Introduction to thermodynamics of phase transformations

Mechanical aspects

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Motivations

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Transformations at the solid state: ubiquitous in processes

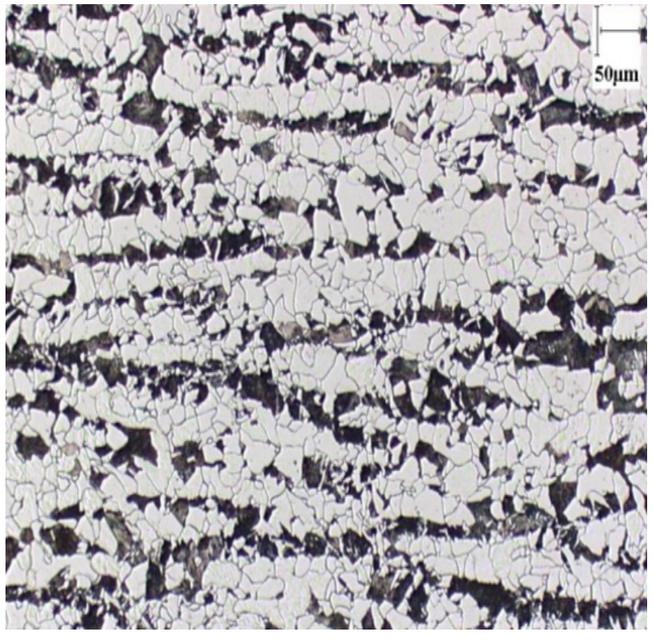
- Heat treatments
- Thermo-mechanical treatments
- Machining
- Additive manufacturing
- Surface treatments
- Hydride precipitation (storage, embrittlement)

Beyond processes: in alloy design (recycling), ageing (sustainability)

Motivations

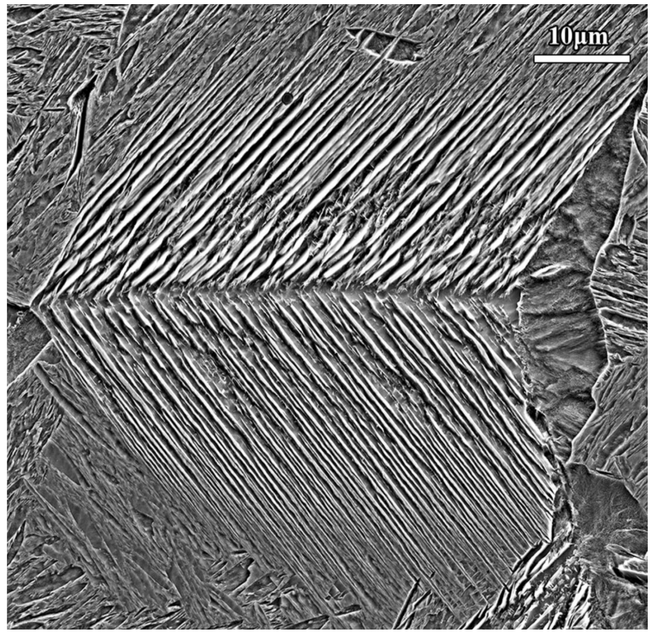
Transformations at the solid state: steels

Ferrite (Fe-C-Mn)



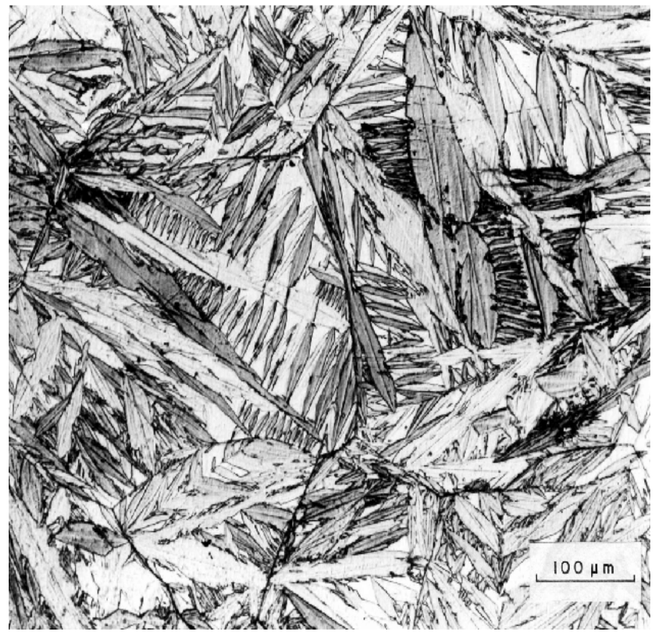
Gouné

Bainite (Fe-0.3C)



Yin, Hillert, and Borgenstam (2017)

Martensite (Fe-31Ni-0.002 C)

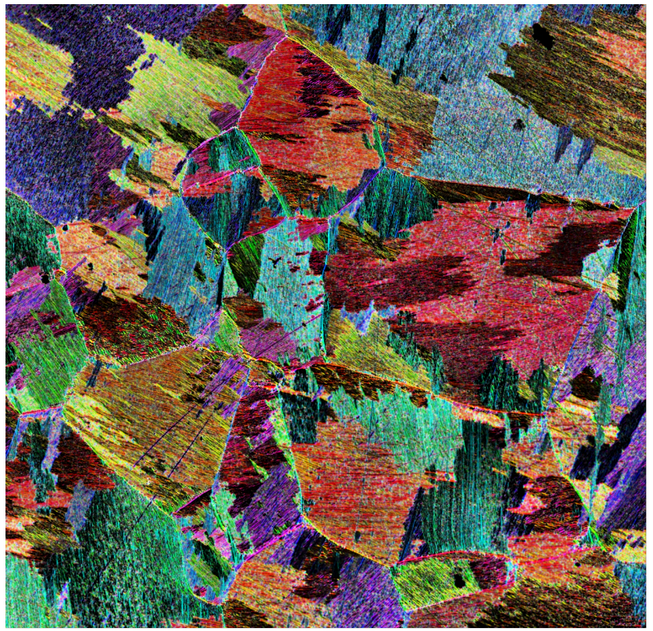


Guimarães and Rios (2015)

Motivations

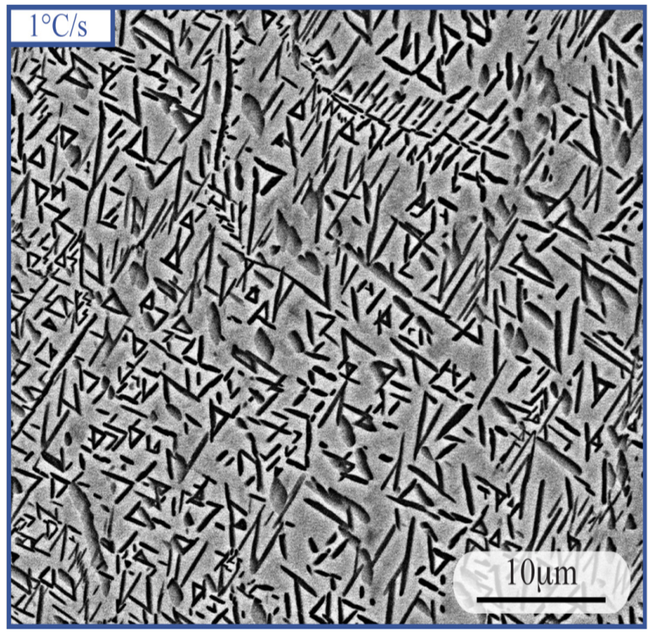
Transformations at the solid state: Ti alloys

Widmanstätten (Ti64)



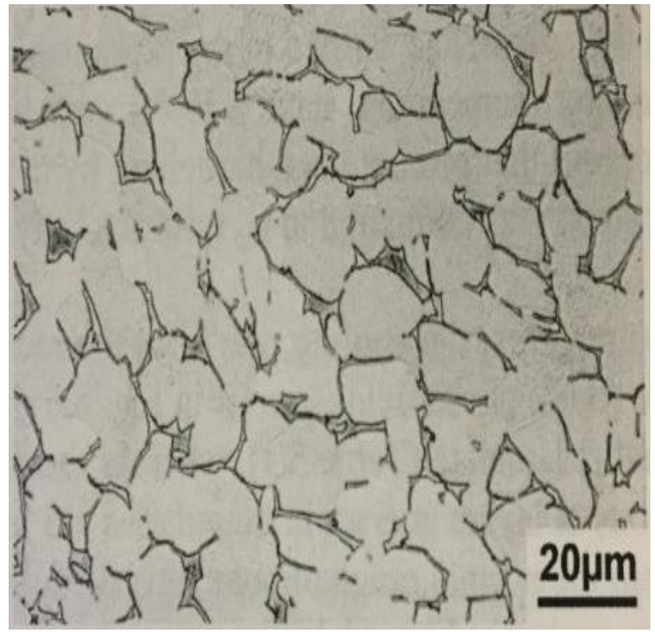
Denand (IJL)

Intragranular (Ti5553)



Settefrati (2012)

Globular (Ti6242)

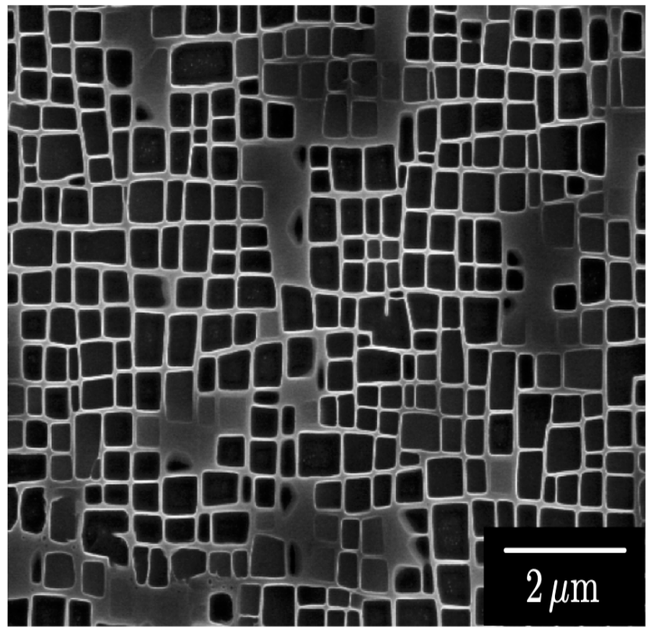


Lütjering and Williams (2003)

Motivations

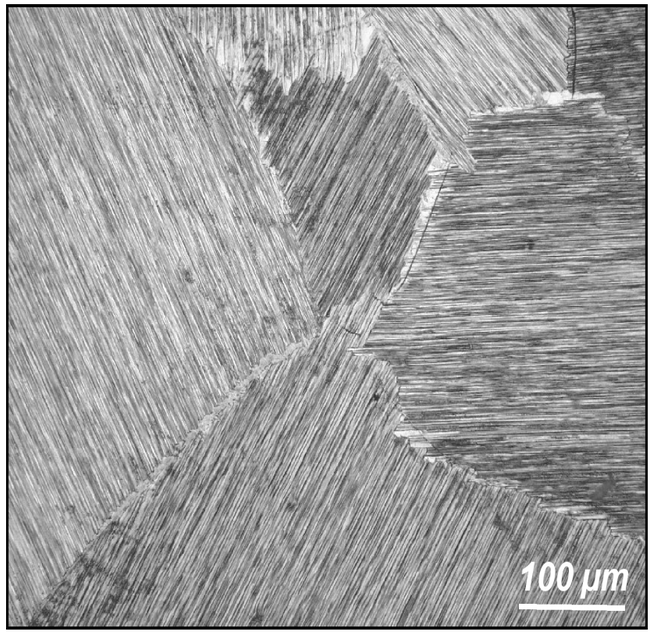
Transformations at the solid state: intermetallics and beyond

Ni-based superalloys



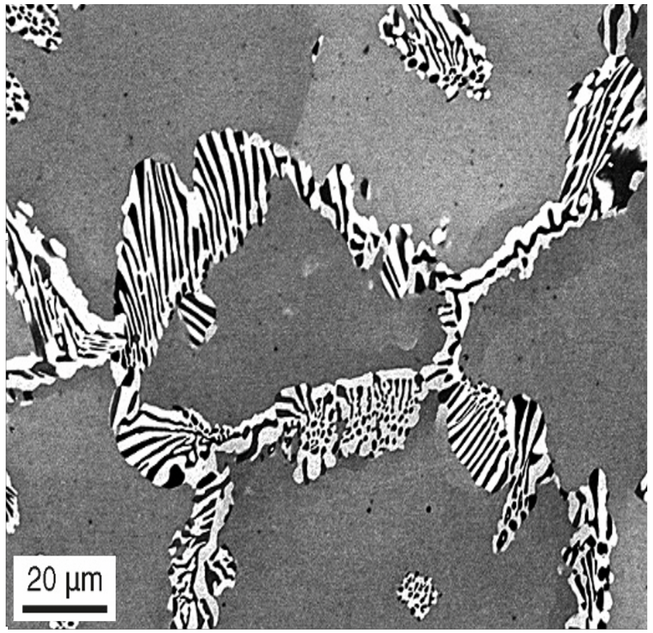
Diologent and Caron (2004)

Ti-Al



Charpentier, Hazotte, and Daloz (2008)

HEA



Poulain (ICMPE)

Motivations

Content

Basic thermodynamic concepts to investigate the role of mechanics on

1. Equilibrium
2. Interfaces
3. Kinetics
4. Morphologies

Many old but excellent (forgotten?) works

Equilibrium

Influence of stress on phase diagrams

Pure materials and displacive transformations

CuAlNi single crystal, from Sittner DoITPoMS, Univ. Cambridge

Influence of stress on phase diagrams

Pure materials and displacive transformations

Driving force for transforming: $\Delta G < 0 \implies T < T_{eq}$

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For alloys, T_{eq} is the T_0 temperature

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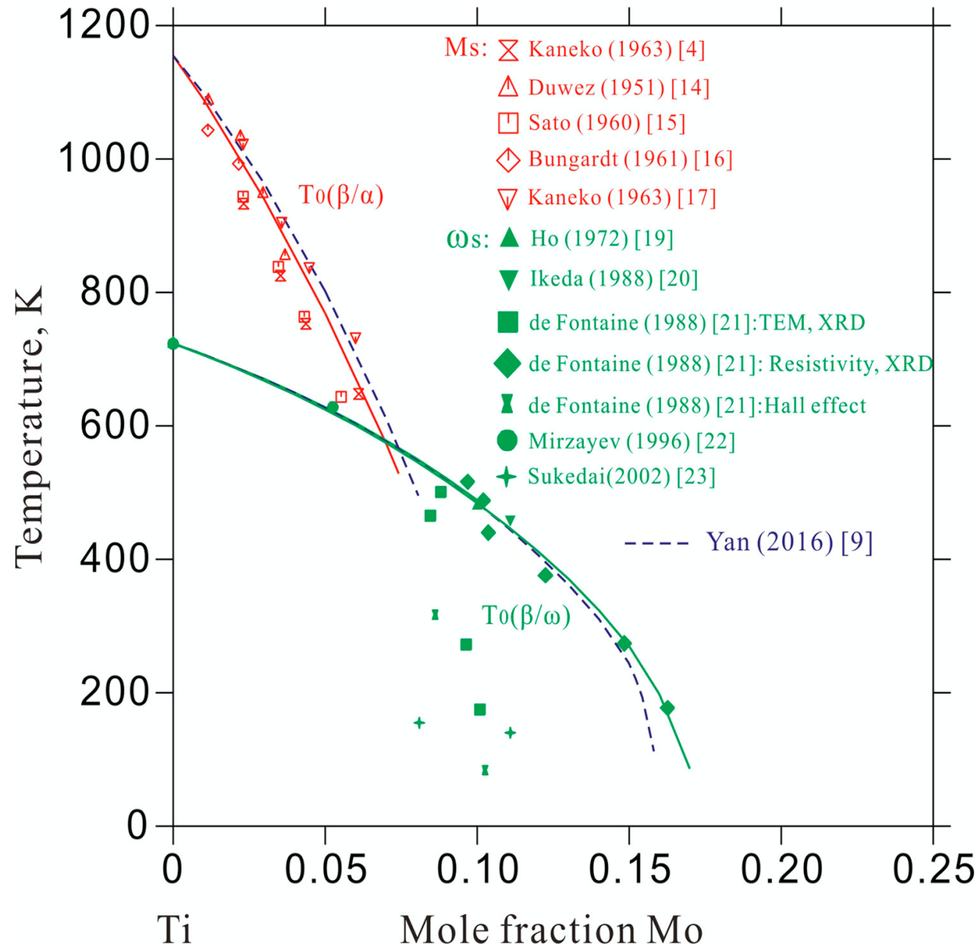


ω phase in Ti-Mo Hu et al. (2018)

Influence of stress on phase diagrams

Pure materials and displacive transformations

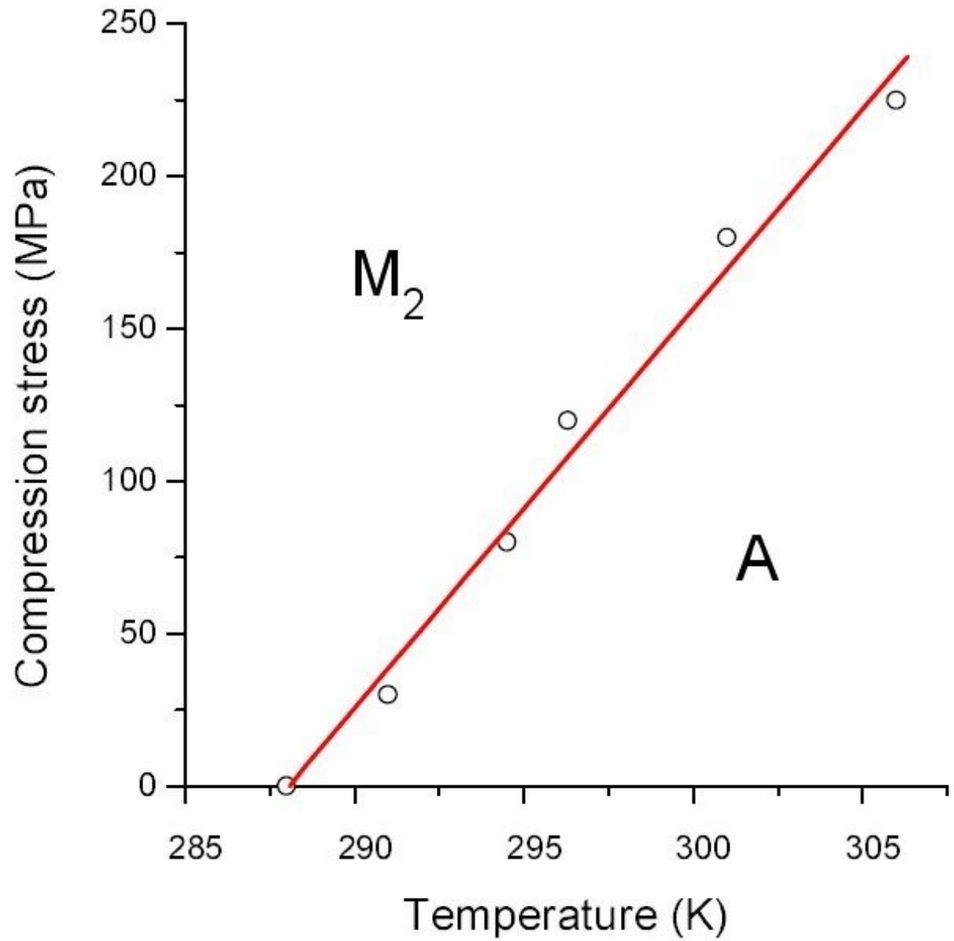
Driving force for transforming: $\Delta G < 0 \implies T < T_{eq}$

