PROPAGATING INSTABILITIES IN PERIODIC MEDIA

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Abstract

Under tension low carbon steels exhibit inhomogeneous deformation. This phenomenon driven by dislocations is called Lüders banding. This instability creates fronts of localized strain that propagate in the structure. To date, only simple geometries, sheets and tubes, have been studied. This emerging class of advanced materials provides new possibilities in terms of mechanical properties. This work deals with such materials with predetermined morphology in order to develop lightweight metallic structures with capability to localize deformation thanks to their geometry. We investigate the effect of the architecture on the global behavior of the entire structure. Especially, how bands can interact with a lattice and how to control initiation and propagation of localized strain with the architecture. Consequently, controlled local instabilities could lead to singular macroscopic behavior. Furthermore, we are concerned by studying the spread of instabilities in those periodic media.

1 Introduction

The need for enhanced material properties in engineering applications, such as high specific strength or large recoverable strain, led to the development of cellular materials. New means of production have accelerated the development of this class of materials for which topology is the key. A subclass of architectured materials are so-called lattice structures, which are made of an assembly of identical unit-cells. A lattice structure can be defined as a tessellation of unit-cells periodically distributed or not. In the following, we are only interested in 2D patterns. Numerous studies focused on the behavior of those architectures and they are classically divided into two different groups depending on their mode of deformation : (i) bending-dominated or (ii) stretch-dominated. (i) Bending-dominated geometries are mechanisms. An example is the hexagon cell Fig. 2.1 c. When loaded, it can deform thanks to the rotation of pin-joint and induces bending in the truss caused by rigidity of the joints. It exhibits low stiffness and low strength. (ii) Stretch-dominated geometries are structures according to [1]. An example is the triangle cell Fig. 2.1 a. When loaded, trusses are either in tension or in compression. Joints are only few solicited in rotation and the deformation is stretch-dominated. Those structures have higher stiffness or strength than bending-dominated lattices.

Lattices structures enhance the specific mechanical properties of their constituent materials thanks to their architectures and plastic deformation can interact with it. Plasticity in architectures has been studied widely with perfect elastoplastic models. A particular type of plastic instabilities is related to the microstructure for some materials, *e.g.* mild steel, aluminum alloy, etc. This is the so-called Piobert-Lüders phenomenon, which appears during the initialization of plasticity and causes localized plastic deformation as bands that propagates along specimens. To the knowledge of the authors, this problem has never addressed for lattice structures. For example, [**?**] studied, numerically and experimentally, the interaction between Lüders banding and buckling of steel bars. [3] emphasized the role of the propagation of Piobert-Lüders bands in the emergence of a propagating non-uniform curvature during the bending of steel tubes. Although the localized nature of the Piobert-Lüders deformation can not only lead to premature collapse if not well understand, it could actually be useful in the context of architectured materials. One can take advantage of these instabilities controlled through architecture. Because they manifest a macroscopic deformation, the entire structure could be shaped thanks to the propagation of localized plastic strain. In this article, we are interested in how Lüders phenomenon interact with 2D-lattices of different topologies covering both bending- and stretch-dominated architectures.

The purpose of the present work is to study the nucleation and propagation of Piobert-Lüders bands in a periodic media with specific architectures and analyze their effect on the macroscopic mechanical response of the structure. This paper is organized as follows. In the next section, the periodic boundary condition problem is formulated after a presentation of the topologies studied. Following this, large deformation framework for plasticity and an elastoplastic model for Piobert-Lüders banding are presented in Section 3. Numerical results and discussions are presented in Section 4. Macroscopic behavior of each chosen lattice is explained in the light of accumulate plastic strain maps. Concluding remarks are presented in Section 5.

2 Mechanical behavior of cellular materials with material instabilities

2.1 Geometries

This study focuses on the in-plane finite strain tensile response of three representative topologies : triangular, diamond and hexagonal as shown in Fig. 2.1. The large range of behavior justifies the choice of those topologies.
The Maxwell rule describes the condition for a structure to be static in term of the number of bars and joints. If the condition is not fulfilled, the lattice is considered a mechanism. According to [4] summarizing pin-jointed kinematics of selected planar trusses, triangular structure does not exhibit any mechanism because of a high nodal connectivity with $Z = 6$. On the contrary, diamond lattice has a lower nodal connectivity of $Z = 4$ presents a possible mechanism in particular direction. Last cell, the hexagon with a connectivity of $Z = 3$ exhibits deformation mechanism whatever the direction of solicitation. The triangular lattice is stiff and stretch-dominated. On the contrary, the diamond and hexagonal lattices are compliant and bending-dominated structures. Whereas when loaded in the direction of the cell walls, diamond lattice becomes stiff.

