

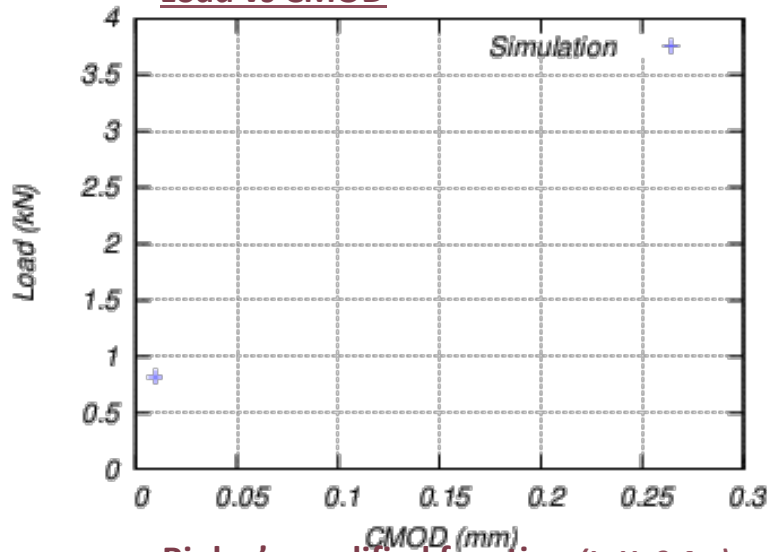


SIMULATION vs EXPERIMENTAL

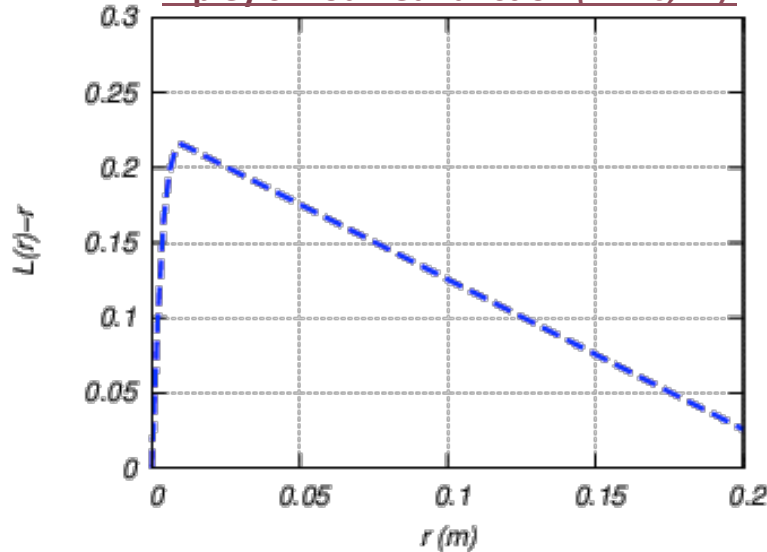


SIMULATION

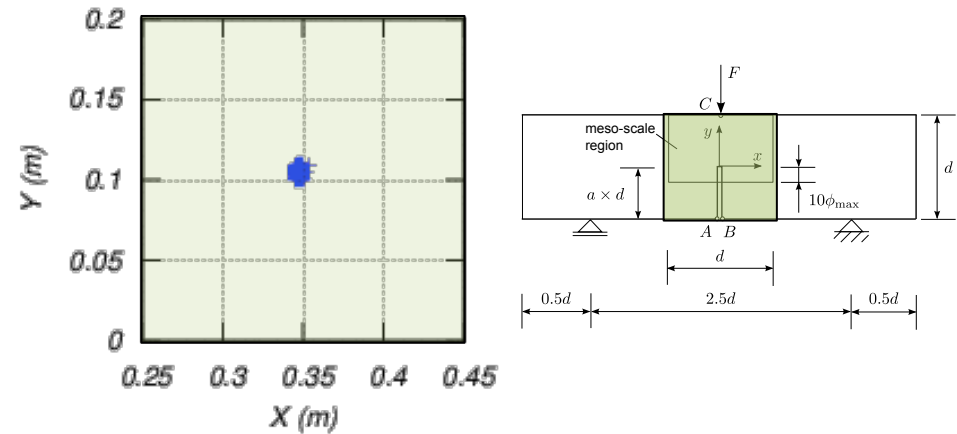
Load vs CMOD



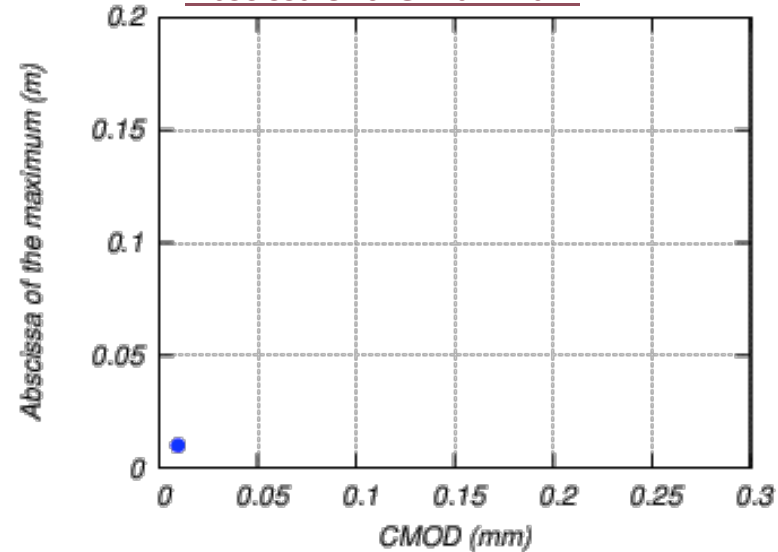