Influence of pressure

$$dg = -sdT + vdp$$

Clausius-Clapeyron

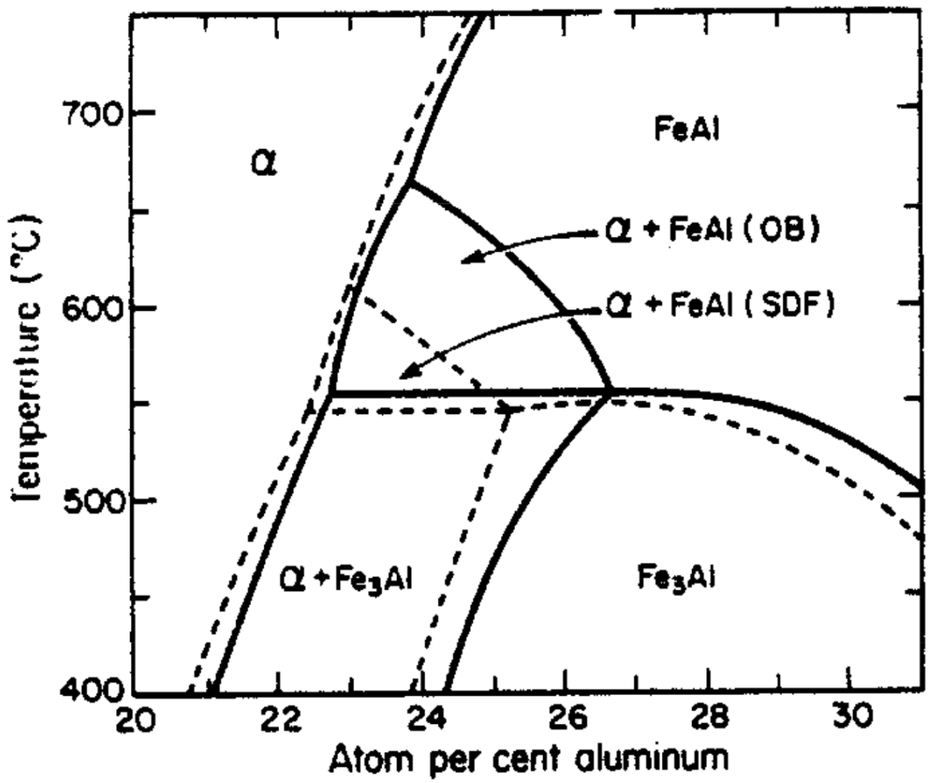
$$\left.\frac{dp}{dT}\right|_{eq} = \frac{\Delta s}{\Delta v} = \frac{\Delta h}{T_{eq}\Delta v}$$

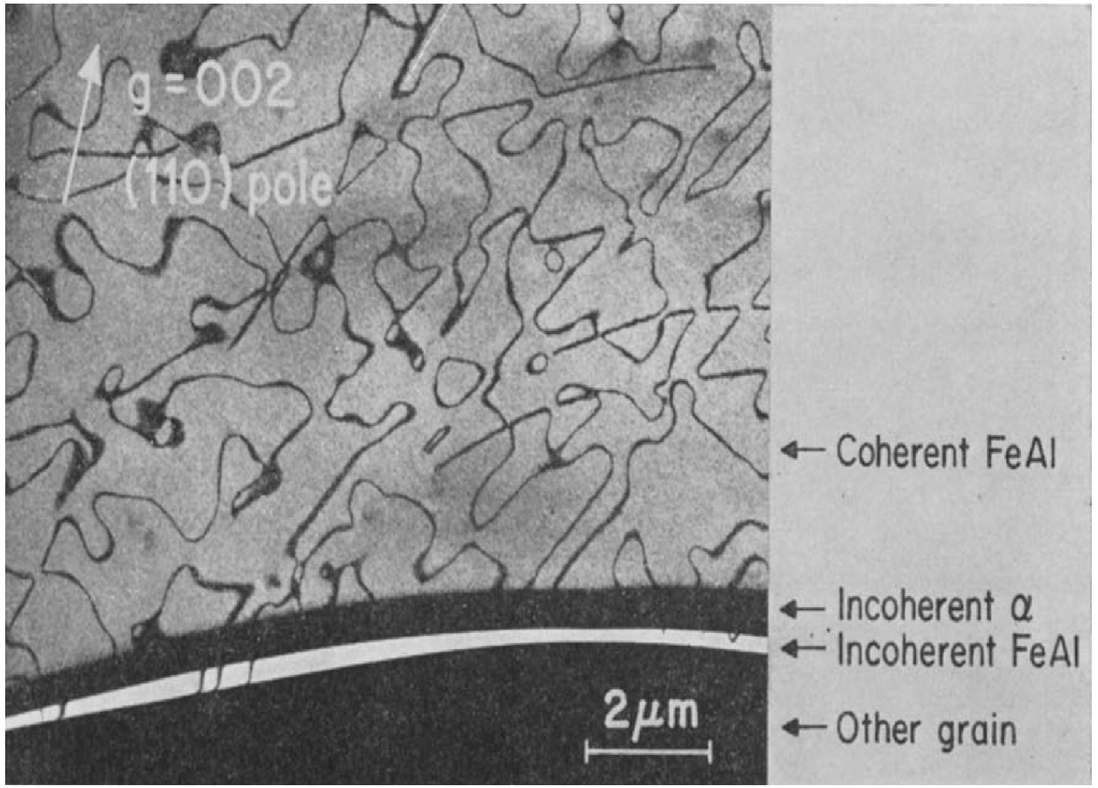


NiMnGa single crystal compressed along [110] Chernenko and L'vov (1999)

Influence of stress on phase diagrams

Diffusional *(here ordering)*





Allen and Cahn (1975)

Influence of stress on phase diagrams

We need simple estimates of stress and strain

In particular, should depend only on

- phase fractions
- concentrations, homogeneous in each phases

Rule out complex microstructures $\Longrightarrow$ simple microstructures

- laminates
- Eshelby ellipsoids

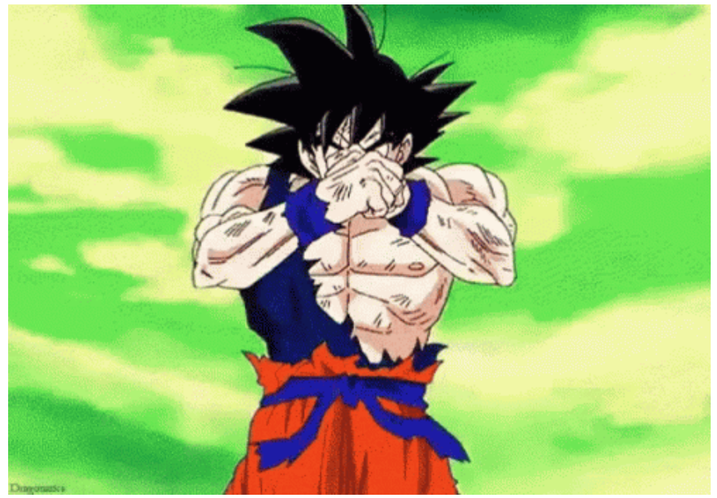
Indeed, simple strain and stress at least in one of the phase

Influence of stress on phase diagrams

Central concept: eigenstrain

Simplest case, much preferred in oldest works: isotropic

$$\varepsilon^\star = \varepsilon^0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



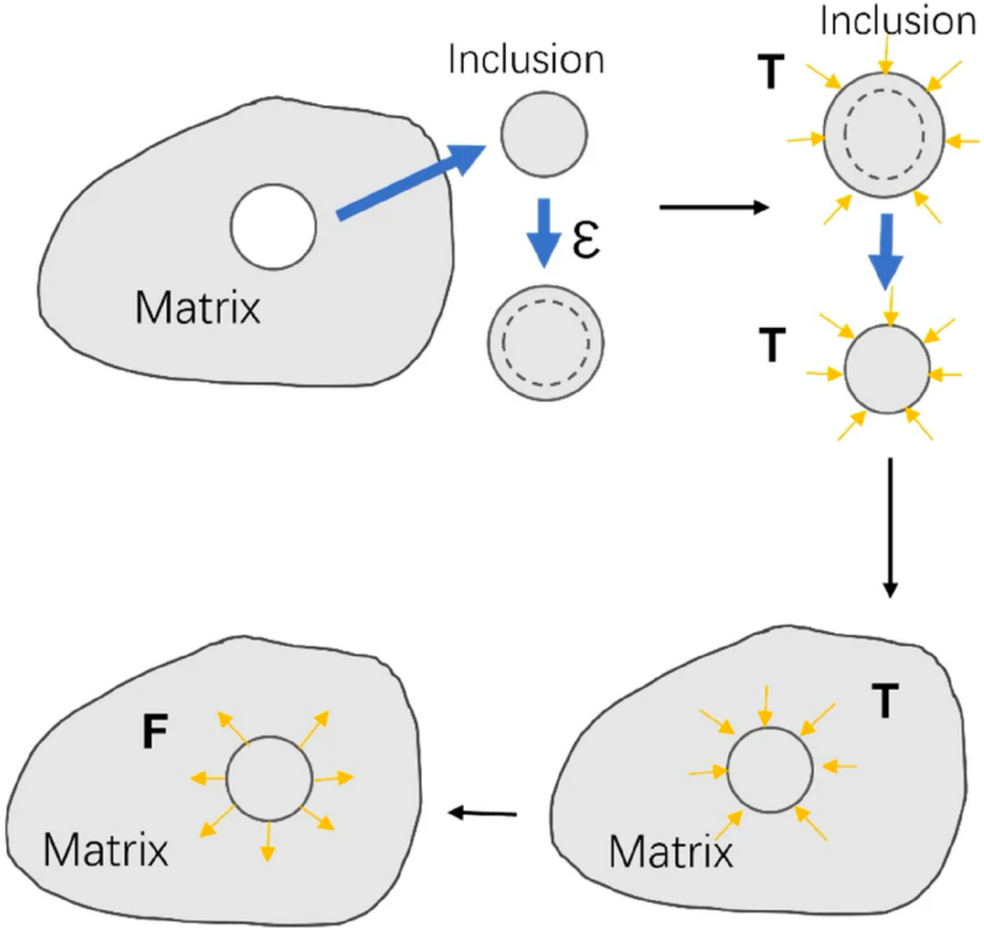
- Relevant for some ordering of cubic phases (bcc $\rightarrow$ B2, fcc $\rightarrow$ $L1_2$)
- Used very often for austenite $\rightarrow$ ferrite (although not the only contribution)

Influence of stress on phase diagrams

Inclusions

$$u_i(\vec{x}) = \int_\Sigma G_{il}(\vec{x} - \vec{x}\,') \, \sigma^{\star}_{lk} n_k \, d\Sigma(\vec{x}\,')$$

with $\sigma^{\star}_{ij} = C_{ijkl} \varepsilon^{\star}_{kl}$



D. Li et al. (2022)

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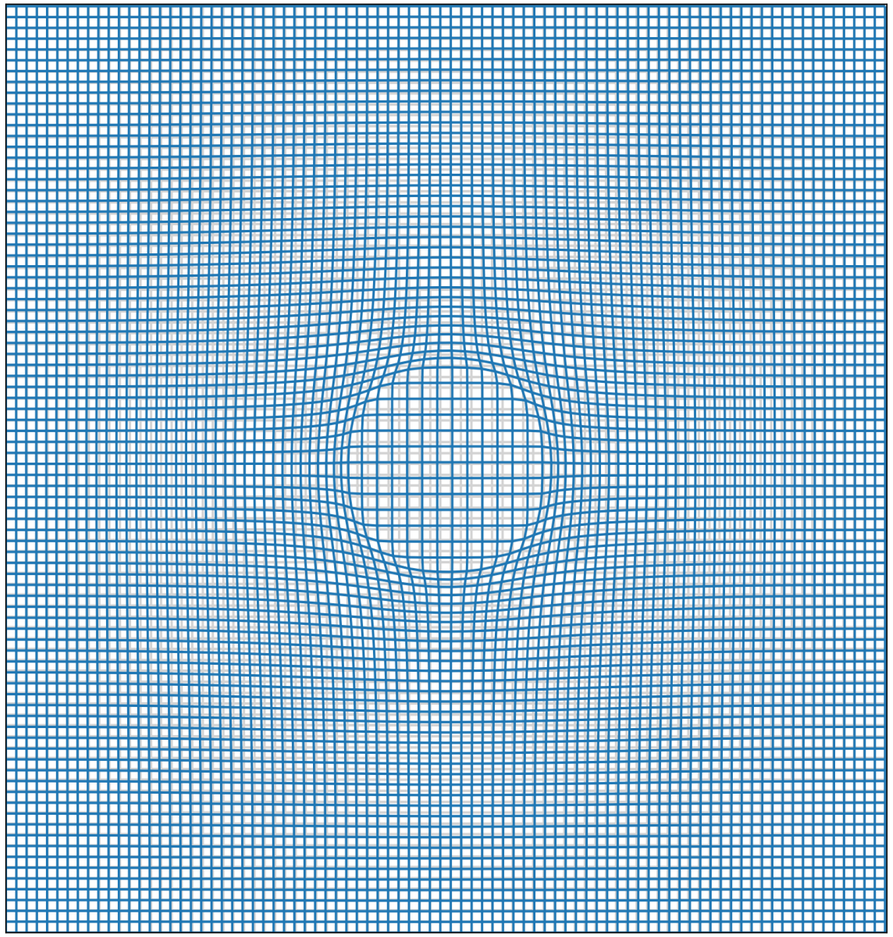
The magic Eshelby solution

strain and stress homogeneous inside the inclusion!

$$\varepsilon^p_{ij} = S_{ijkl} \varepsilon^\star_{kl}$$



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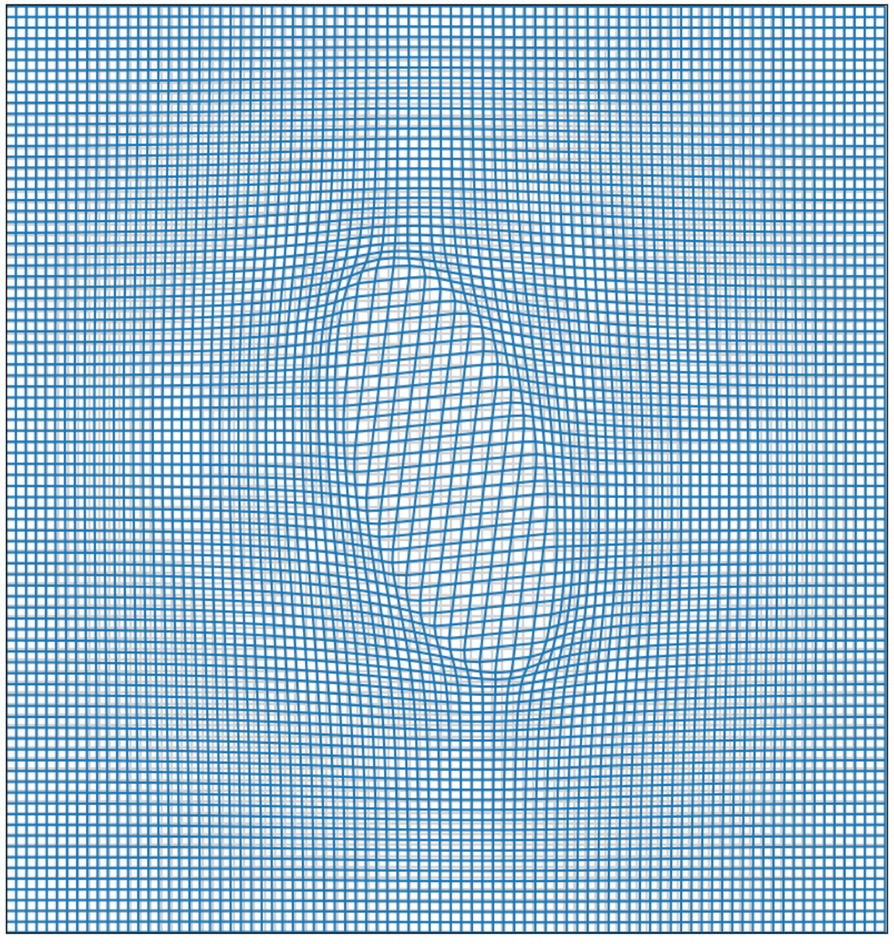

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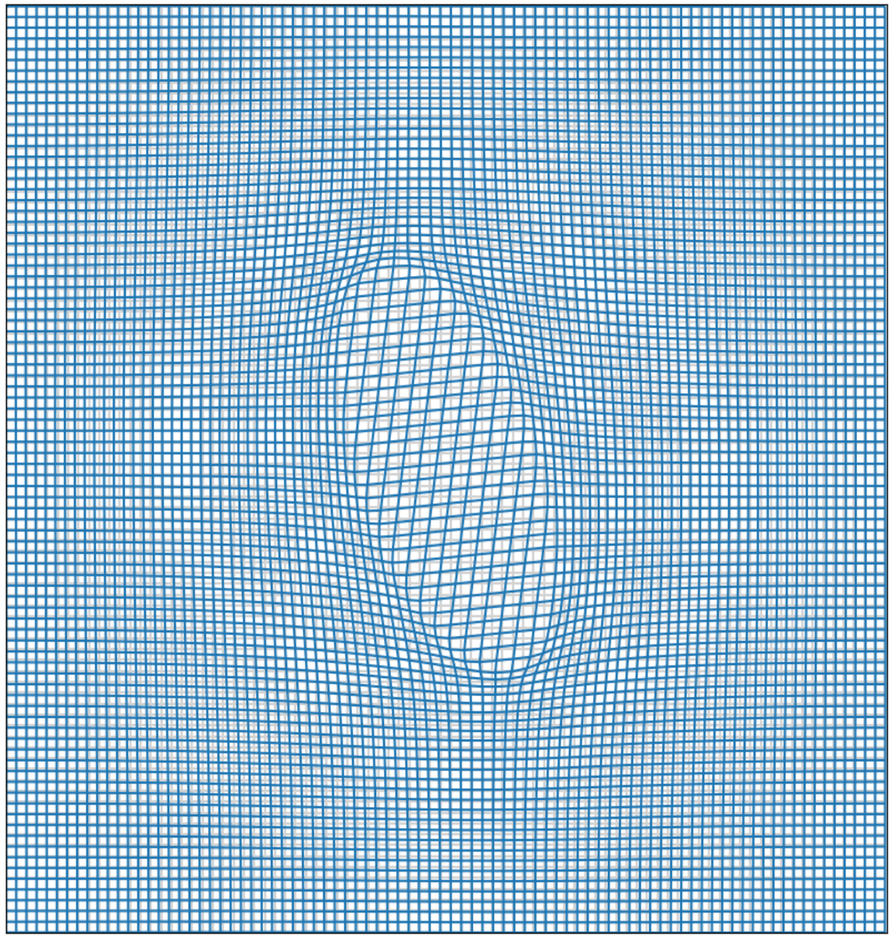
Elastic energy

$$\mathrm{E_{el}} = \frac{1}{2}\int_V \sigma_{ij}\varepsilon^{e}_{ij} dV = -\frac{1}{2}\int_{V_0} \sigma_{ij}\varepsilon^{\star}_{ij} dV = -\frac{V_0}{2}\sigma_{ij}\varepsilon^{\star}_{ij}$$



Influence of stress on phase diagrams

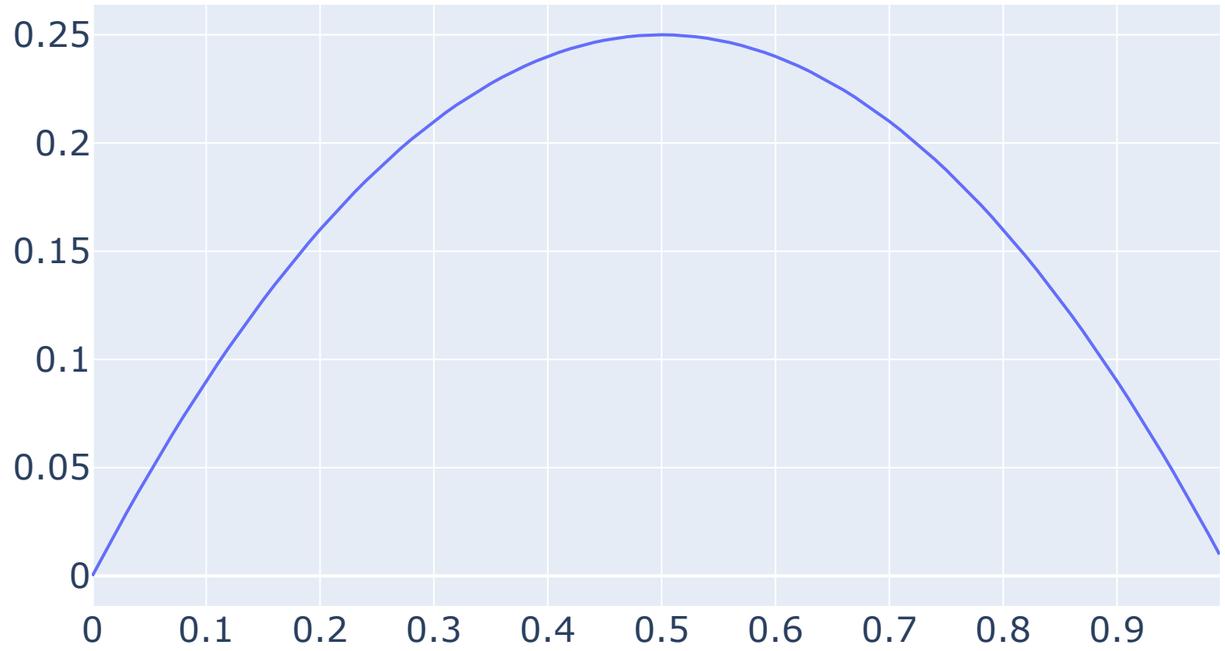
Elastic energy versus phase fraction

Bitter-Crum theorem e.g. Fratzl, Penrose, and Lebowitz (1999) or Khachaturyan (1983)