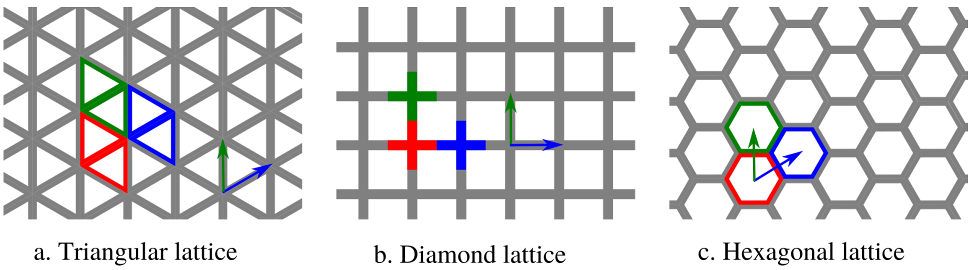

FIGURE 1 – Lattices topologies and their corresponding unit cell employed in FE analyse.

Those infinite media are periodic so they can be described from a unit cell tessellating the plane along to periodicity vectors. Figure 2.1 describes for each lattice the unit cell in red and the two vectors of periodicity in blue and green with the corresponding translated cells. Volumic fraction and the length of cell walls has been arbitrarily fixed to 30% and 1 respectively. Infinite periodic media is modeled from specific boundary conditions applied on the unit cell of each geometries.

2.2 Periodic boundary conditions

Investigated lattices are periodic in the two directions of the plane X and Y. Homogenized behavior of the structure found from the unit-cell with periodic boundary conditions. The purpose is to establish the resulting local strain field $\underset{\sim}{\varepsilon}(\underline{\boldsymbol{u}})$ and the local stress field $\underset{\sim}{\boldsymbol{\sigma}}(\underline{\boldsymbol{u}})$, when applying a boundary macroscopic strain $\underset{\sim}{\boldsymbol{E}}$. They are defined by the spatial average over the representative volume element.

$$\begin{aligned} \underset{\sim}{\boldsymbol{\Sigma}} &= \frac{1}{V}\int_V \underset{\sim}{\boldsymbol{\sigma}} dV \\ \underset{\sim}{\boldsymbol{E}} &= \frac{1}{V}\int_V \underset{\sim}{\boldsymbol{\varepsilon}} dV \end{aligned} \tag{1}$$

Because the geometry is periodic along the periodicity vectors, the solution fields $\underset{\sim}{\boldsymbol{\varepsilon}}$ and $\underset{\sim}{\boldsymbol{\sigma}}$ verify this property. Periodic problem over the unit cell describes the displacement field $\underline{\boldsymbol{u}}$ as the sum of a macroscopic part and a periodic fluctuation part :

$$\underline{\boldsymbol{u}} = \underset{\sim}{\boldsymbol{E}}.\underline{\boldsymbol{x}} + \underline{\boldsymbol{v}} \tag{2}$$

This decomposition is also verified for the strain field since $\underset{\sim}{\boldsymbol{\varepsilon}}$ derives from $\underline{\boldsymbol{u}}$. Applying the macroscopic strain $\underset{\sim}{\boldsymbol{E}}$, one computes the fluctuation vector on the unit cell. The uniform strain distribution E would be the strain of the medium if it were homogeneous and $\underline{\boldsymbol{v}}$ represents an in-plane periodic fluctuation due to local inhomogeneities of the material and in this case directly to architecture. Thus, the local strain and stress fields vary in a periodic manner about their mean value E and Σ. In terms of boundary conditions, periodicity means that on opposite sides of the cell displacement vectors $\underline{u}$ are periodic and stress vectors $\sigma.\underline{\boldsymbol{n}}$ are anti-periodic.