Ripley's modified function (L=H=0,4m)



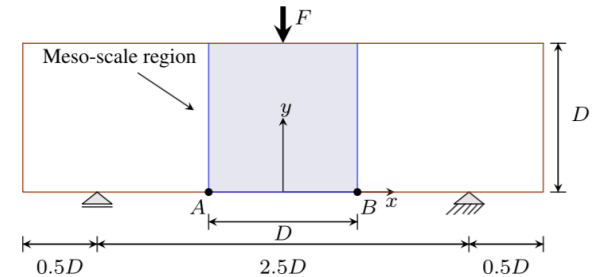
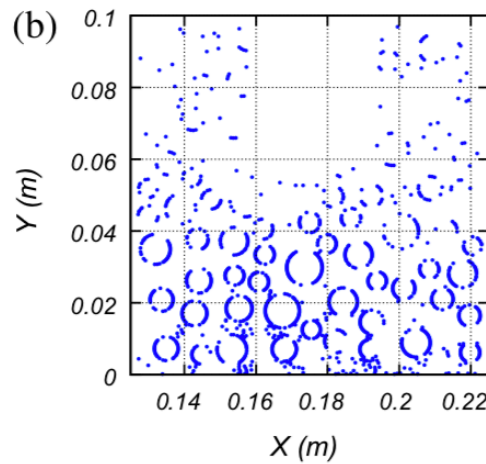
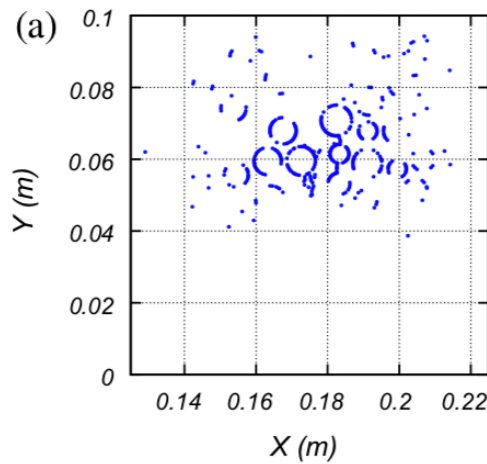
Position of events occurring during the time step