- homogeneous elastic constants
- homogeneous eigenstrain
- infinite system (vanishing average stress)
- isostropic $\varepsilon^\star$

$$f_{\text{el}} = z(1-z)\frac{E}{1-\nu}(\varepsilon^0)^2$$

It does not depend on the pattern, only on the phase fraction



Influence of stress on phase diagrams

Common tangent construction = constrained minimization of f

$$f = z f^{\alpha}(x) + (1 - z) f^{\beta}(y)$$

$$\mathrm{L} = f - \lambda \left(w - zx - (1 - z)y \right)$$

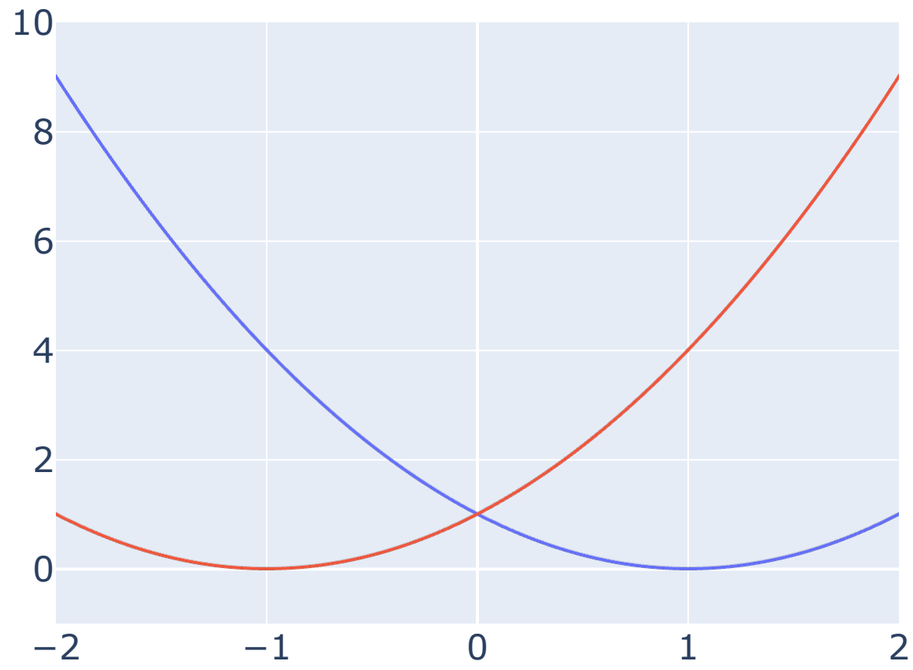
Extremize $\implies \delta \mathrm{L} = 0$

$$\frac{\partial \mathrm{L}}{\partial x} = 2(1 - x)z - \lambda z = 0$$

$$\frac{\partial \mathrm{L}}{\partial y} = 2(1 + y)(1 - z) + \lambda(1 - z) = 0$$

$$\frac{\partial \mathrm{L}}{\partial z} = (1 - x)^2 - (1 + y)^2 + \lambda(x - y) = 0$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = w - zx - (1 - z)y = 0$$



The result does not depend on the nominal concentration in binary alloys because it is linear with respect to z

Influence of stress on phase diagrams

Common tangent construction = constrained minimization of f

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$$\mathrm{L} = f - \lambda \left(w - zx - (1 - z)y \right)$$

Extremize $\implies \delta \mathrm{L} = 0$

$$\lambda = 2(1 - x)$$

$$\lambda = 2(1 + y)$$

$$\lambda = 0$$

$$z = \frac{1}{2} + \frac{w}{2}$$

The result does not depend on the nominal concentration in binary alloys because it is linear with respect to z

Influence of stress on phase diagrams

Common tangent construction = constrained minimization of f

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Extremize $\implies \delta \mathrm{L} = 0$

$$x = +1$$

$$y = -1$$

$$z = \frac{1}{2} + \frac{w}{2}$$

$$\lambda = 0$$

The result does not depend on the nominal concentration in binary alloys because it is linear with respect to z

Influence of stress on phase diagrams

What happens when elasticity is accounted for?

Williams (1984) Cahn and Larché (1984)

$$f = z f^{\alpha}(x) + (1 - z) f^{\beta}(y) + f_{\mathrm{el}}(z)$$

$$\mathrm{L} = f - \lambda (w - zx - zy)$$

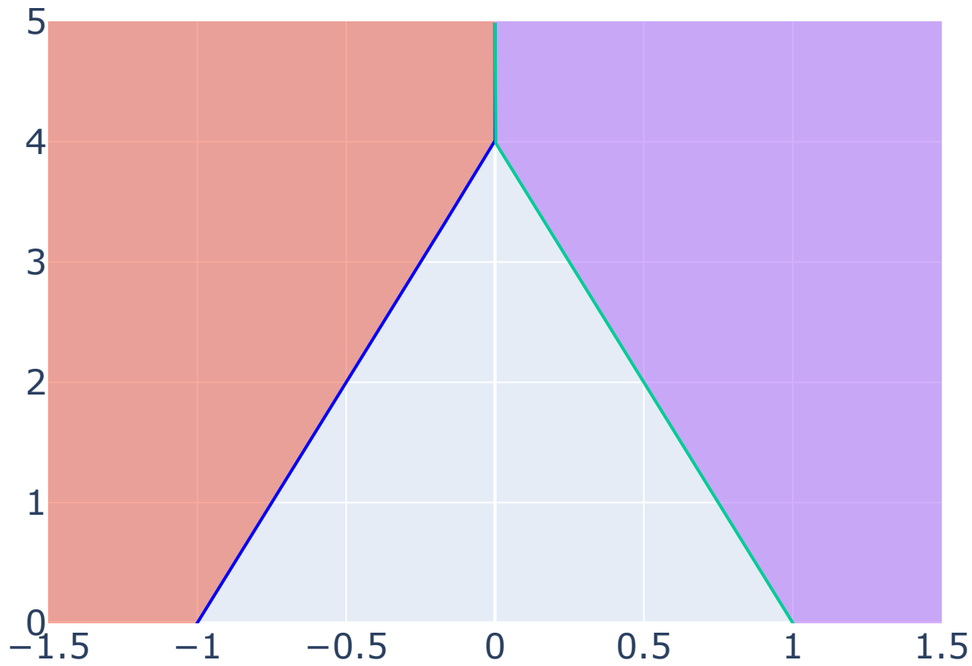
Extremize $\implies \delta \mathrm{L} = 0$

$$\frac{\partial \mathrm{L}}{\partial x} = 2(1 - x)z - \lambda z = 0$$

$$\frac{\partial \mathrm{L}}{\partial y} = 2(1 + y)(1 - z) + \lambda(1 - z) = 0$$

$$\frac{\partial \mathrm{L}}{\partial z} = (1 - x)^2 - (1 + y)^2 + \lambda(x - y) + A(1 - 2z) = 0$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = w - zx - (1 - z)y = 0$$



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$$\mathrm{L} = f - \lambda (w - zx - zy)$$

Extremize $\implies \delta \mathrm{L} = 0$

$$z = \frac{1}{2} + \frac{2w}{4 - A}$$

$$x = +1 - \frac{Aw}{4 - A}$$

$$y = -1 - \frac{Aw}{4 - A}$$

Influence of stress on phase diagrams

Questions/issues (personal opinion)

- Are the effects really significant?
- Should we throw out the low temperature parts of phase diagrams?
- Are the rules to read the phase diagrams no longer relevant?
- Is CalPhaD the Alpha and Omega of alloy design?
- Does annealing really remove all these effects?

Issues rising again with HEA (e.g. Lass (2022))

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Beyond (before) the phase diagrams

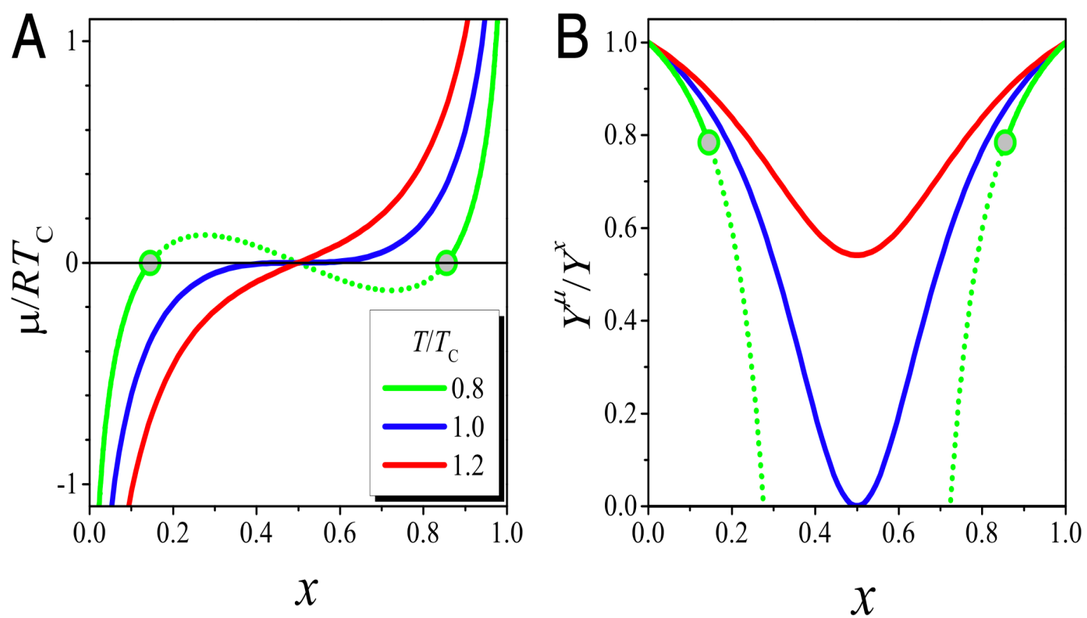
Open system elasticity Larché and Cahn (1985)

Verifying Larché–Cahn elasticity, a milestone of 20th-century thermodynamics

Shi, Markmann, and Weissmuller (2018)

$$G^{\mu} = G^{c}$$

$$E^{\mu} = \frac{E^{c}}{1 + \chi\eta^{2}E^{c}}$$



Interfaces

Influence of stress on interfaces

Dislocation emission from an austenite lath growing in a ferrite matrix Du, Mompiou, and Zhang (2017)

Influence of stress on interfaces

Variational calculation of Larché and Cahn (1978) see also Voorhees and Johnson (2004)

Insulated system with 2 phases containing 2 substitutional species joined at a coherent interface

Minimize (everything in the reference frame)

$$\mathrm{E} = \int_{V^\alpha} e^\alpha(F^\alpha, s^\alpha, c_A^\alpha, c_B^\alpha)\, dV + \int_{V^\beta} e^\beta(F^\beta, s^\beta, c_A^\beta, c_B^\beta)\, dV + \int_\Sigma e^\Sigma(s^\Sigma)\, d\Sigma$$

constrained by:

- constant total entropy (insulated system) $\mathrm{S} = \int_{V^\alpha} s^\alpha\, dV + \int_{V^\beta} s^\beta\, dV + \int_\Sigma s^\Sigma\, d\Sigma$
- constant numbers of A and B atoms (no interface segregation) $N_i = \int_{V^\alpha} c_i^\alpha\, dV + \int_{V^\beta} c_i^\beta\, dV$

Lagrangian to extremize

$$\delta\mathrm{L} = \delta\mathrm{E} - \theta\,\delta\mathrm{S} - \mu_A\,\delta N_A - \mu_B\,\delta N_B = 0$$

with θ, μ_A and μ_B Lagrange multipliers

Influence of stress on interfaces

Variational calculation of Larché and Cahn (1978) see also Voorhees and Johnson (2004)

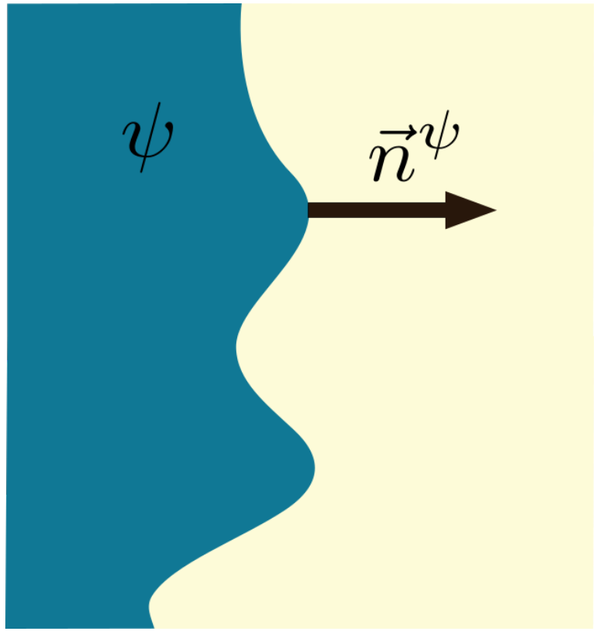
For any A and ψ

$$\delta \mathrm{A}^{\psi} = \int_{V^{\psi}} \delta a^{\psi}\, dV + \int_{\Sigma} a^{\psi}\, \delta y^{\psi}\, d\Sigma$$

For the interface Σ

$$\delta \mathrm{A}^{\Sigma} = \int_{\Sigma} \delta a^{\Sigma}\, d\Sigma + \int_{\Sigma} a^{\Sigma}\, 2\bar{\kappa}\, \delta y^{\beta}\, d\Sigma$$

for β convex



$$\delta y^{\psi} = \delta \vec{y}^{\psi} \cdot \vec{n}^{\psi}$$

Influence of stress on interfaces

Variational calculation of Larché and Cahn (1978) see also Voorhees and Johnson (2004)

First variation of internal energy (any phase)

$$\delta e^{\psi} = \theta^{\psi}\,\delta s^{\psi} + T^{\psi}_{ij}\,\delta F^{\psi}_{ij} + M^{\psi}_{AV}\delta c^{\psi}_{A} + M^{\psi}_{BV}\delta c^{\psi}_{B}$$

$$\delta e^{\Sigma} = \theta^{\Sigma}\,\delta s^{\Sigma}$$

- where diffusion of A and B is mediated by vacancies V
- where the Gibbs dividing surface eliminates $\partial e^{\Sigma}/\partial \kappa_i$

The mechanical work can be integrated by parts

$$\int_{V^{\psi}} T^{\psi}_{ij}\delta F^{\psi}_{ij}\,dV = \int_{\Sigma} T_{ij}\,\delta u_i n_j\,d\Sigma - \int_{V^{\psi}} \frac{\partial T_{ij}}{\partial x_j}\delta u_i\,dV$$

Influence of stress on interfaces

Variational calculation of Larché and Cahn (1978) see also Voorhees and Johnson (2004)

Putting all together and because all variations are independent

$$0 = \int_{V^\psi} dV \left[(\theta^\psi - \theta)\, \delta s^\psi + (M^\psi_{AV} - \mu_A)\, \delta c^\psi_A + (M^\psi_{BV} - \mu_B)\, \delta c^\psi_B - T^\psi_{ij\,,j} \delta u^\psi_i \right]$$

$$0 = \int_\Sigma d\Sigma (\theta^\Sigma - \theta)\, \delta s^\Sigma$$

$$0 = \int_\Sigma d\Sigma \left[\omega^\alpha\, \delta y^\alpha + \omega^\beta\, \delta y^\beta + 2\bar{\kappa}\, \gamma\, \delta y^\beta + T^\alpha_{ij}\, n^\alpha_j \delta u^\alpha_i + T^\beta_{ij}\, n^\beta_j \delta u^\beta_i \right]$$

with grand potentials $\omega^\psi = e - \theta s^\psi - \mu_A c^\psi_A - \mu_B c^\psi_B$ and interface energy $\gamma = e^\Sigma - \theta^\Sigma$

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with grand potentials $\omega^\psi = e - \theta s^\psi - \mu_A c^\psi_A - \mu_B c^\psi_B$ and interface energy $\gamma = e^\Sigma - \theta^\Sigma$

Usual equilibrium conditions

$$\theta^\alpha = \theta^\beta = \theta^\Sigma = \theta$$

$$M^\alpha_{AV} = M^\beta_{AV} = \mu_A$$

$$M^\alpha_{BV} = M^\beta_{BV} = \mu_B$$

$$\frac{\partial T^\psi_{ij}}{\partial x_j} = 0 \quad \forall\, \psi$$

Influence of stress on interfaces

Variational calculation of Larché and Cahn (1978) see also Voorhees and Johnson (2004)

At the interface

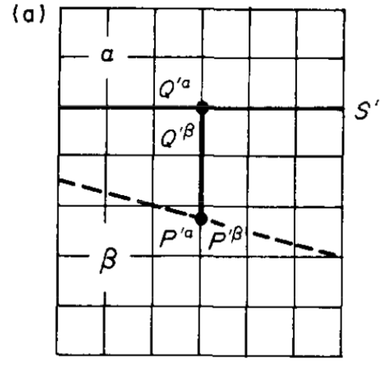
$$0 = \int_\Sigma d\Sigma \left[\omega^\alpha \delta y^\alpha + \omega^\beta \delta y^\beta + 2\bar{\kappa}\gamma \delta y^\beta + T_{ij}^\alpha n_j^\alpha \delta u_i^\alpha + T_{ij}^\beta n_j^\beta \delta u_i^\beta \right]$$

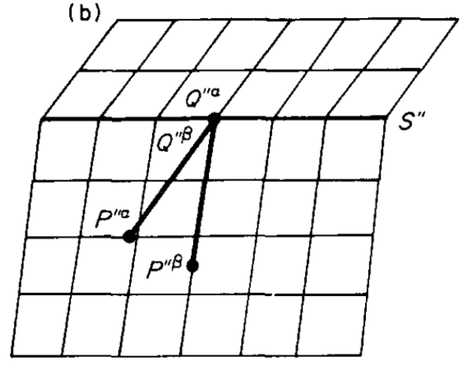
- We have $\delta y^\alpha = -\delta y^\beta$
- and the $\delta\vec{u}^\psi$ are not independent

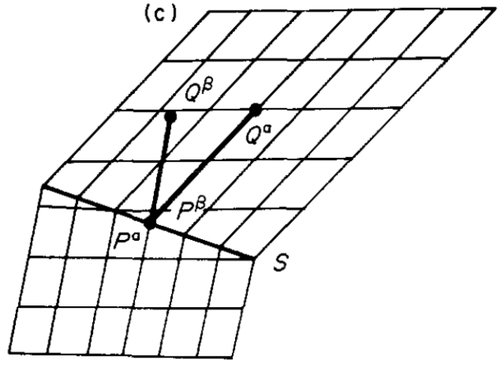
For a coherent interface $\delta u_i^\alpha = \delta u_i^\beta + \left(F_{ij}^\beta - F_{ij}^\alpha \right) n_j^\beta \delta y^\beta$

$$0 = \int_\Sigma d\Sigma \left[T_{ij}^\alpha n_j^\alpha + T_{ij}^\beta n_j^\beta \right] \delta u_i^\alpha$$

$$0 = \int_\Sigma d\Sigma \left[\omega^\beta - \omega^\alpha + T_{ij}^\alpha n_j^\alpha \left(F_{ik}^\beta - F_{ik}^\alpha \right) n_k^\beta + 2\bar{\kappa}\gamma \right] \delta y^\beta$$







Influence of stress on interfaces

Variational calculation of Larché and Cahn (1978) see also Voorhees and Johnson (2004)

Finally

$$T_{ij}^{\alpha} n_j^{\beta} = T_{ij}^{\beta} n_j^{\beta}$$

$$\omega^{\beta} - \omega^{\alpha} - T_{ij}^{\beta} n_j^{\beta} \left(F_{ik}^{\beta} - F_{ik}^{\alpha} \right) n_k^{\beta} + 2\bar{\kappa}\gamma = 0$$

Small strain version

$$\sigma_{ij}^{\alpha} n_j^{\beta} = \sigma_{ij}^{\beta} n_j^{\beta}$$

$$\omega^{\beta} - \omega^{\alpha} - \sigma_{ij}^{\beta} \left(\varepsilon_{ij}^{\beta} - \varepsilon_{ij}^{\alpha} \right) + 2\bar{\kappa}\gamma = 0$$