$$\forall(\underline{x}^-;\underline{x}^+) \in (\partial\Omega^-;\partial\Omega^+), \qquad \underline{\boldsymbol{v}}\,(\underline{\boldsymbol{x}}^-) = \underline{\boldsymbol{v}}\,(\underline{\boldsymbol{x}}^+) \tag{3}$$

$$\forall(\underline{\boldsymbol{x}}^-;\underline{\boldsymbol{x}}^+) \in (\partial\Omega^-;\partial\Omega^+), \qquad \underset{\sim}{\boldsymbol{\sigma}}(\underline{\boldsymbol{x}}^-).\underline{\boldsymbol{n}}\,(\underline{\boldsymbol{x}}^-) = -\underset{\sim}{\boldsymbol{\sigma}}(\underline{\boldsymbol{x}}^+).\underline{\boldsymbol{n}}\,(\underline{\boldsymbol{x}}^+) \tag{4}$$

2.3 Homogenized uniaxial mechanical responses

As explained in [5], the triangular lattice and hexagonal lattices possess a 60° rotational symmetry. They display isotropic behavior in their in-plane linear response, but anisotropy for non-linear behavior. While the diamond lattice is strongly anisotropic in both its linear and non-linear responses. Uniaxial tensile tests along X-direction were performed for each orientation in the symmetry range of the corresponding cell up to a 10 % of total strain. The three architectures are loaded in all possible directions in-plane. Thanks to the rotational symmetries, Y-direction test are not necessary. Therefore, it is interesting to study which direction of loading promote the propagation of plastic strain bands. More specifically, how in those topologies, either bending- or stretch-dominated, Lüders effect can be trigged and controlled by tension state of stress. In addition to this, the current study focuses of instabilites on the homogenized mechanical behavior of lattices. The mascroscopic uniaxial stress versus strain response is obtain for each of the three lattices in order to be linked to the propagation pattern of instabilities and its bending- or stretch-dominated behavior.

3 Simulation of the Piobert-Lüders phenomenon

3.1 A large deformation model for plasticity

The Piobert-Lüders instability characterizes the elastic to plastic deformation in low carbon steel. Macroscopic effect is the emergence and subsequent propagation of the plastic deformation bands. An elastoplastic material model is used in this work in order to simulate the Piobert-Lüders band formation and propagation. The model considers large deformation framework because of strain levels undergone by the different topologies studied in section 4. The large deformation method for isotropic nonlinear material behavior is based on a co-rotational transformation of the stress-strain problem into a local objective referential. This framework developed in [6, 7, 8] allows the extension of constitutive laws from infinitesimal strain to large deformation without modification. Simulation of Luders phenomenon relies a non-monotonic hardening function. Lüders band propagation occurs thanks to a softening-hardening behavior.

3.2 Numerical approach of the Piobert-Lüders phenomenon

Tsukahara and Iung [9] introduced a local behavior modeling the Piobert-Lüders behavior by FE method. It consists in a decription of the work-hardening material function as linear softening branch followed by a linear hardening branch. Later, [10] smoothen this behavior using an exponential local behavior also decreasing then increasing introduced here :

$$R(p) = R_0 + Q_1(1 - e^{-b_1 p}) + Q_2(1 - e^{-b_2 p}) + Q_3(1 - e^{-b_3 p}) \quad (5)$$

R_0 (MPa)	Q_1 (MPa)	b_1	Q_2 (MPa)	b_2	Q_3 (MPa)	b_3
100	-100	80	400	10	5	500

Table 1 – Parameters for the phenomenological plasticity model

The corresponding values of the parameters are given in Table 1. For elastic behavior, the Young modulus is $100MPa$ and Poisson's ratio is 0.3. For plastic behavior, an additional softening term $(Q_2; b_2)$ supplement a Voce strain hardening law described by the yield stress R_0 and the potential $(Q_1; b_1)$ with $Q_1 > 0$ and $Q_2 < 0$. Static strain aging is model by the negative coefficient Q_2. The third potential $(Q_3; b_3)$ is added to the initial model to round off the stress peak and helps for better convergence [11]. It leads to replicate phenomenologically formation and propagation of Lüders bands. In the case of the tension of a plate, the initial softening due to the negative potential induces plastic strain localization in a band giving a stress peak in the stress-strain curve. The following hardening behavior lead to the propagation of the band until the whole sample is filled. Macroscopically, it gives a plateau at a lower stress than the peak. After the propagation of the band throughout the specimen, homogeneous hardening takes place.