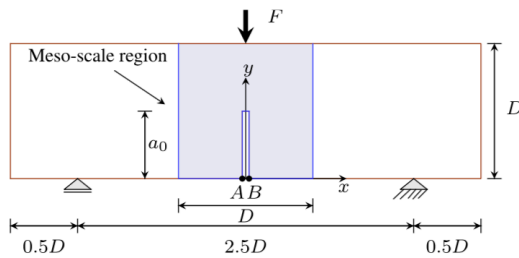
Abscissa of the maximum



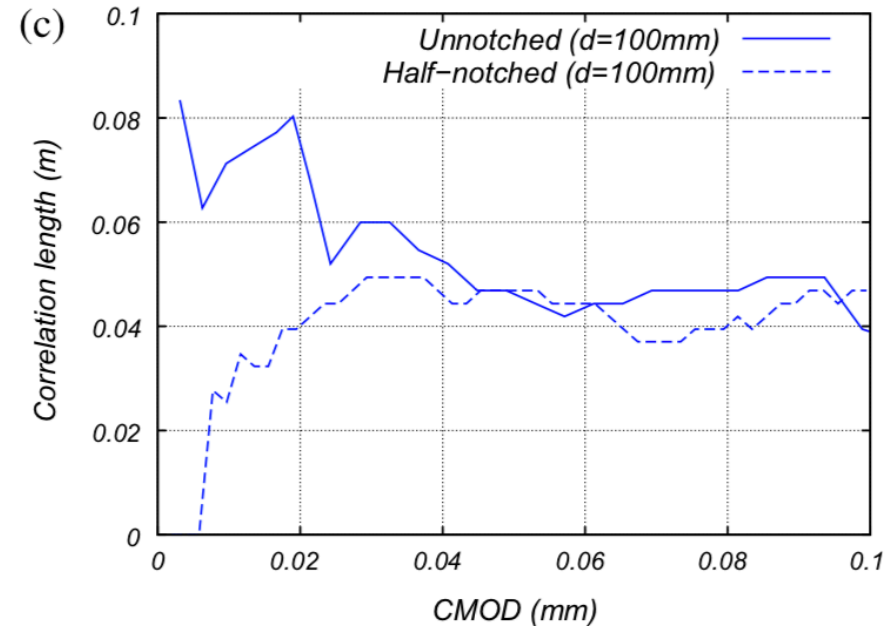
• Comparision between unnotched and notched specimen