- The grand potential contains the elastic energy
- Extra energy required to keep coherency upon accretion

Influence of stress on interfaces

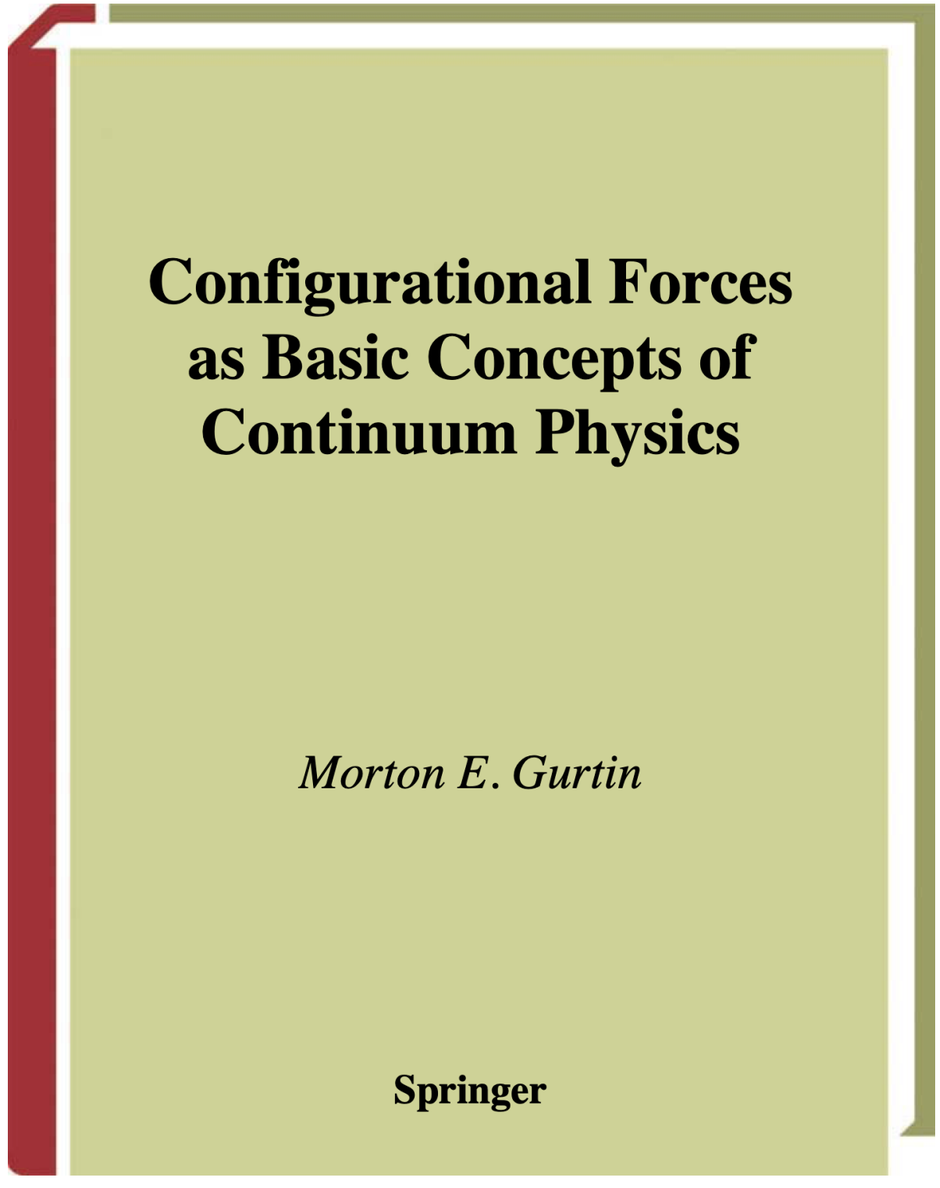
Beyond variational calculations

1. In fact $\omega\delta_{ij} - F_{ki}T_{kj}$ is the energy-momentum Eshelby tensor (Peach-Kohler, J integral) $\Longrightarrow$ configurational mechanics

   Read Gurtin and Voorhees (1996), Gurtin (2000), Maugin (2011) and Fischer et al. (2008)

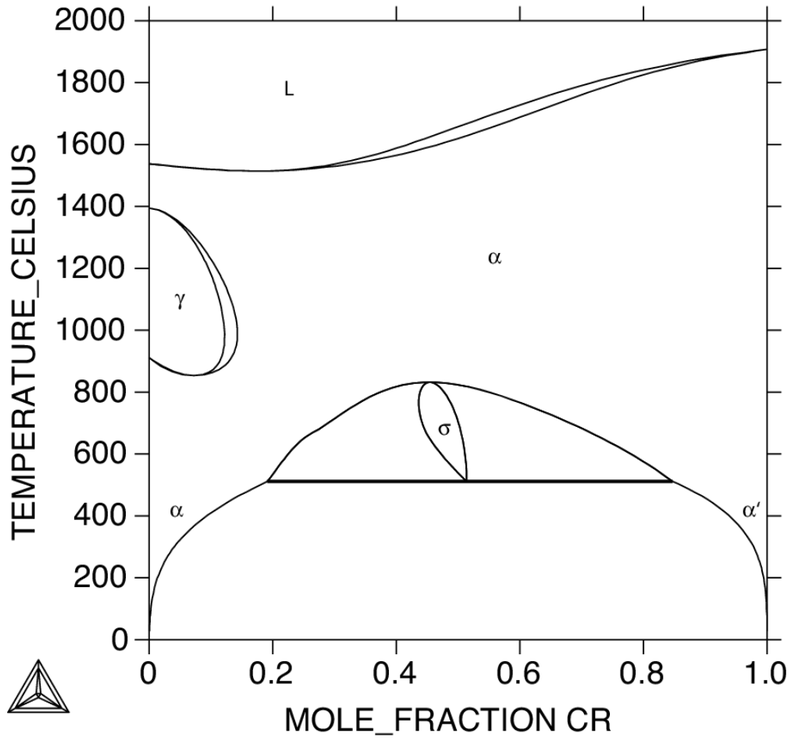
2. Nonetheless, computing the dissipation combining 1st and 2nd principles with a moving sharp interface is sufficient for finding the usual interfacial constitutive laws

Talk to J.F. Ganghoffer (in the room) and read his old papers

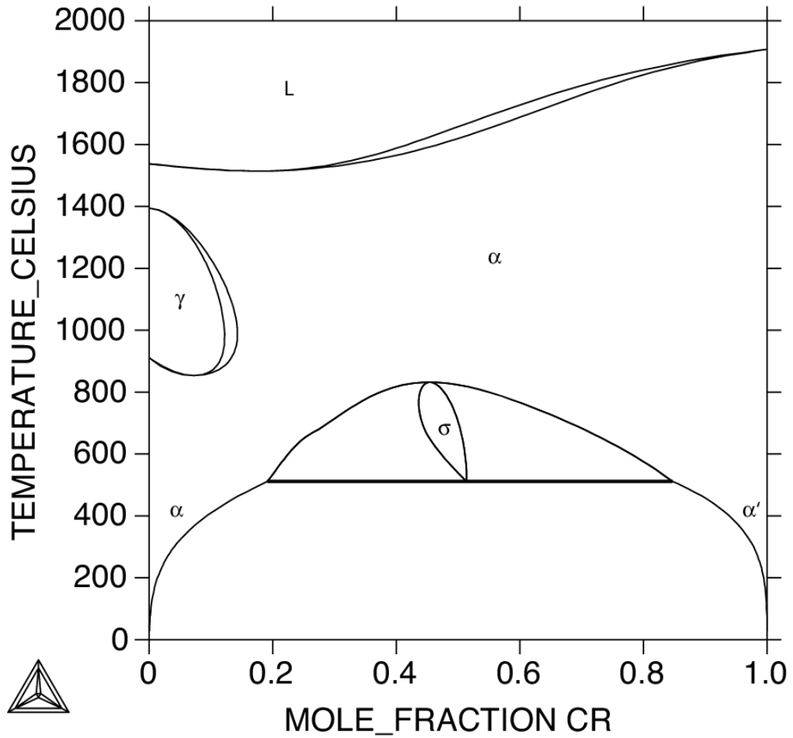


Kinetics

Nucleation or spinodal decomposition?

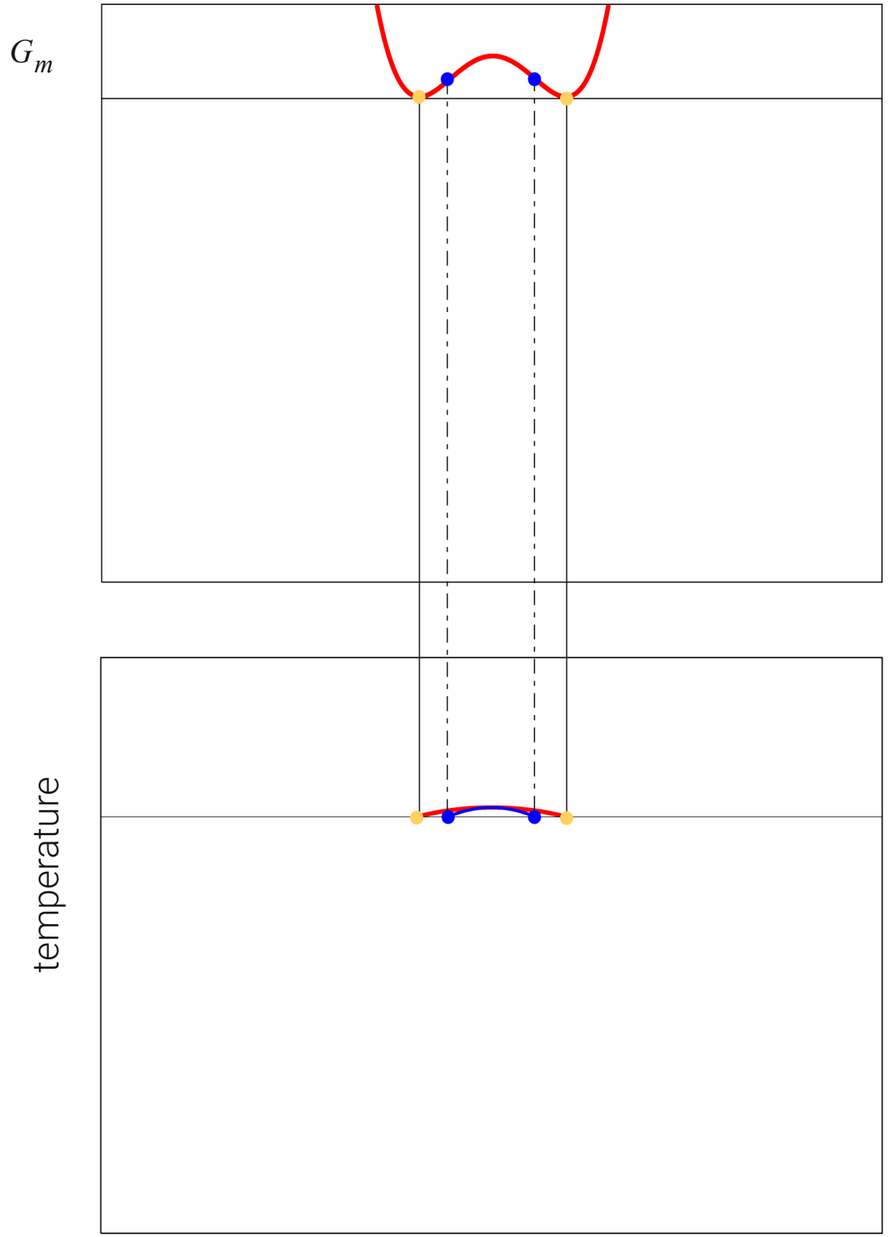


Nucleation or spinodal decomposition?

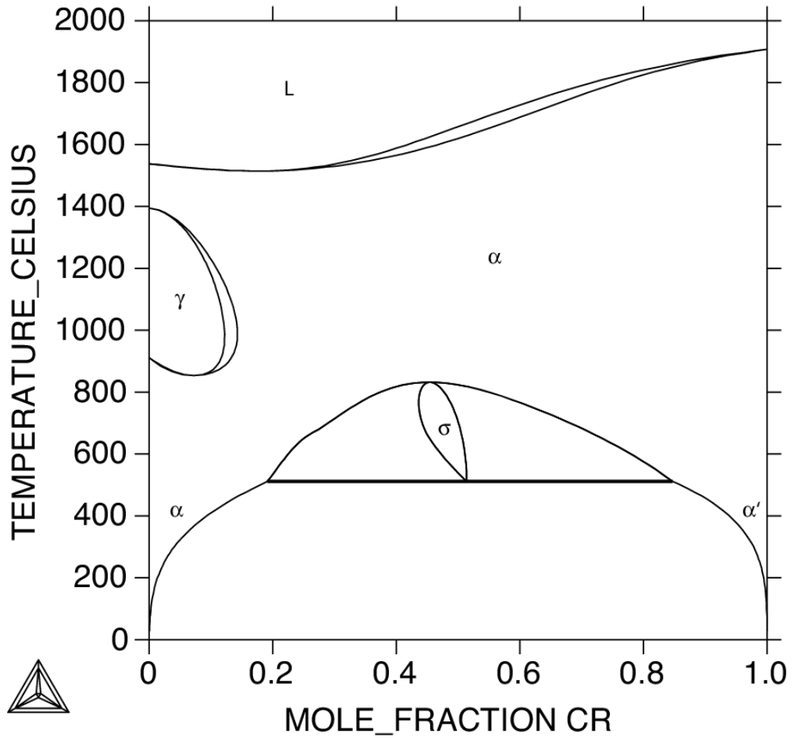


For ex. regular solid solution

$G_m = RT[c\ln c + (1-c)\ln(1-c)] + Ac(1-c)$



Nucleation or spinodal decomposition?



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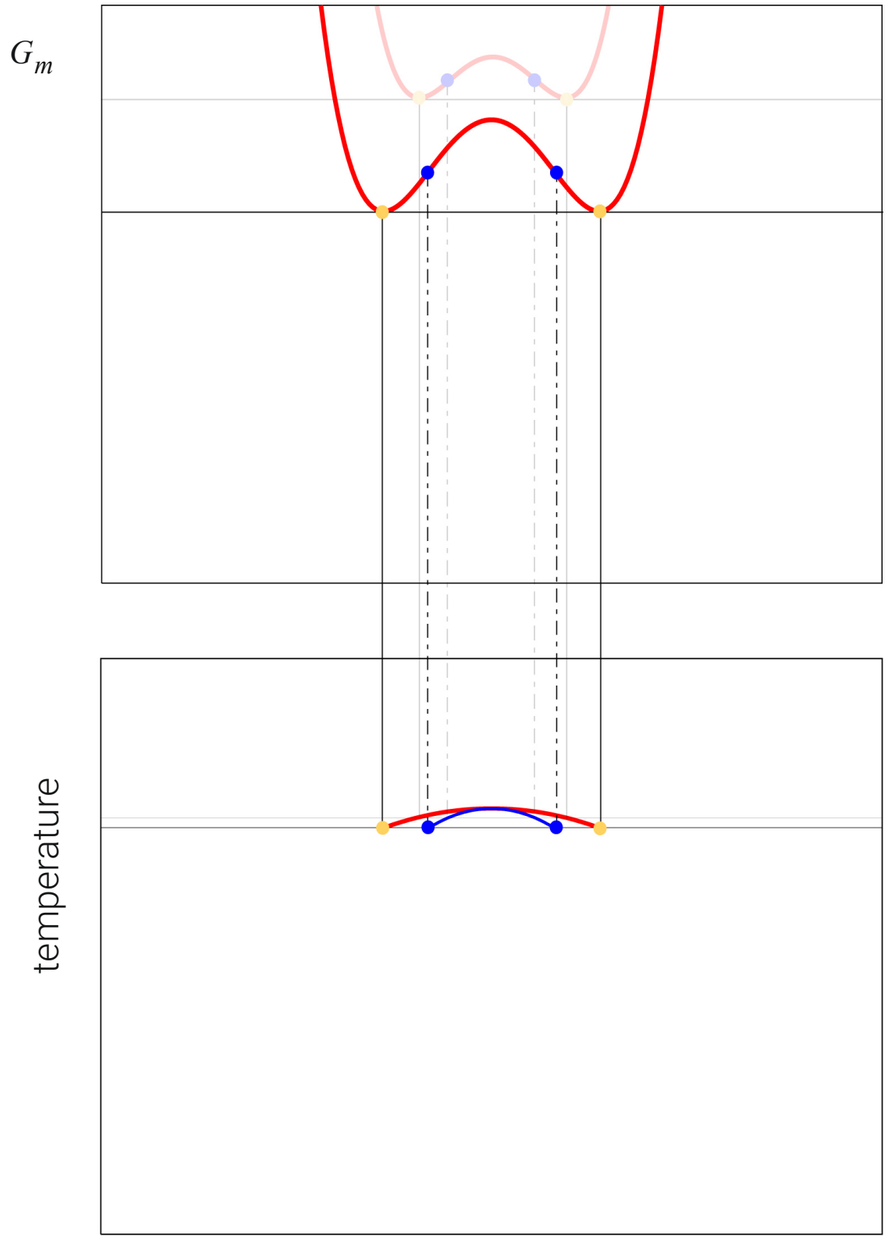

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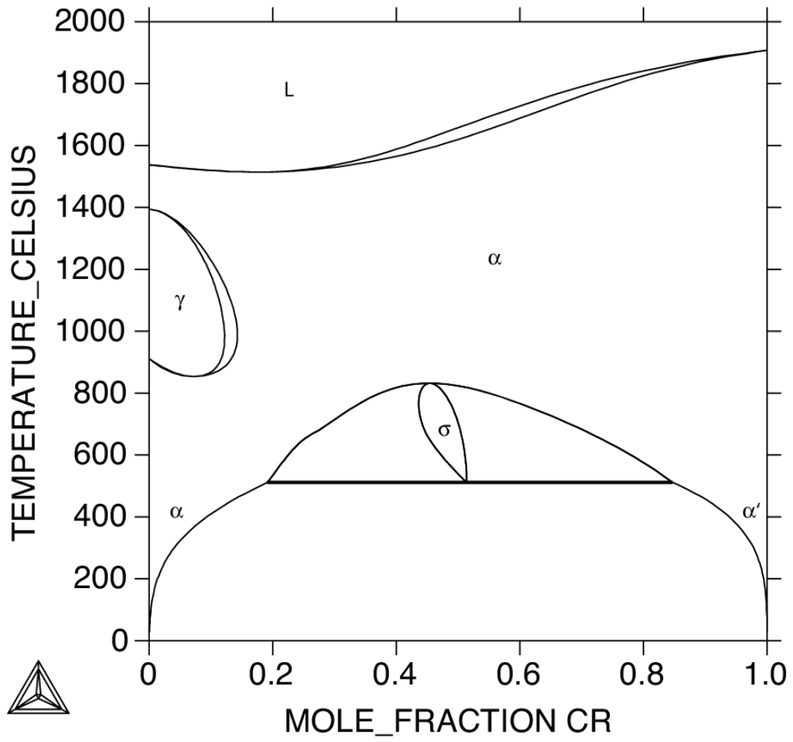