4 Mechanical properties of cellular structures with material instabilities

Full finite element solutions are reported for the elastoplastic response of the three lattices considered. A finite strain frame has been been considered, taking into account effects of large deformation with plasticity. Selected FE results are reported until a macroscopic strain load of 10% and the accumulative plastic strain (epcum) evolution is given for the most relevant orientations.

4.1 Triangle lattice

Global behavior : The macroscopic mechanical response for the triangular lattice has a global shape identical for each orientation, except level of stress and drop of stress for particular orientation 5°, 10° and 15°. The response exhibits an initial stiff and linear behavior corresponding to the linearly elastic regime of the material. The stress attains a local maximum value when appears the localization of the plastic strain. Then stress slowly decreases to reach a plateau, indicating that the propagation of plastic strain in the branches of the lattice has begun. Finally a hardening behavior takes place after the bands have fully crossed the structure.

Propagation of instabilities : The difference of stress level and the local instability which occurs for orientations 5°, 10° and 15° can be explained through the analyze of the proper propagation of Lüders instabilities in the architecture oriented differently. Lüders deformation initiates in the middle of the cell wall of the triangle and in the ones where the stress exceed yield stress. This nucleation is affected by the orientation. For orientation 0°, Lüders deformation appears in all branch at the same time. In fact, they all have the exact same orientation compared to the loading direction. For all other configurations, Lüders deformation appears, first, only in the external branches of the cell. Then, as the macroscopic stress loading rises, bands propagate along the branches symmetrically (Fig. 3). Once

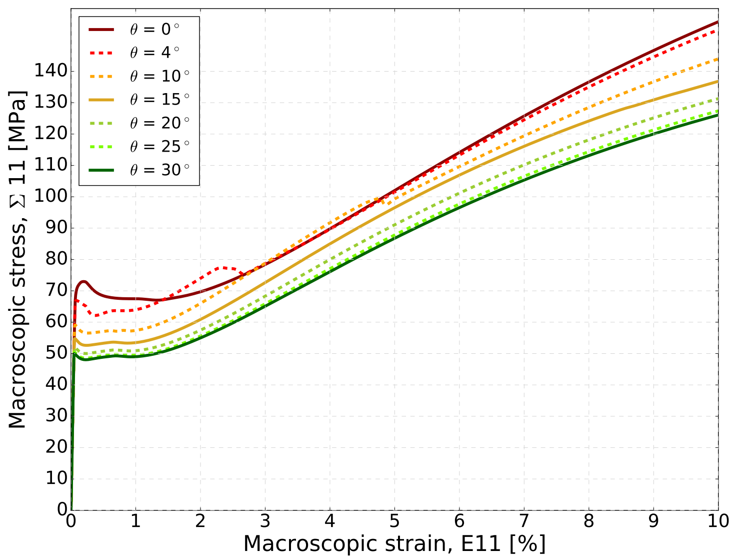

FIGURE 2 – Macroscopic behavior of the triangular lattice in all the in-plane directions.