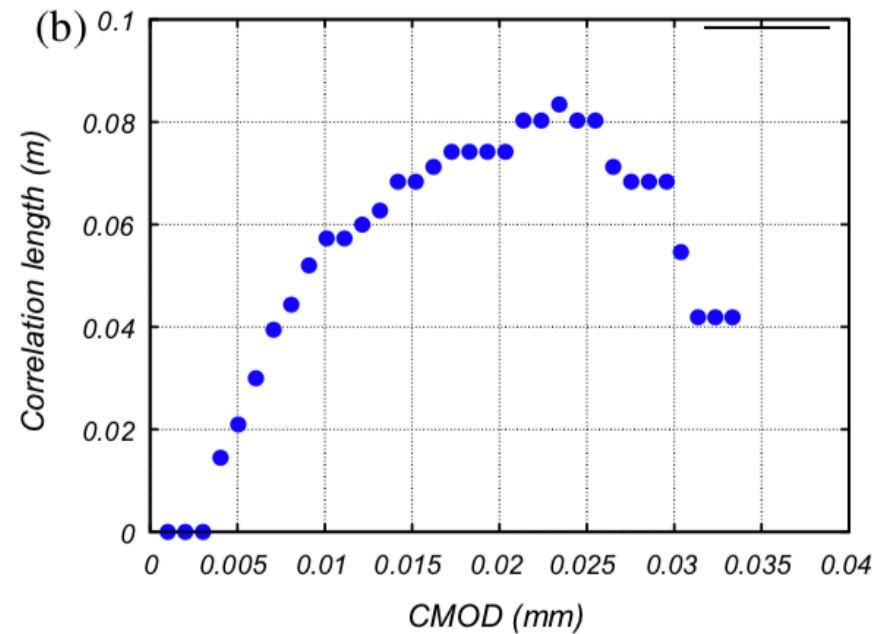
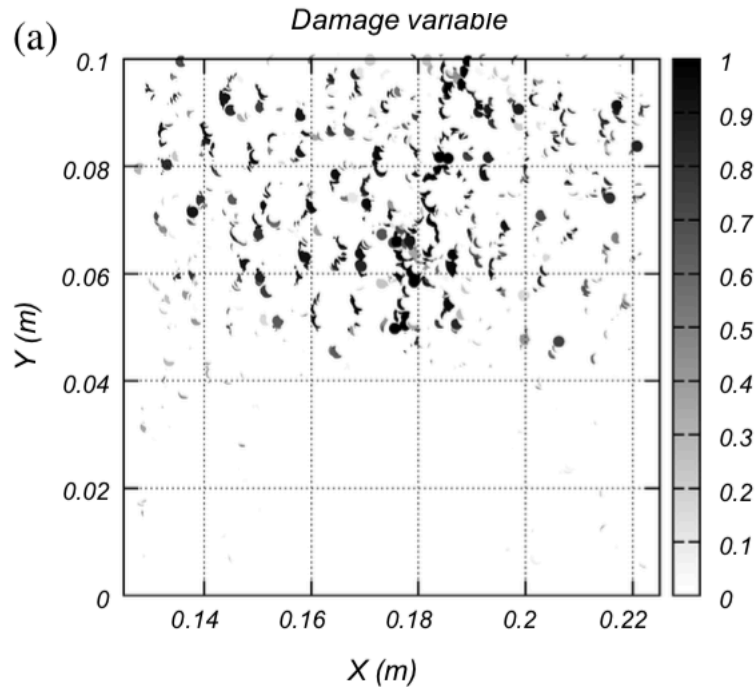
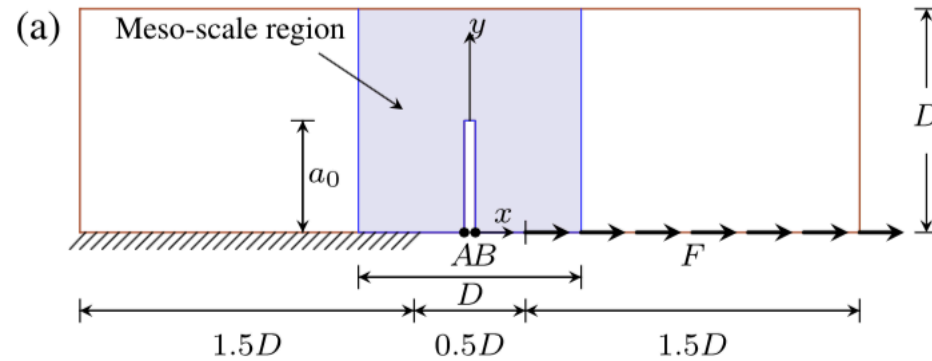
(b) Unnotched beam



(a) Half-Notched beam



- Direct tension



Outline

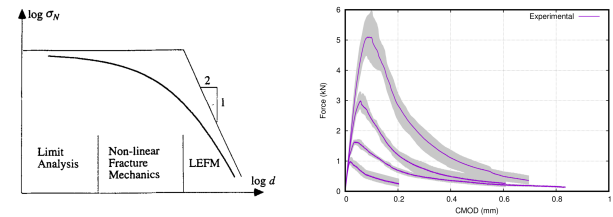
– Introduction and context: non-local models for quasi-brittle failure

– Indirect calibration methods

Pick load size effect laws

Softening curves of different specimen sizes (1, 3 or 4)

Example of calibration failure or success

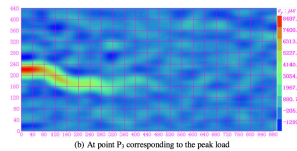


– Direct calibration method ?

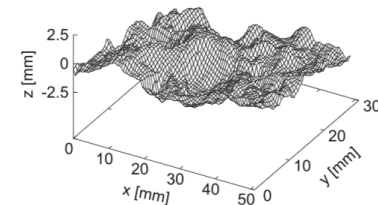
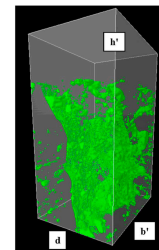
Digital image correlation

X-ray tomography

Fracture surface roughness



(b) At point P₁ corresponding to the peak load

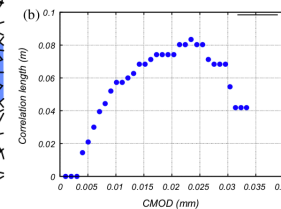
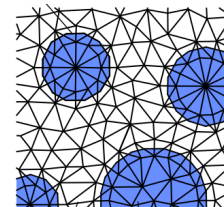
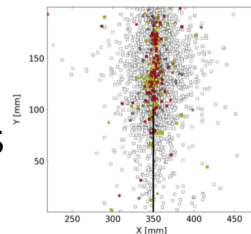