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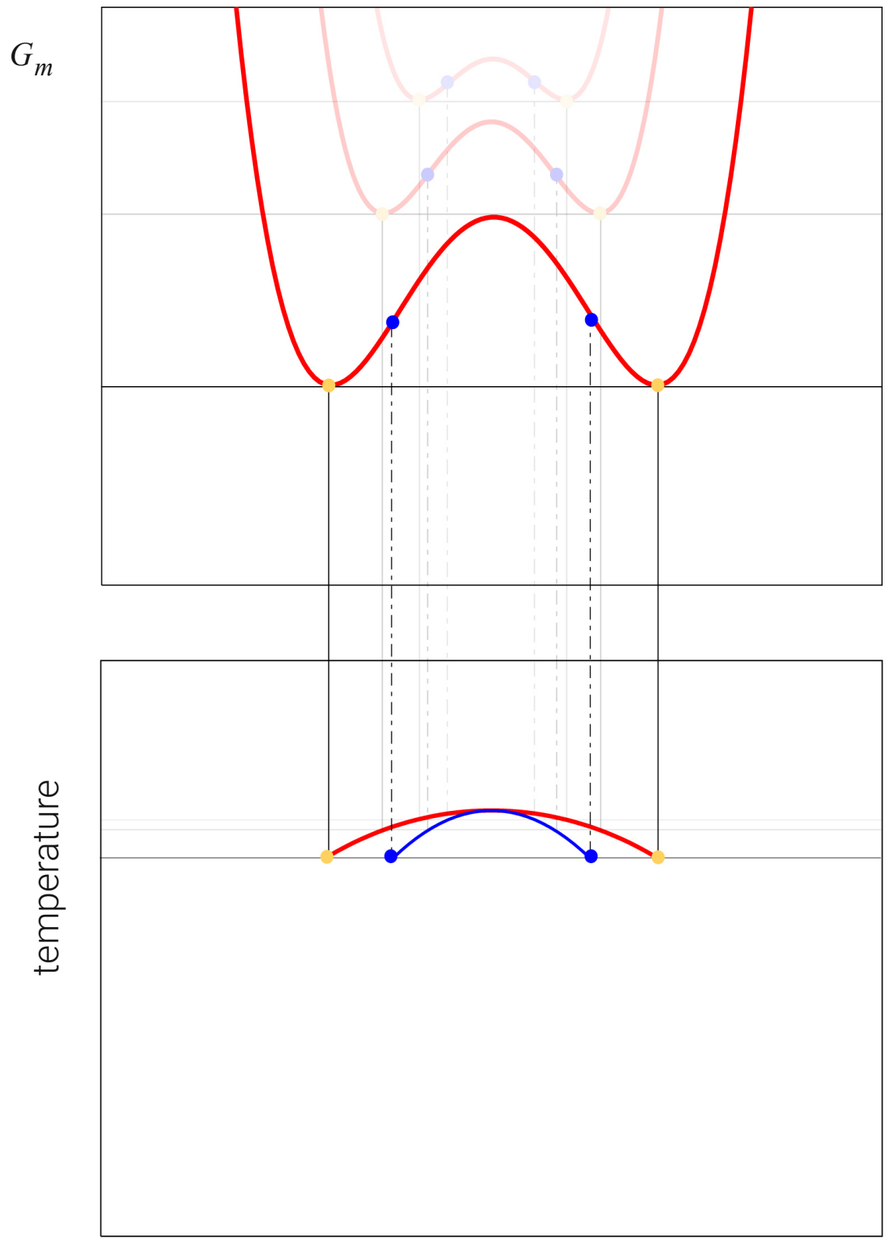

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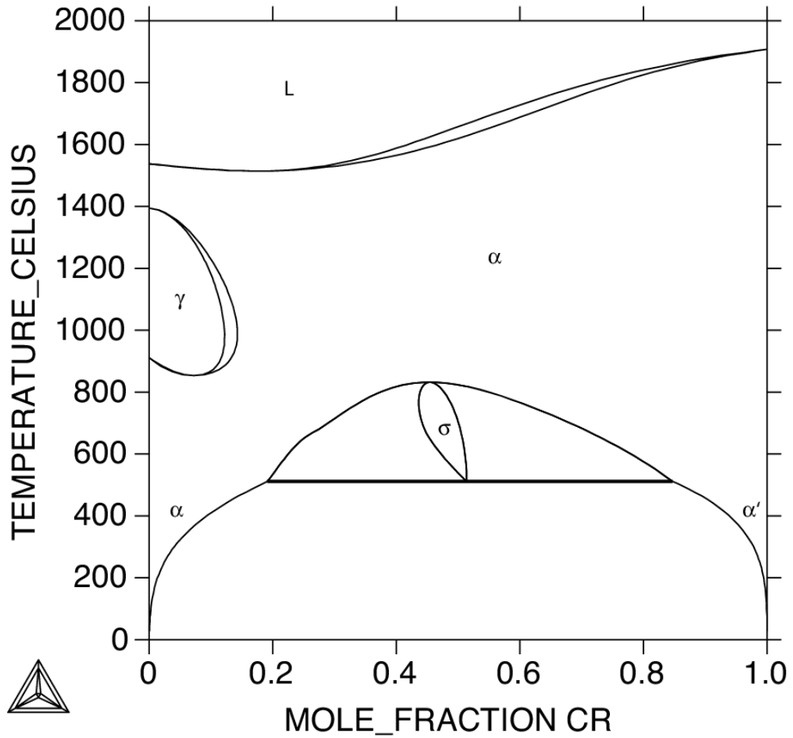

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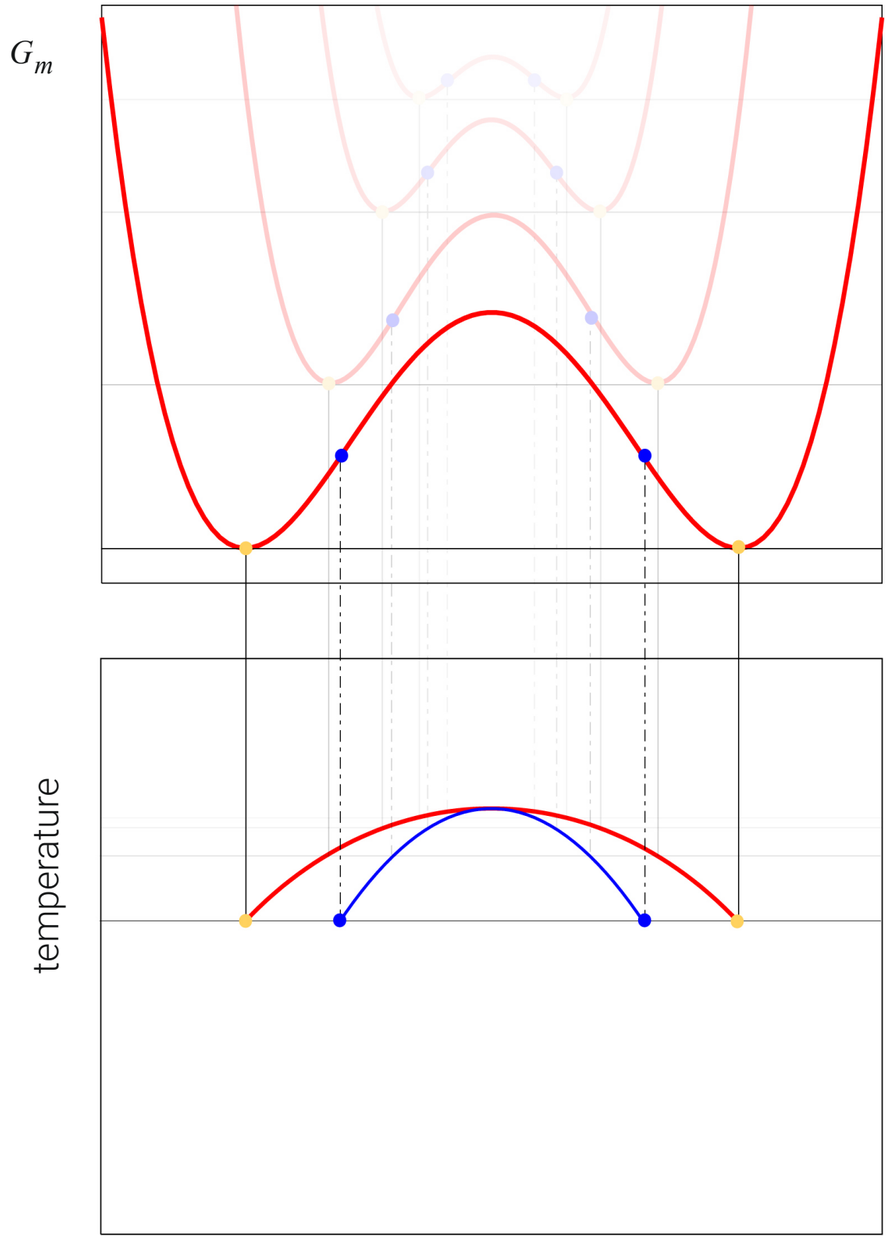
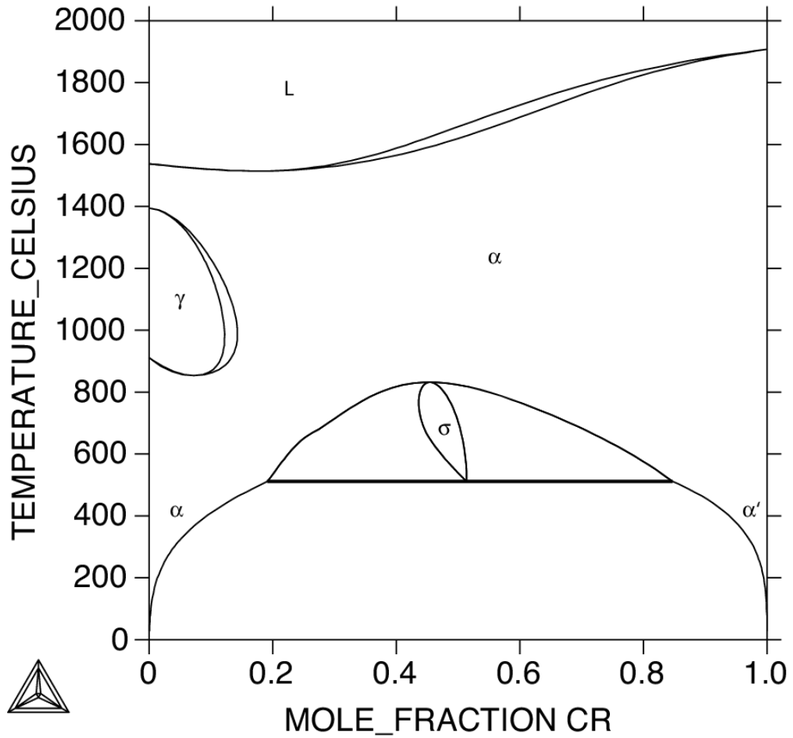


Nucleation or spinodal decomposition?



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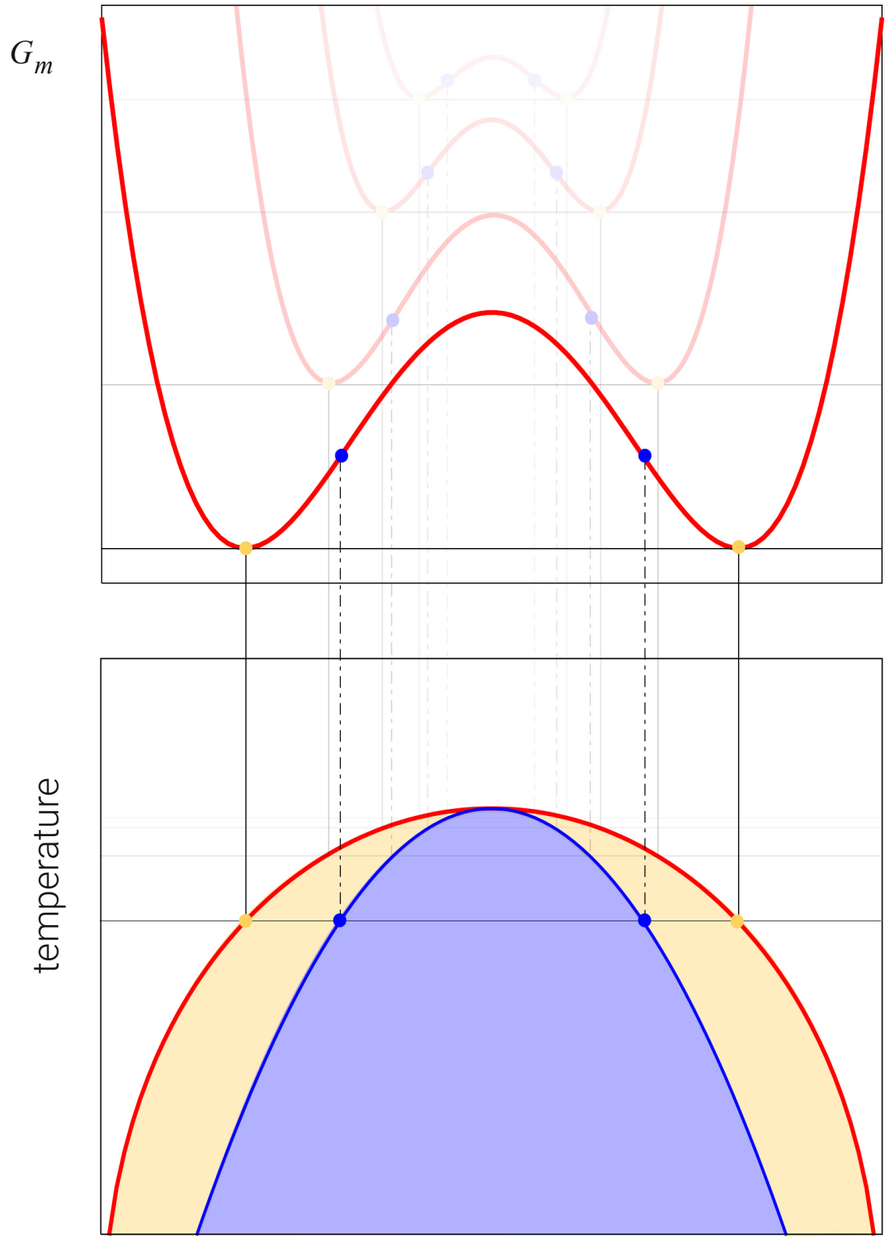
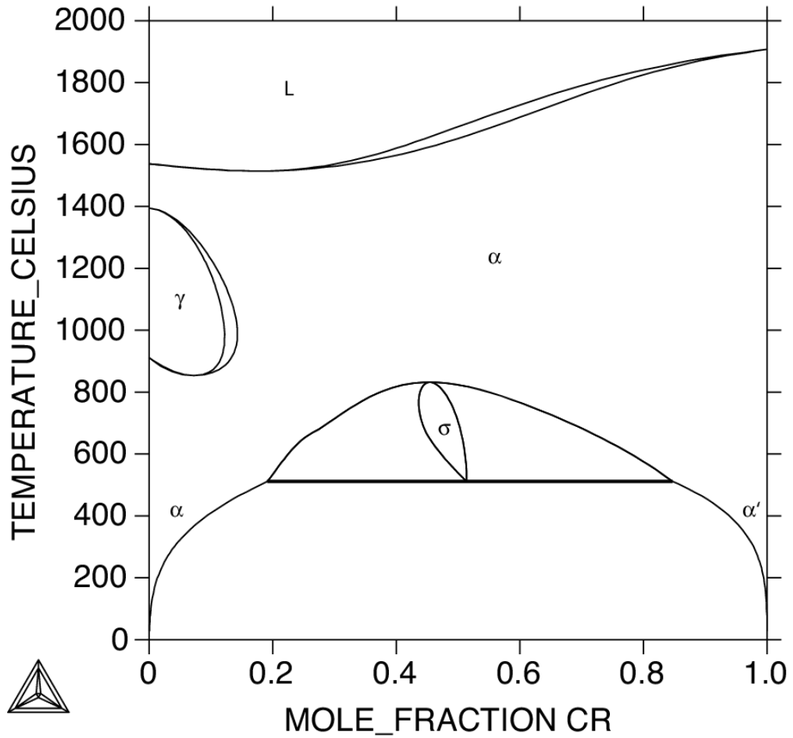
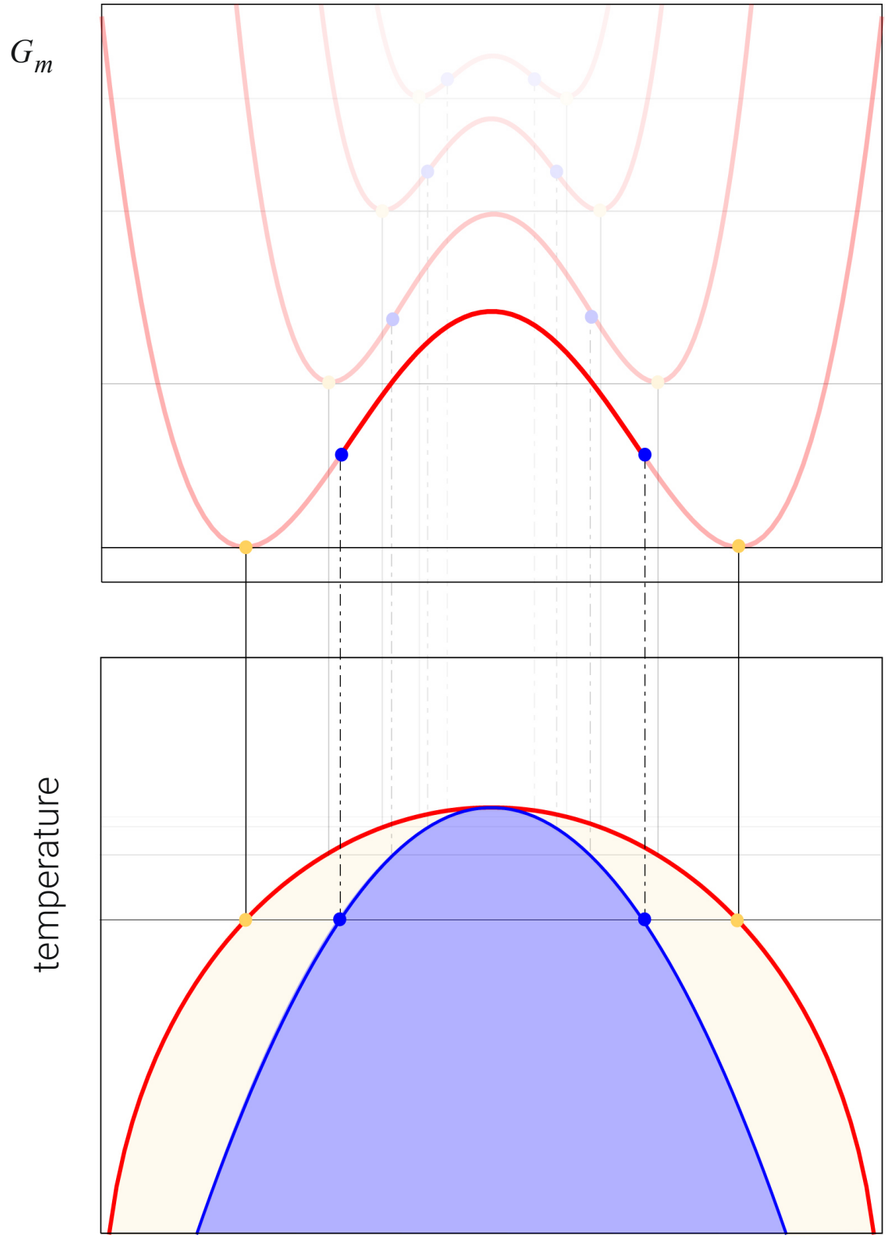


Nucleation or spinodal decomposition?



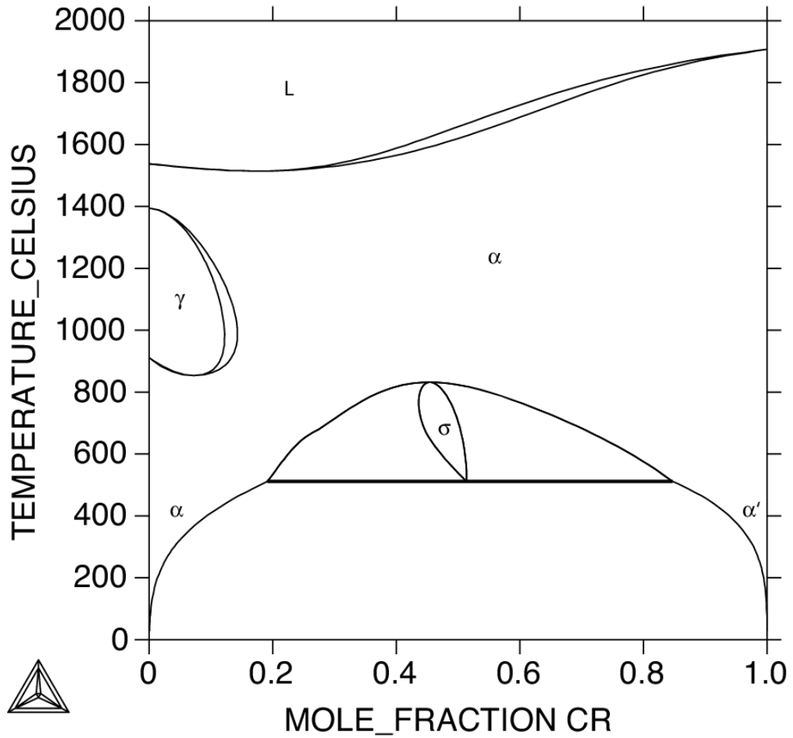


For ex. regular solid solution

$G_m = RT[c\ln c + (1-c)\ln(1-c)] + Ac(1-c)$

- Inside the spinodal $\partial^2 G_m / \partial c^2 < 0$: unstable homog. syst.