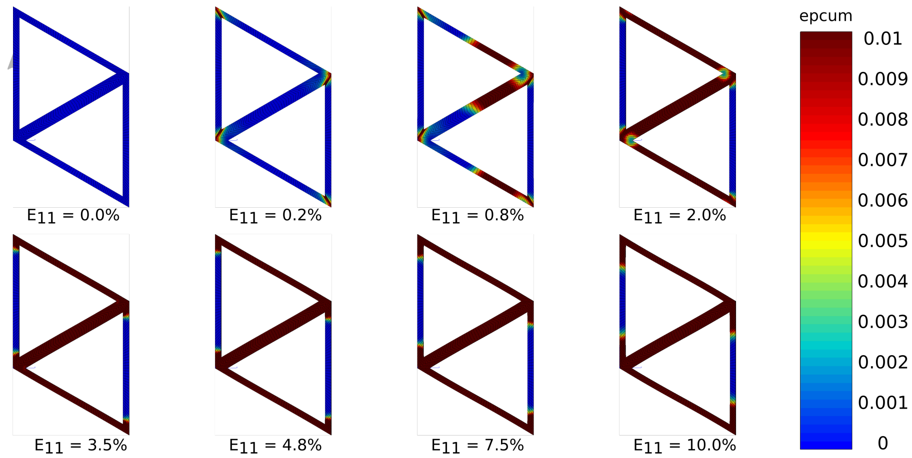

FIGURE 3 – Triangle cell loaded in X direction angle 0

all branches are fully localized, cell deforms macroscopically. Propagation and deformation of the cell explains the plateau stress followed by a hardening behavior on the macroscopic curves. Nonetheless, for orientations 5°, 10° and 15°, after macroscopic strain has increased from the first localization, plastic strain localizes an other time in the middle branch. This explains the instability in the third zone on the macroscopic stress-strain curve. Because of the hight-connectivity of the cell, the branches mainly solicited in tension propagates Lüders bands. For every orientation of the cell, there are always branches submitted to tension.

4.2 Diamond lattice

Global behavior : Diamond lattice behaves very differently from the triangular lattice and its both linear and non-linear behaviors are strongly anisotropic. From the macroscopic stress-strain curve, two extreme mechanical responses can be identified. The first one, corresponding to the orientation 0° is close to the localization-propagation behavior. When the second one for orientation 45°, relies on the elastic then plastic bending of the struts. The analyze of the diamond lattices is divided into two parts : stretching-dominated regime when struts are less than 5° disoriented from the loading direction and bending-dominated regime from 5° to 45°.

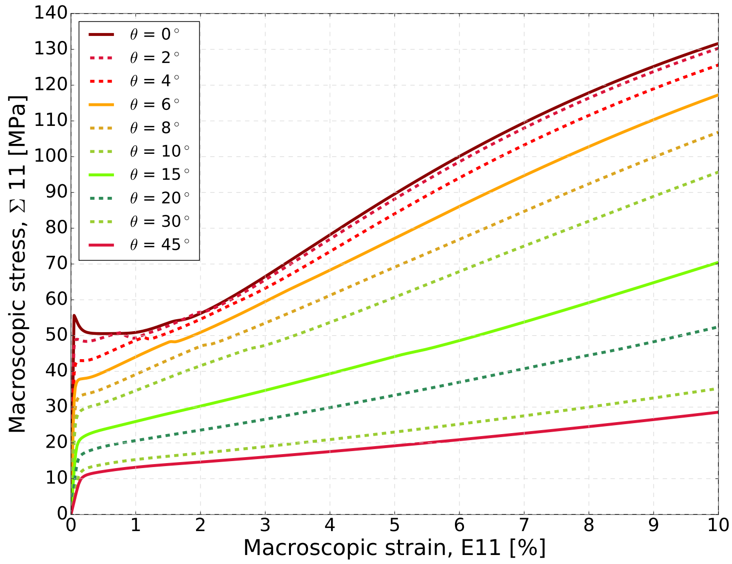

FIGURE 4 – Macroscopic behavior of the diamond lattice solicited in all the in-plane directions.