– Toward the calibration of an evolving characteristic length

Acoustic emission

Mesoscale modelling

Spatial ecology and Ripley's functions



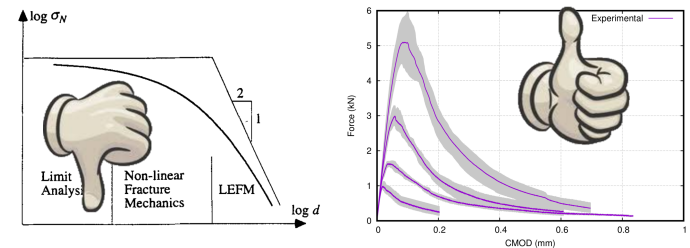
– Conclusion and perspectives



Conclusion

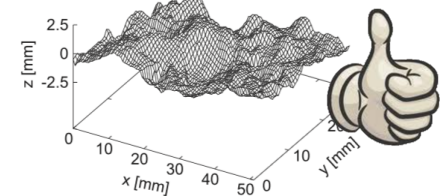
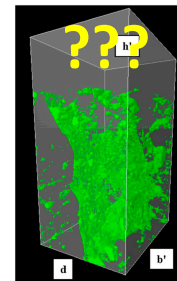
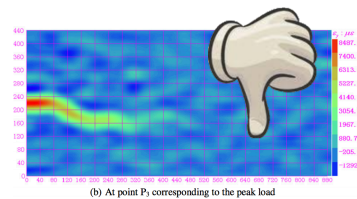
– Indirect calibration methods

- Only universal USEL law may be used but softening curves are far from preferable
- Comprehensive database now exist: PLEASE USE IT !!!



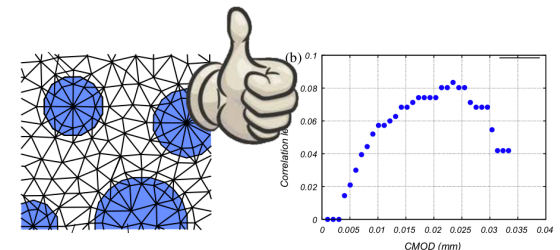
– Direct calibration methods

- Digital image correlation and X-ray tomography are not yet enough for calibration
- Fracture surface roughness seems very promising



– Toward the calibration of an evolving characteristic length

- The mesoscale approach is consistent globally and locally
- Ripley's functions provide indicators of the randomness of a distribution of events
- An varying characteristic length may be directly extracted using such Ripley's functions



• Non-exhaustive state of the art...

- (Alam et al., 2012) S. Alam, A. Loukili, and F. Grondin, "Monitoring size effect on crack opening in concrete by digital image correlation," *European Journal of Environmental and Civil Engineering*, vol. 16, pp. 818-836, 2012.
- (Bažant and Yu, 2009) Bažant Z, Yu Q. Universal size effect law and effect of crack depth on quasi-brittle structure strength. *Journal of Engineering Mechanics* 2009; 135:78.
- (Bažant, 1984) Z. P. Bažant, "Size effect in blunt fracture: concrete, rock, metal," *Journal of engineering mechanics*, vol. 110, pp. 518-535, 1984.
- (Delaplace et al., 1996) Delaplace, A., Pijaudier-Cabot, G., Roux, S., 1996. Progressive damage in discrete models and consequences on continuum modelling. *J. Mech. Phys. Solids* 44, 99–136.
- (Diggle, 1991) Diggle PJ, Chetwynd AG. Second-order analysis of spatial clustering for inhomogeneous populations. *Biometrics* 1991;47(3):1155–63.
- (Dixon, 2002) Dixon PM. Ripley's K function. In: El-shaarawi AH, Piegorisch WW, editors. *Encyclopedia of environmetrics*, vol. 3. Chichester: John Wiley & Sons, Ltd.; 2002. p. 1796–803.
- (Duncan, 1993) Duncan RP. Flood disturbance and the coexistence of species in a lowland podocarp forest, South Westland, New Zealand Author(s). *J Ecol* 1993;81(3):403–16.
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