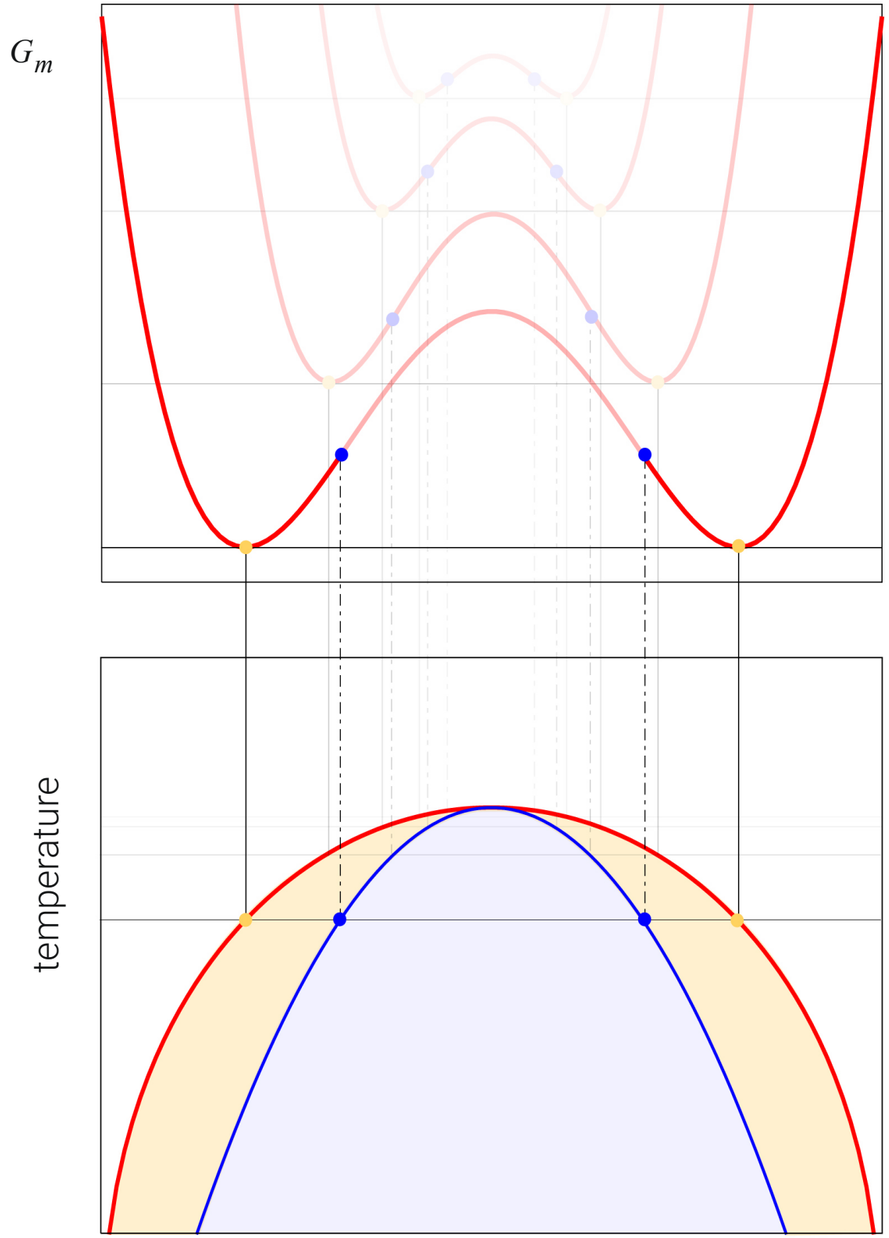
Nucleation or spinodal decomposition?



For ex. regular solid solution

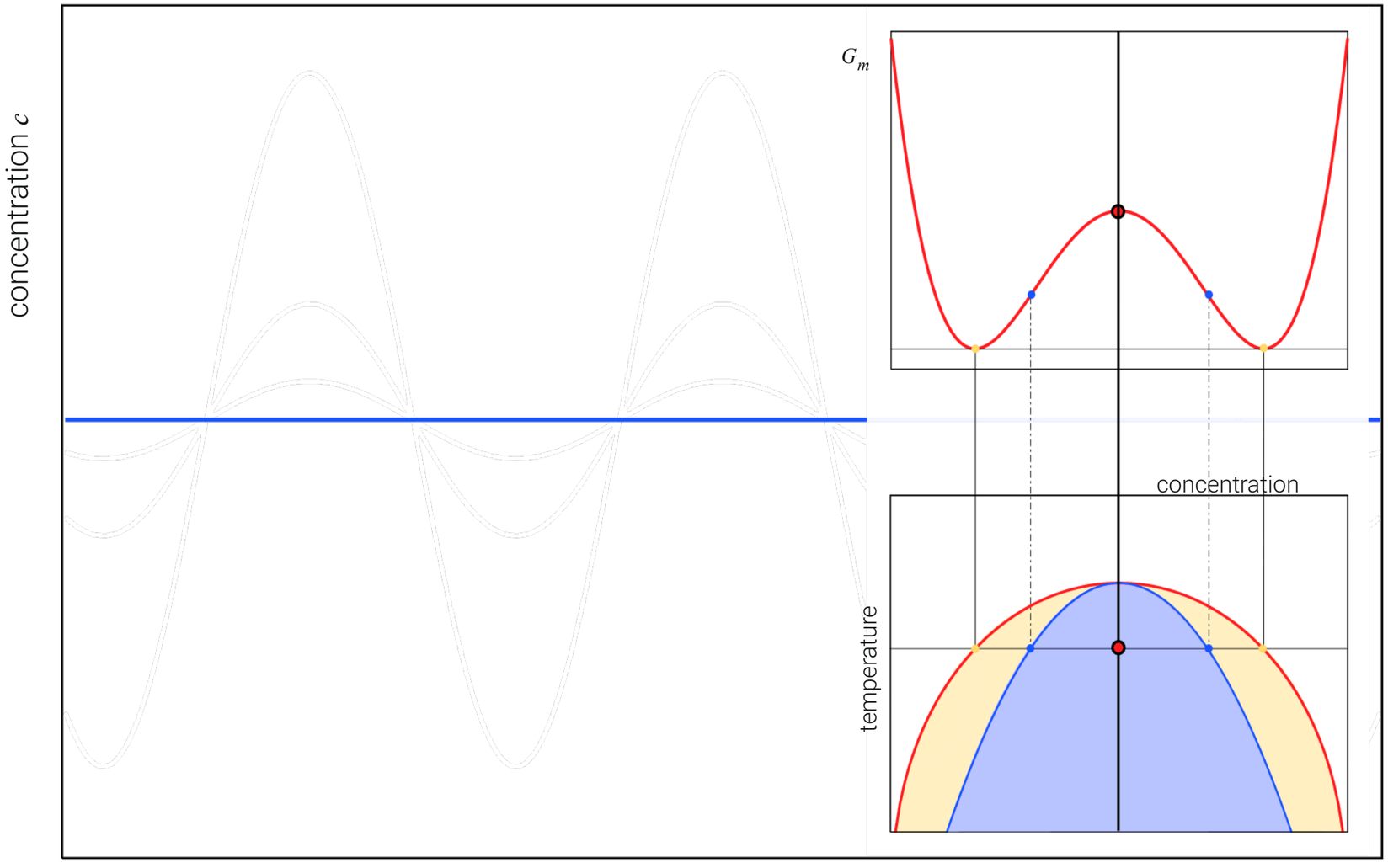
$G_m = RT[c\ln c + (1-c)\ln(1-c)] + Ac(1-c)$

- Inside the spinodal $\partial^2 G_m / \partial c^2 < 0$: unstable homog. syst.
- In between the solubility gap and the spinodal $\partial^2 G_m / \partial c^2 > 0$: metastable homog. syst.



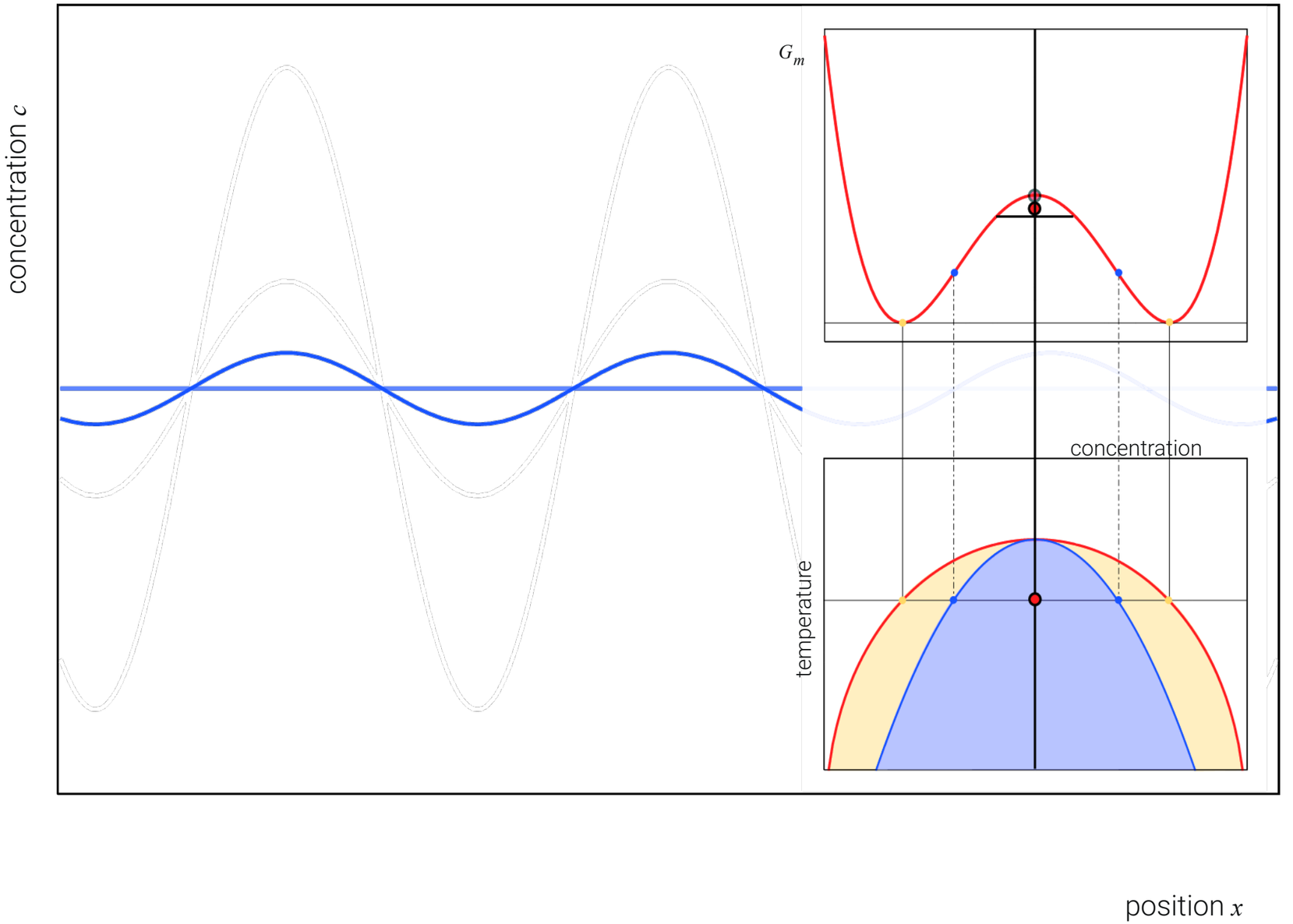
Nucleation or spinodal decomposition?

Inside the spinodal



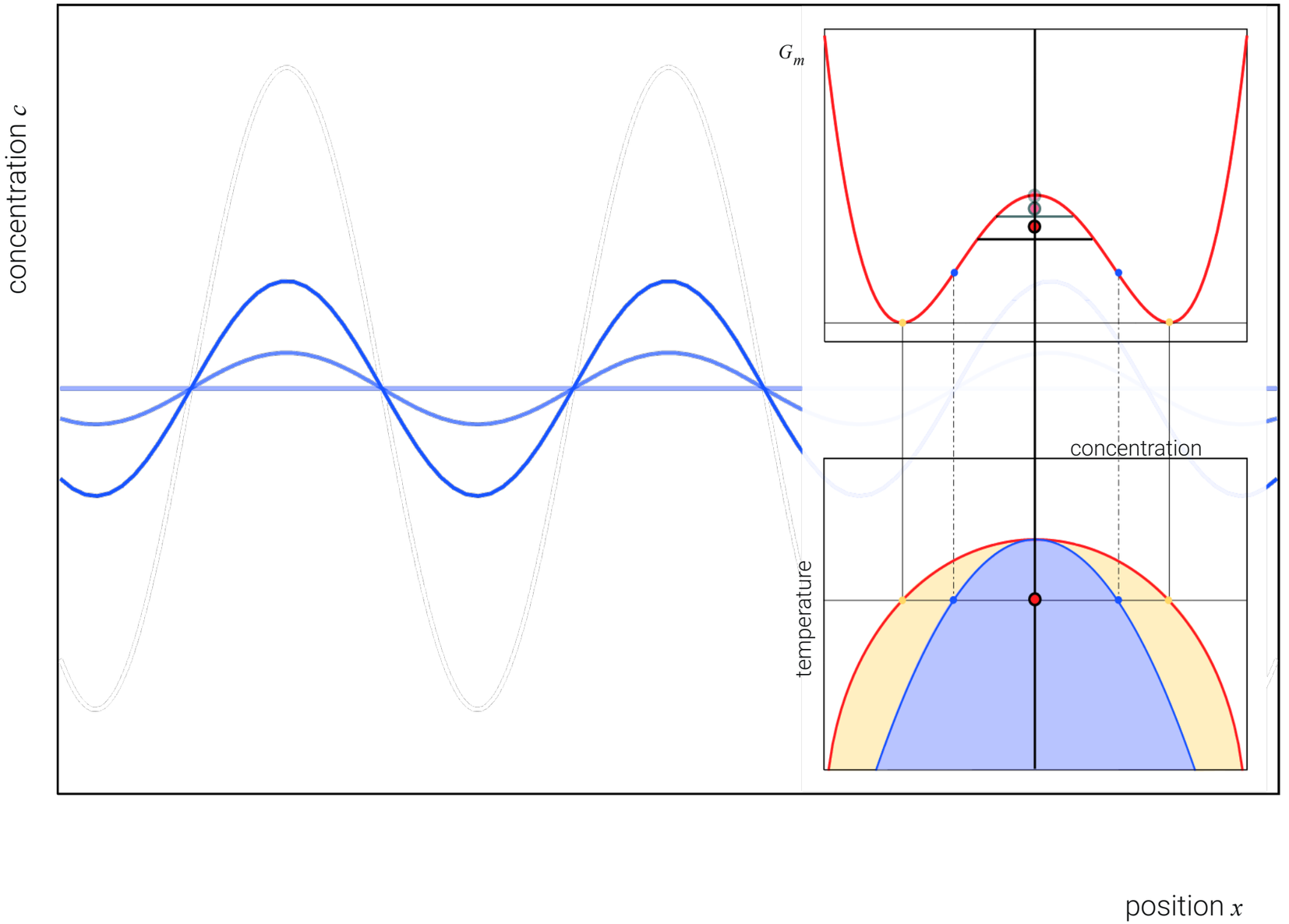
Nucleation or spinodal decomposition?

Inside the spinodal



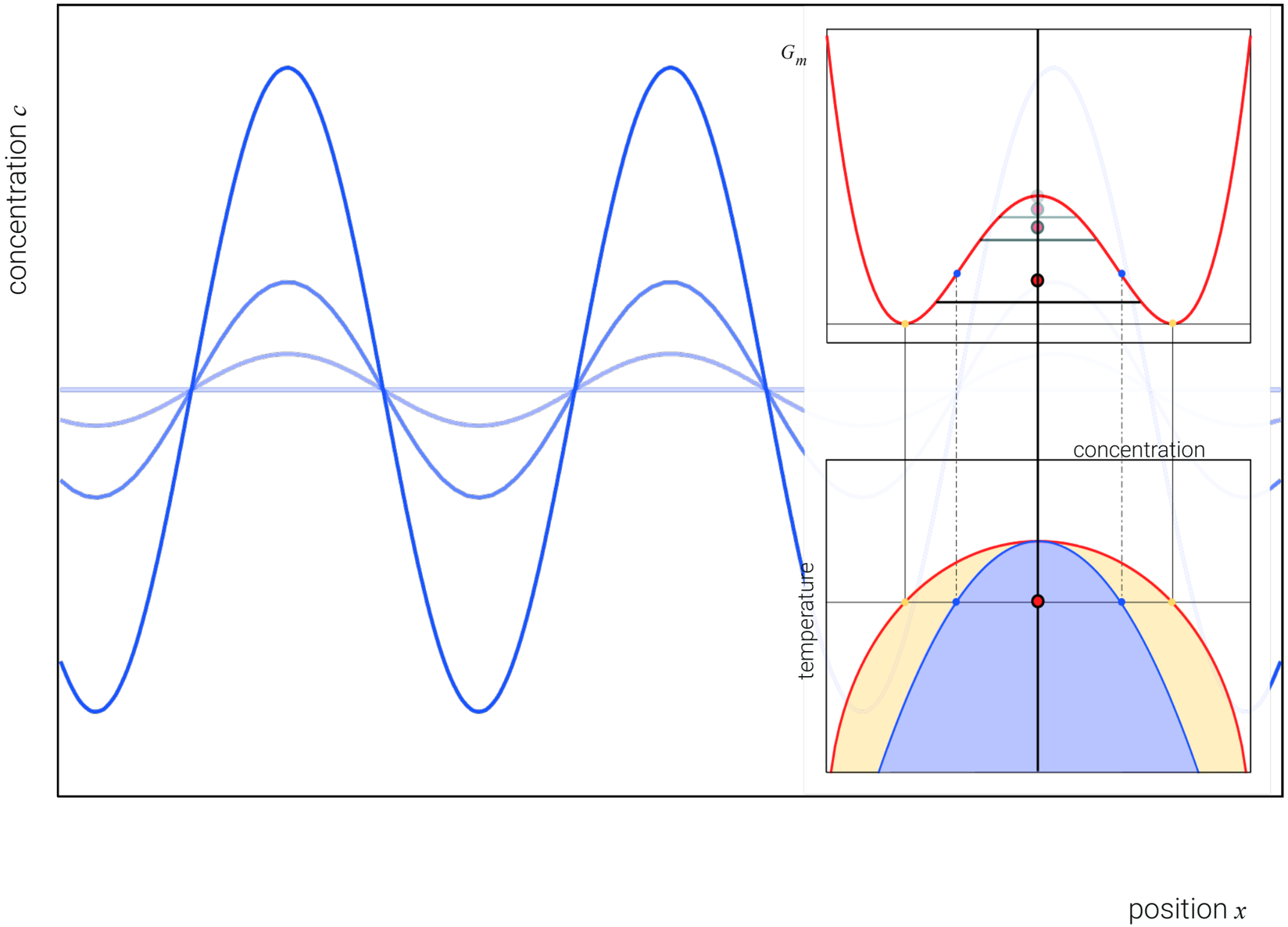
Nucleation or spinodal decomposition?