Bending-dominated regime from $5°$ **to** $45°$

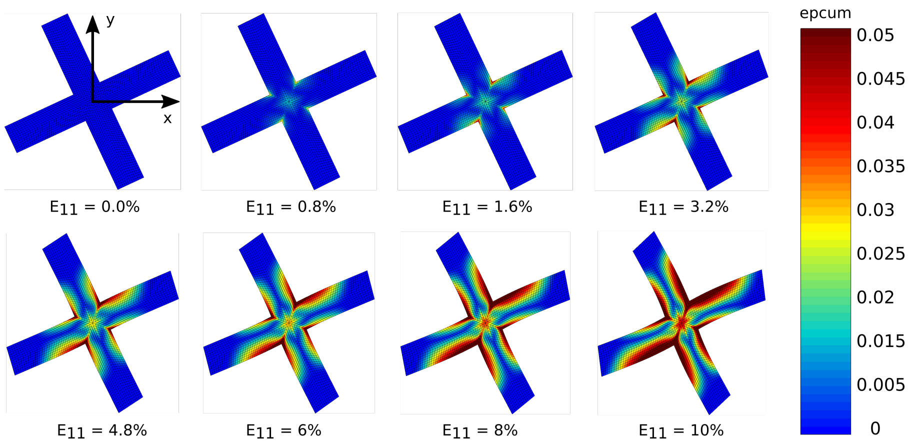

FIGURE 5 – Diamond lattice rotate from an angle of 25° and loaded in X direction

Two zones in the behavior of the diamond cell are described.
Zone 1 : Elastic bending of struts. FE analysis capture the elastic bending of each struts. The effective young modulus depends on the orientation of the branch to the loading direction.
Zone 2 : Plastic bending of struts. Plasticity propagates in the struts of the cell and elastic bending becomes plastic bending. The plastic part enhances a behavior hardened because of the alignment of cells walls with the loading direction. For the 45°-oriented cell, Young modulus and yield stress equal to $10MPa$. Both macroscopic yield stress and Young modulus increase as the cell bars are better oriented in the loading direction.
Propagation of Lüders instability : While struts undergo elastic bending, plasticity appears early in the corners at the joints where stress concentration are maximum. From there, localized plastic strain spread towards the tensioned border of the bended branches.

Stretching-dominated regime for $0°$
Cell walls are exactly aligned with the loading direction. No mechanism are solicited in the lattice, neither bending of the walls nor plastic hinges. Conditions are the closest from a tension test on a single plate made off material with Lüders instability. The same three zones are identified. *Zone 1 :* Stress peak. The initial response of the lattice is a linear part until the yield stress is passed meanwhile

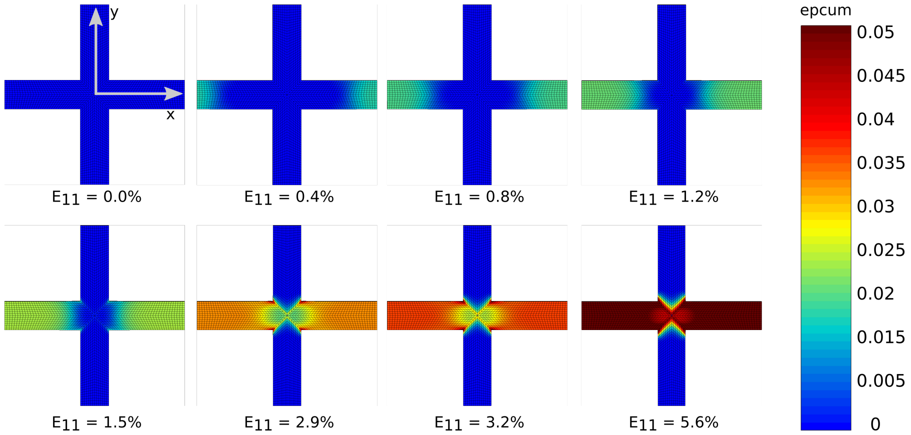

FIGURE 6 – Diamond lattice rotate from an angle of 0° and loaded in X direction

plastic strain localized in the thickness of the branch. This affect the localization which take the shape of a horizontal band.
Zone 2 : Stress plateau. Once the two bands of localized plastic strain, one in each half of the tensioned branch, they both propagate one towards the other until the all strut is plasticized. The results is a stress plateau on the macroscopic stress-strain curve.
Zone 3 : Hardening behavior of the lattice. After, the Lüders instability has propagated through the horizontal branches of the diamond lattice, the lattice undergoes a hardening behavior described by the constitutive law.

Stretching-dominated regime from 1° to 5°
Cell walls are in this case closely oriented in the loading direction. The three zones of the 0°-oriented case can be identified except some differences during the localization and propagation of Lüders instability. Those orientations achieve a harmonious transition between the two behaviors of the diamond lattice. The differences are explained by the disorientation with the loading direction that do not places the strut in pure tension but also in bending.