Inside the spinodal



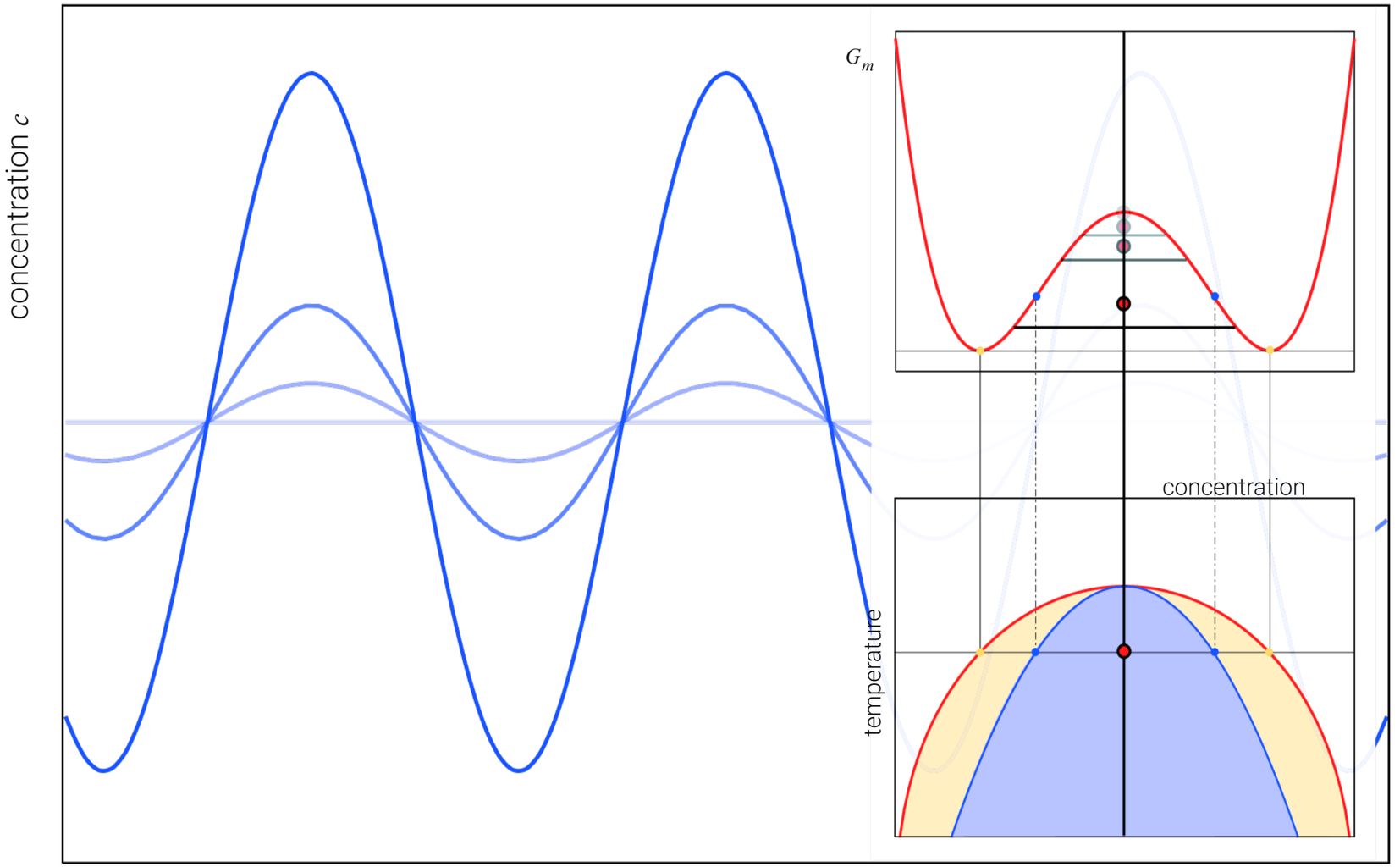
Nucleation or spinodal decomposition?

Inside the spinodal



Nucleation or spinodal decomposition?

Inside the spinodal



Nucleation or spinodal decomposition?

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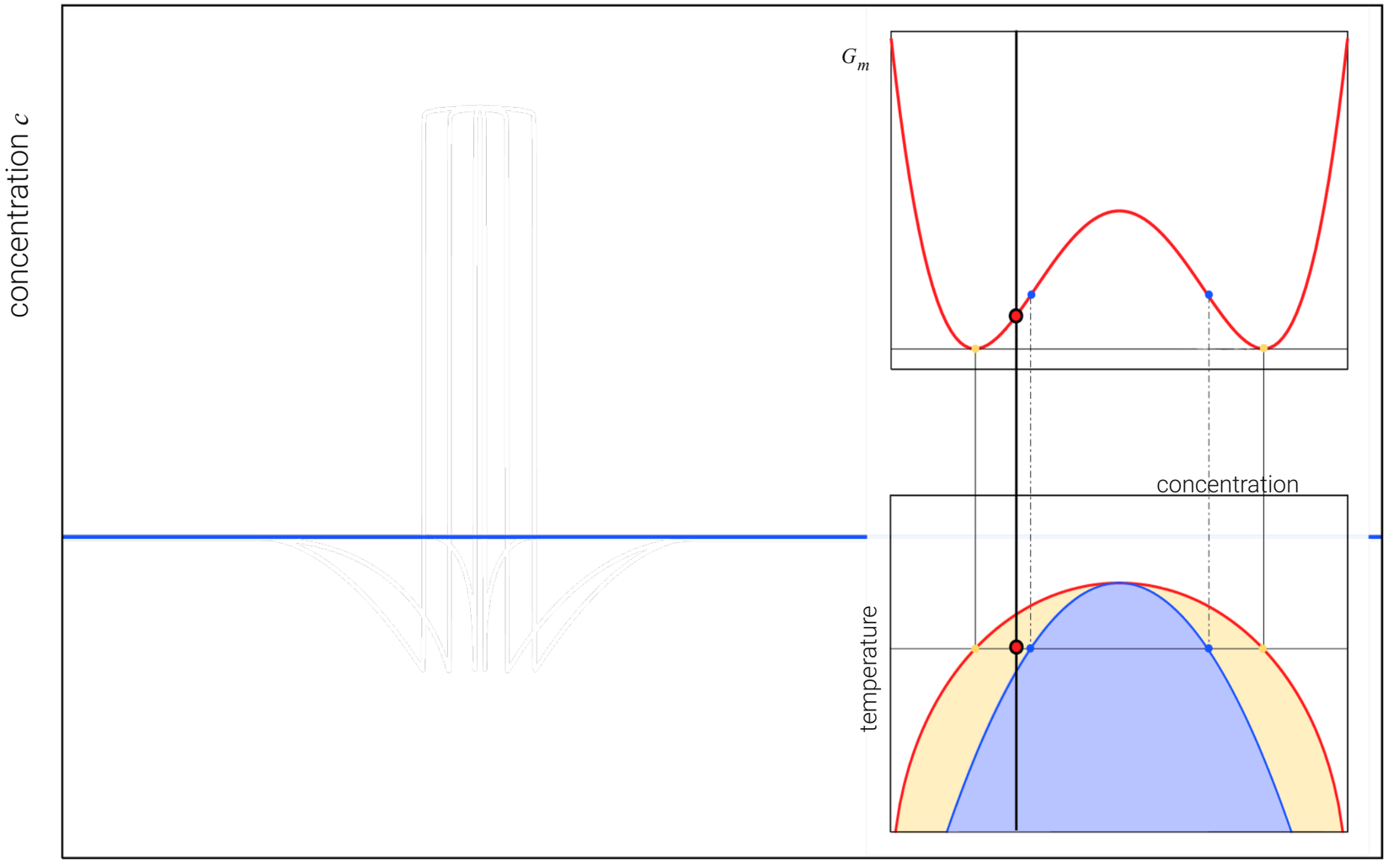
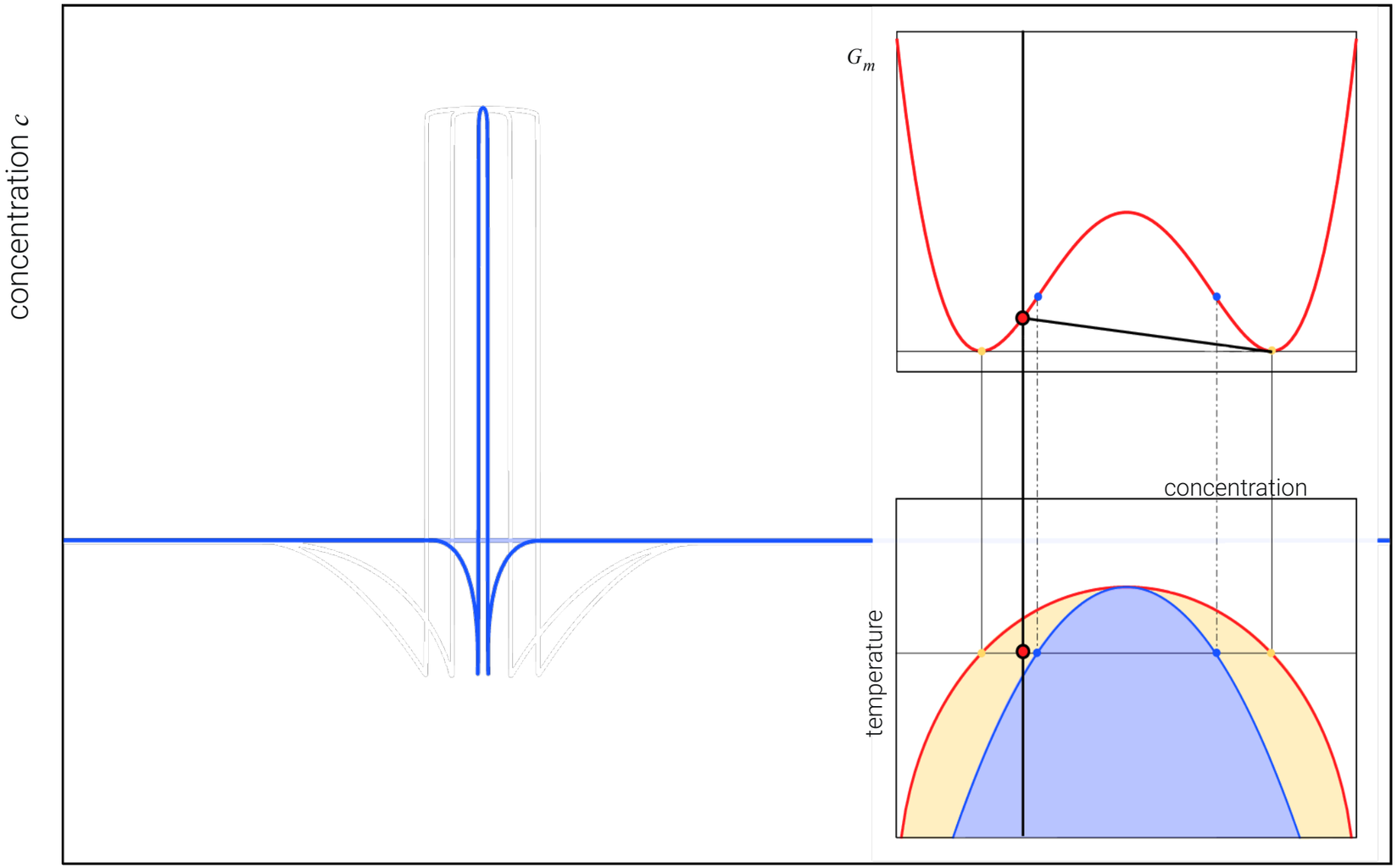


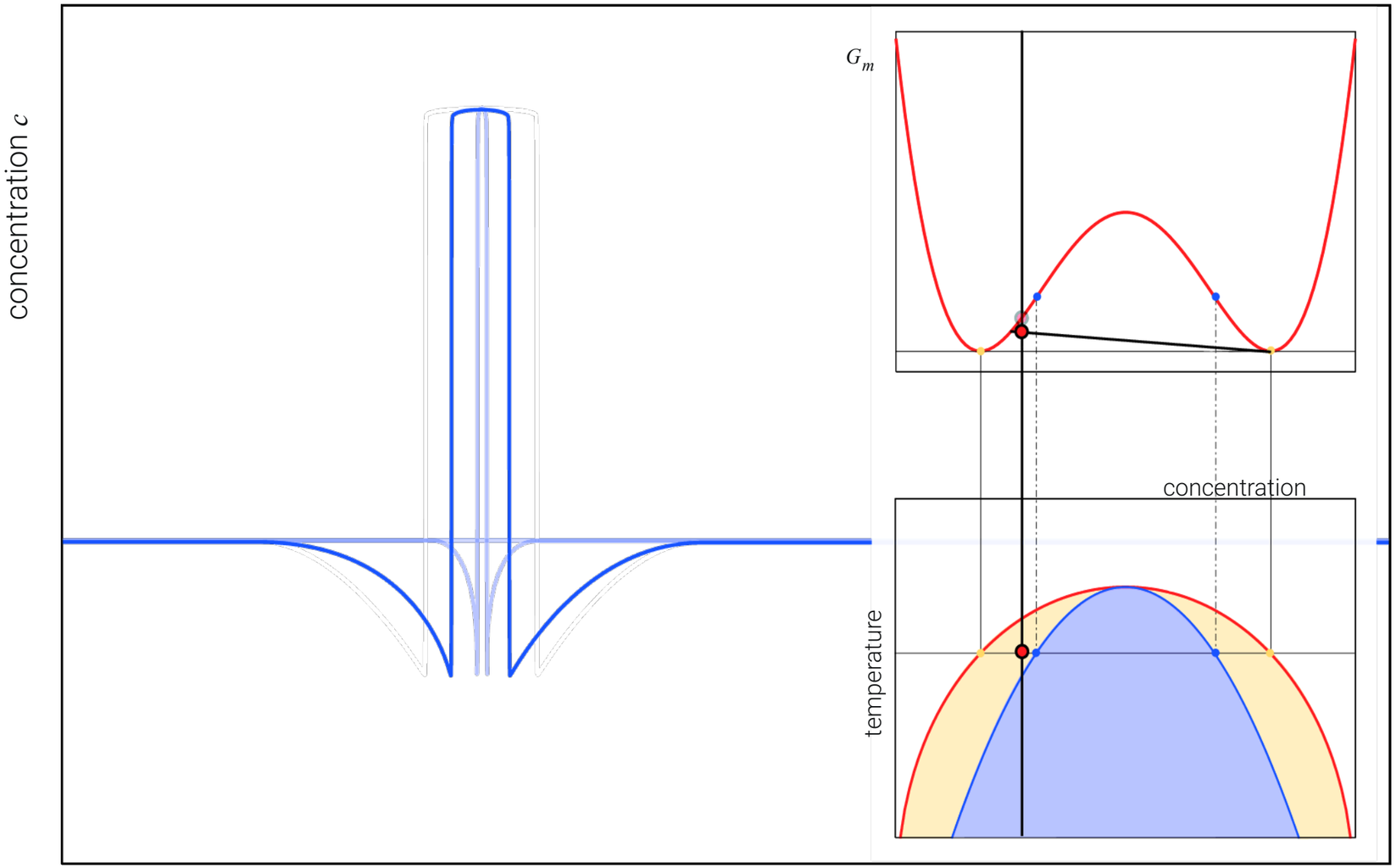
Nucleation or spinodal decomposition?

Outside the spinodal



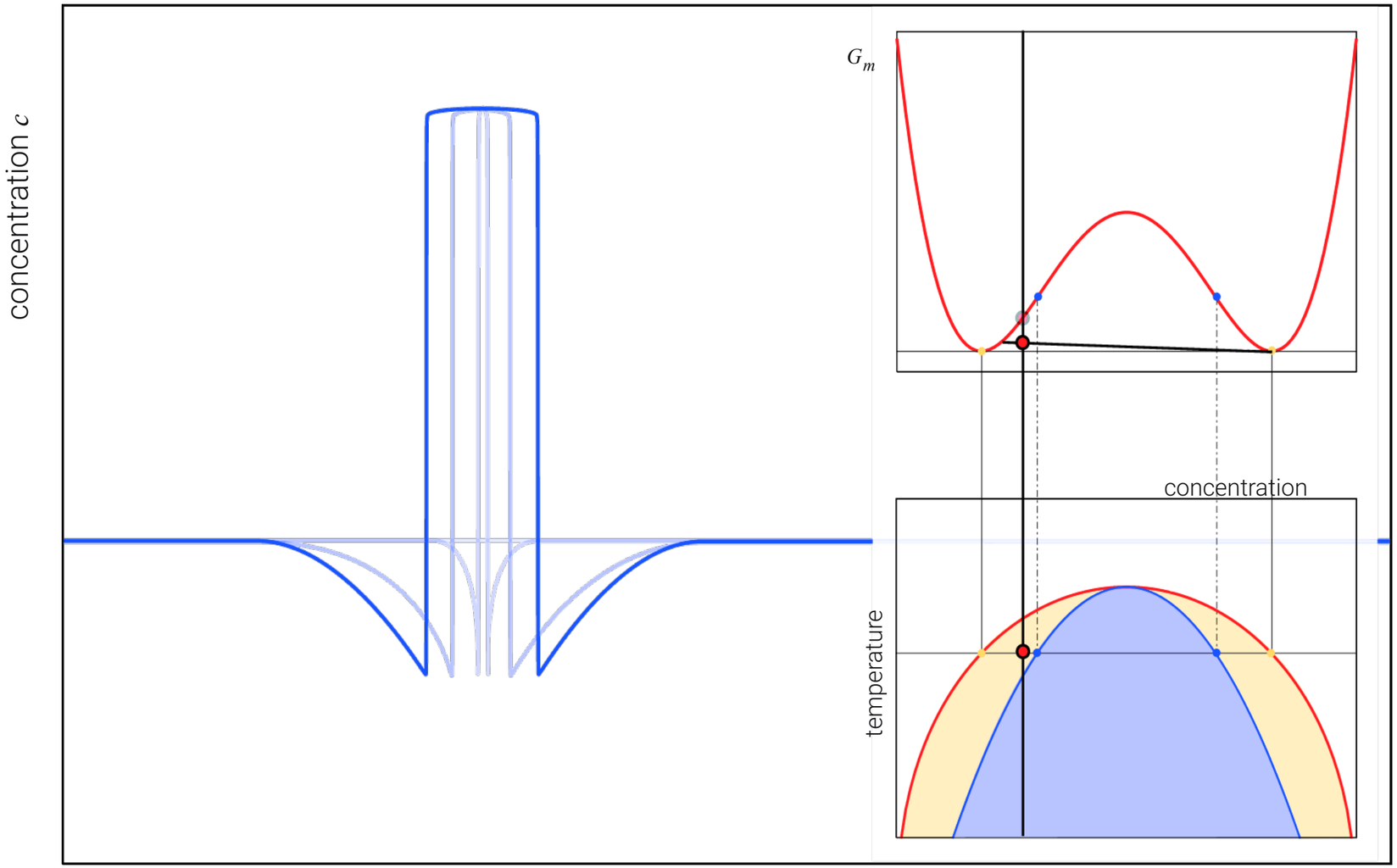
Nucleation or spinodal decomposition?

Outside the spinodal



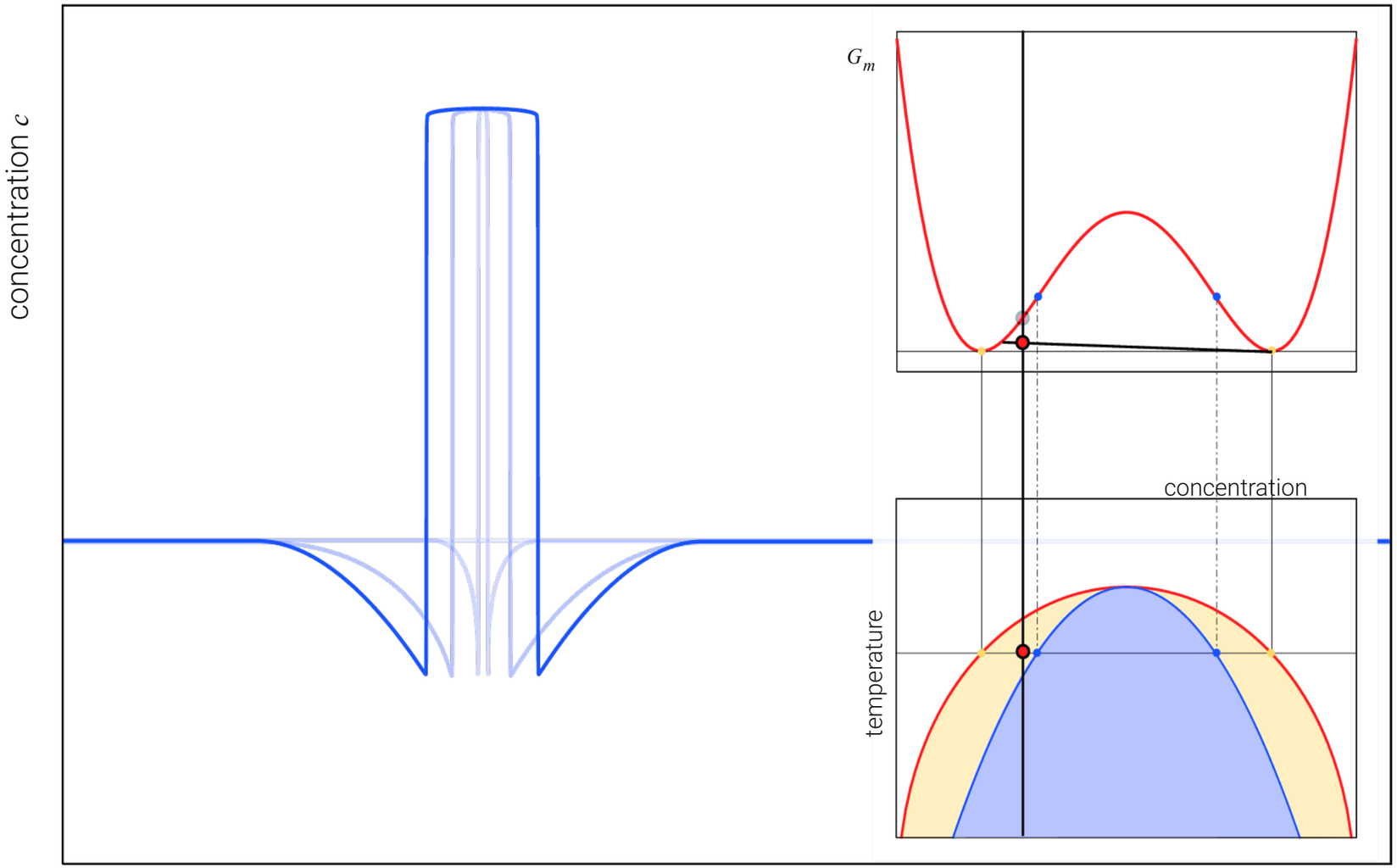
Nucleation or spinodal decomposition?