4.3 Hexagonal lattice

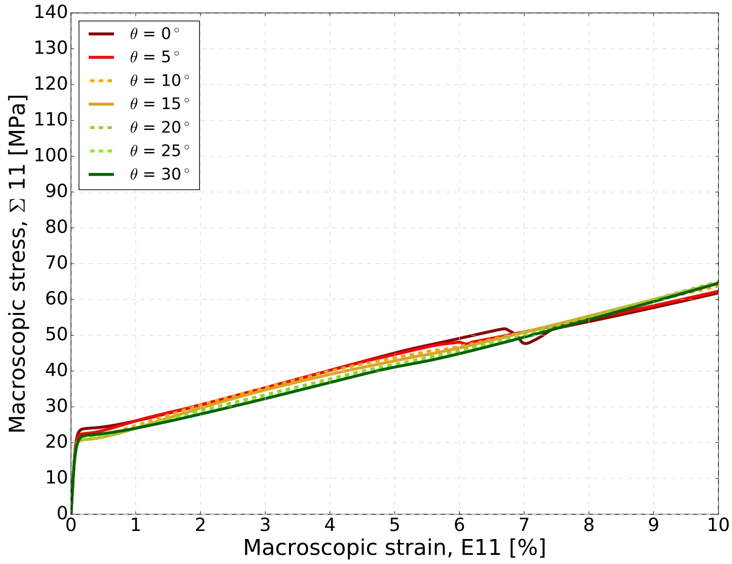

FIGURE 7 – Macroscopic behavior of the hexagonal lattice solicited in all the in-plane directions.

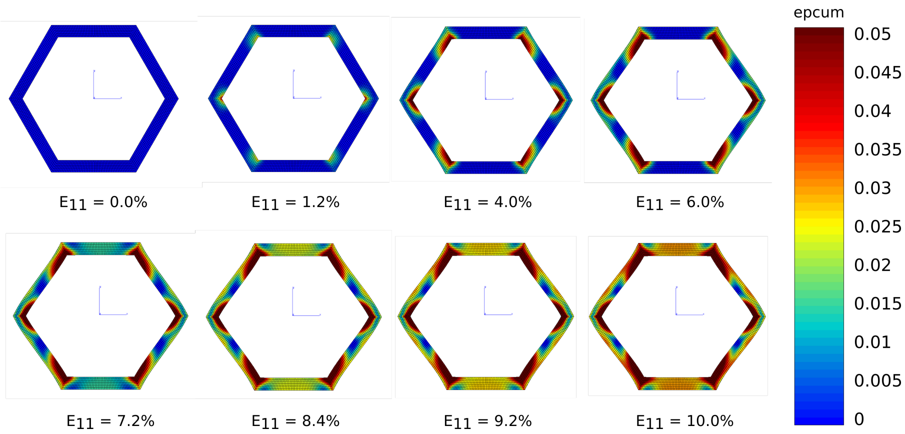

FIGURE 8 – Hexagonal lattice rotate from an angle of 0 degree and loaded in X direction

The isotropic behavior of the hexagonal cell in elasticity is verified while the plasticity shows instabilities depending on the orientation. Because of the connectivity of the hexagonal cell, this lattice is a mechanism. After a linear a response of the material identical for each direction, deformation becomes plastic. With conventional material, plasticity in hexagonal lattice is confined in the plastic hinges. Two phenomenons cause the deformation of the cell : the plastic bending of the cell edges and the stretching of the cell faces. The instability visible for the 0-oriented cell is due to the propagation of a Lüders band in the horizontal cell edges. During this propagation, the cell does not deform elsewhere than in the horizontal edges. The isotropic global behavior of hexagonal lattice is true except an inconspicuous anisotropy caused by material instability. In fact, the nucleation and the propagation of instabilities are influenced by the orientation of the cell. Mechanism behavior cells do not undergo important propagation of Lüders bands, proof is the global stress-strain curve that do not exhibits stress plateau.

5 Conclusion

The in-plane directions show non-linear behavior for each types of structures. It is the result of combined non-linear behaviors from the structure and from the material. Nucleation and propagation of localized plasticity are dependent not only of the architecture but also of its orientation. From the global stress-strain curves for the three different geometries, two Lüders instability behaviors can be identified : propagating and non propagating. Those different mechanisms can be related to the stretch- and bending-dominated analysis. A stretch-dominated structure will propagates bands through its cell walls, while a bending-dominated lattice will not.From the cumulated plastic strain maps, we can distinguish that non propagating Lüders bands undergo a important global deformation and exhibit a low stiffness. On the contrary, propagating cells show high stiffness.
Those findings can lead to control the initiation and the propagation of Lüders bands. The choice of the oriented architecture can be made with the objective to concentrate initiation of plasticity in specific region or to avoid plasticity from other region.

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