Outside the spinodal



Nucleation or spinodal decomposition?

Outside the spinodal



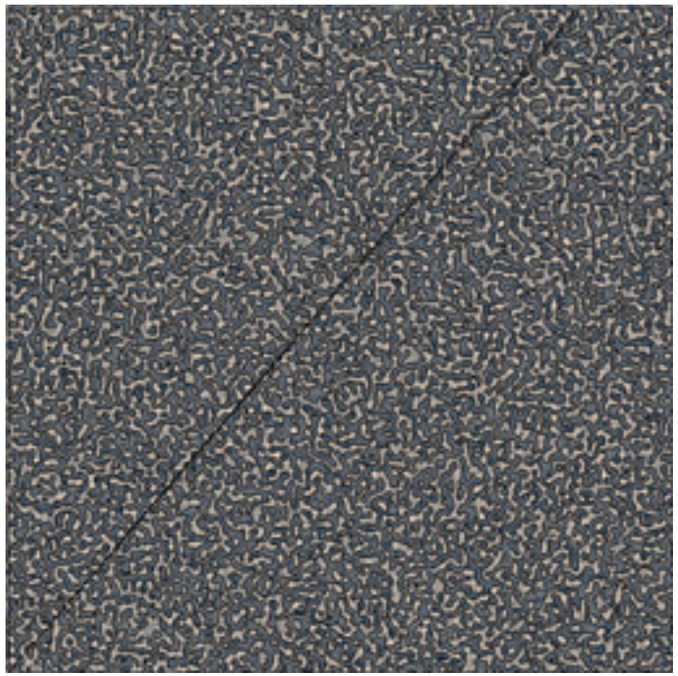
Nucleation or spinodal decomposition?

Inside the spinodal

Cahn-Hilliard model (Cahn and Hilliard 1958)

$$\mathrm{F} = \int f_0(c) + \frac{\alpha}{2} |\nabla c|^2 dV$$

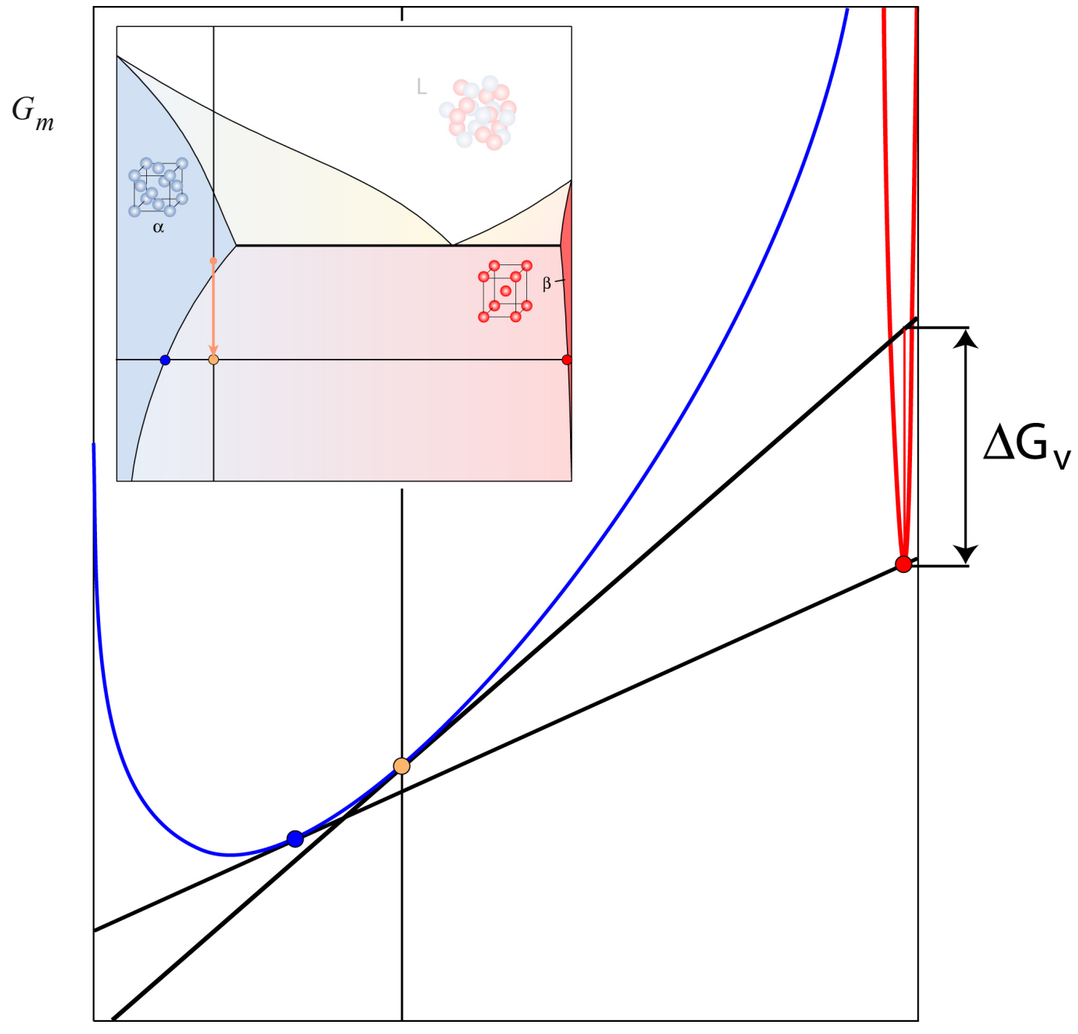
Homework

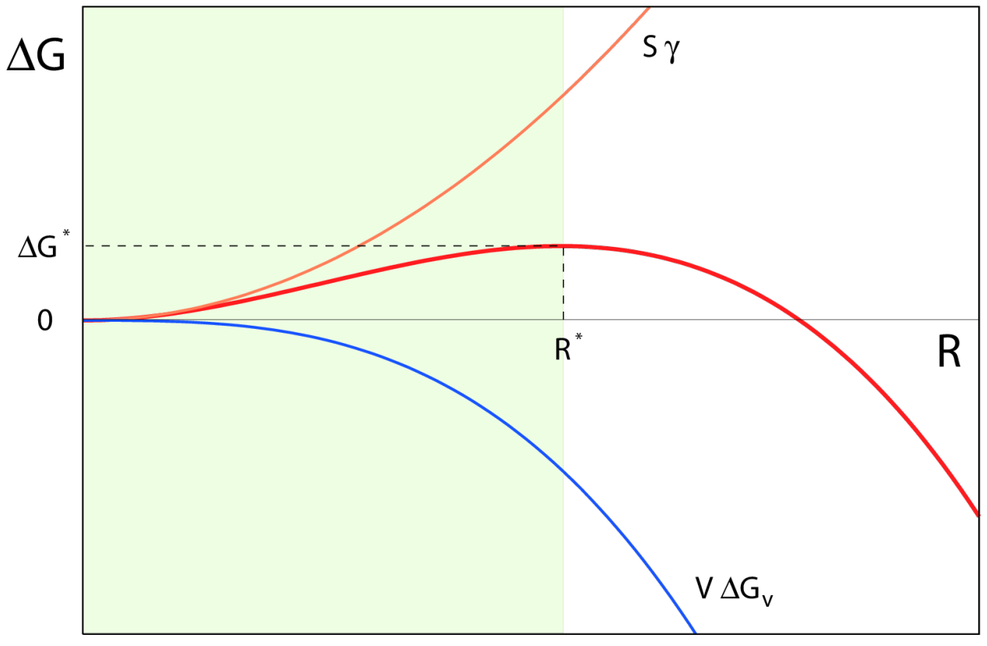


Nucleation or spinodal decomposition?

Outside the spinodal

Classical Nucleation Theory: thermodynamics [Volmer, Weber 1926]



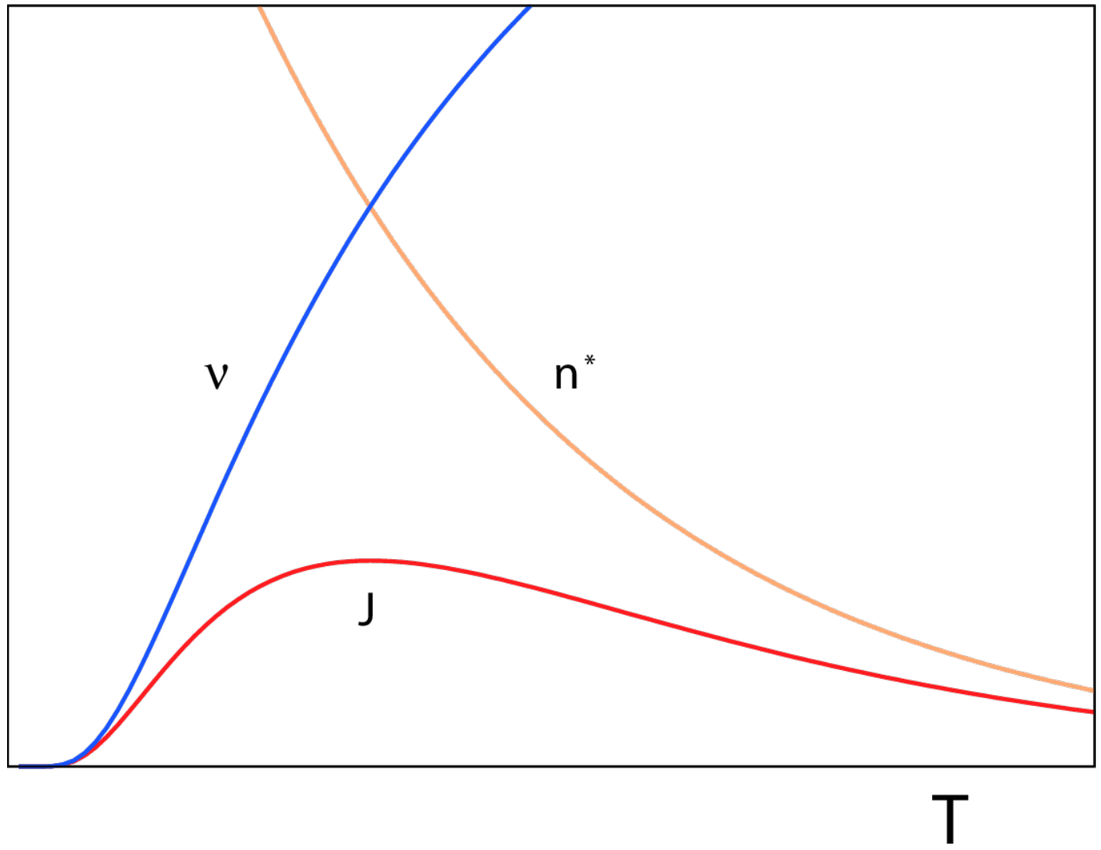


Homework

Nucleation or spinodal decomposition?

Outside the spinodal

Classical Nucleation Theory: kinetics [Becker, Döring 1935] CR



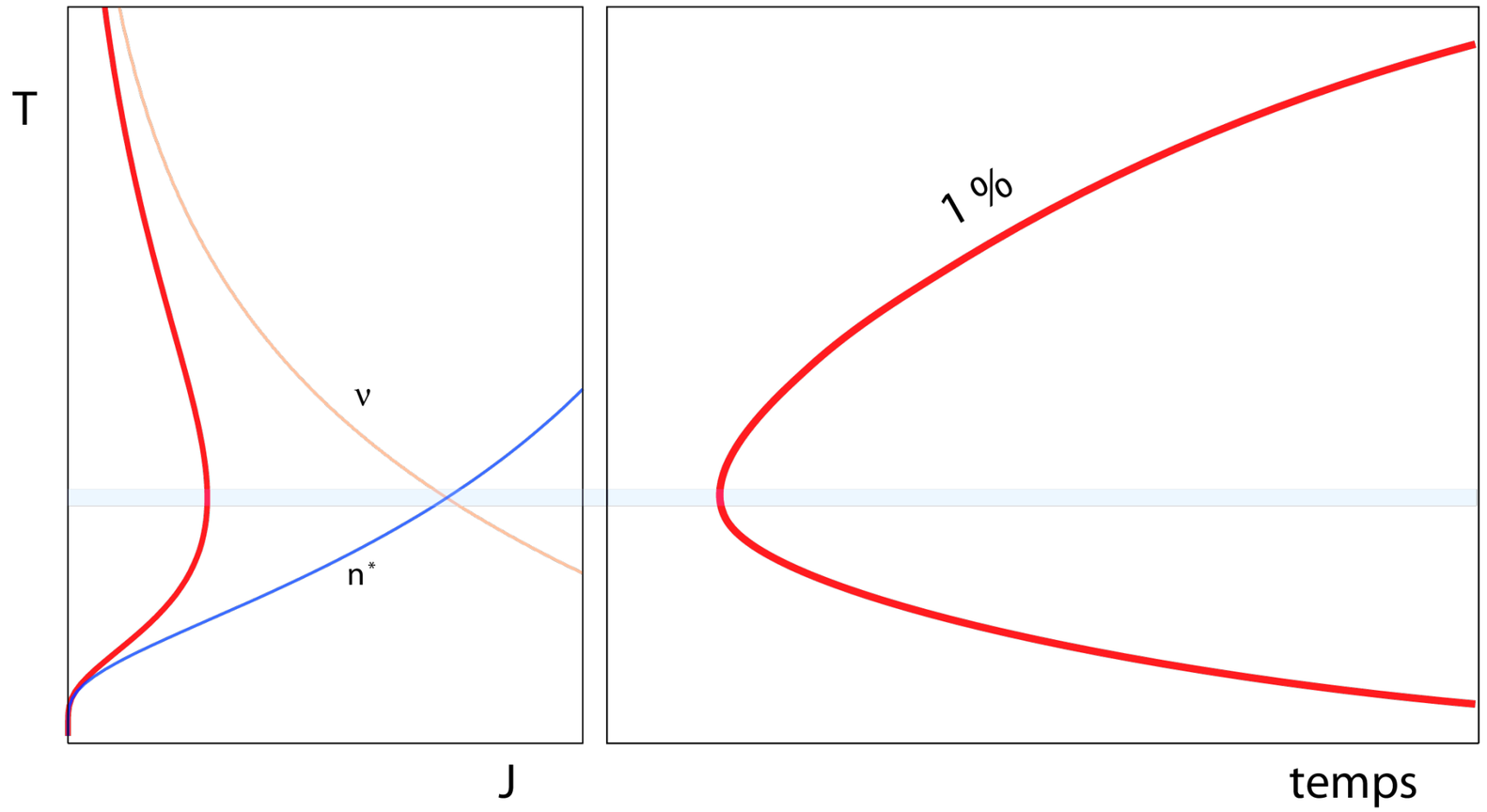
- Nucleation rate $J \propto \nu n^{\star}$
- Attachment frequency $\nu = \frac{D}{\lambda^2} = \nu_0 \exp\left(\frac{-Q}{RT}\right)$
- Number of critical nuclei $n^{\star} = n^0 \exp\left(\frac{-\Delta G^{\star}}{RT}\right)$

Validity [Clouet 2010] [Ryu, Cai 2010]

Nucleation or spinodal decomposition?

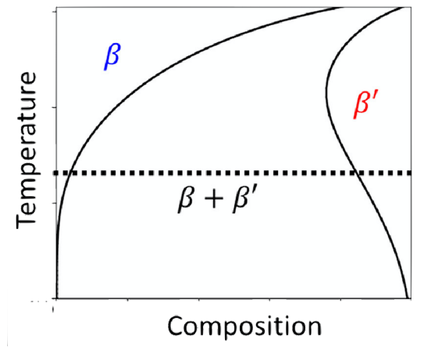
Outside the spinodal

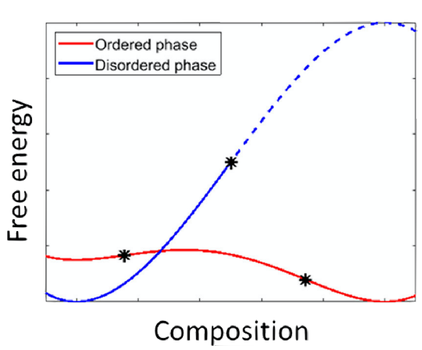
Classical Nucleation Theory: kinetics [Becker, Döring 1935]

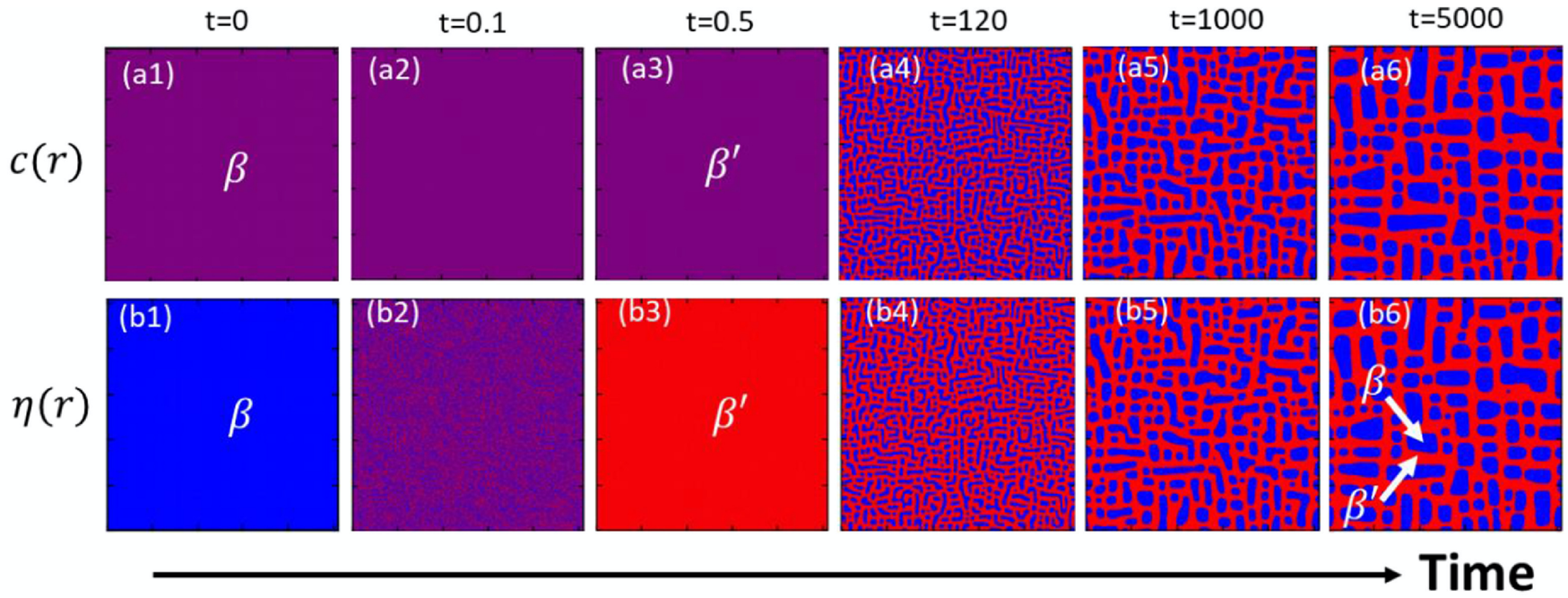


Nucleation or spinodal decomposition?

bcc → bcc in B2 (e.g. *Al-Mo$_{0.5}$-Nb-Ta$_{0.5}$-Ti-Zr*) *Kadirvel, Fraser, and Wang (2023)*







Configuration of the free energies would explain bcc in B2

Influence of stress on nucleation

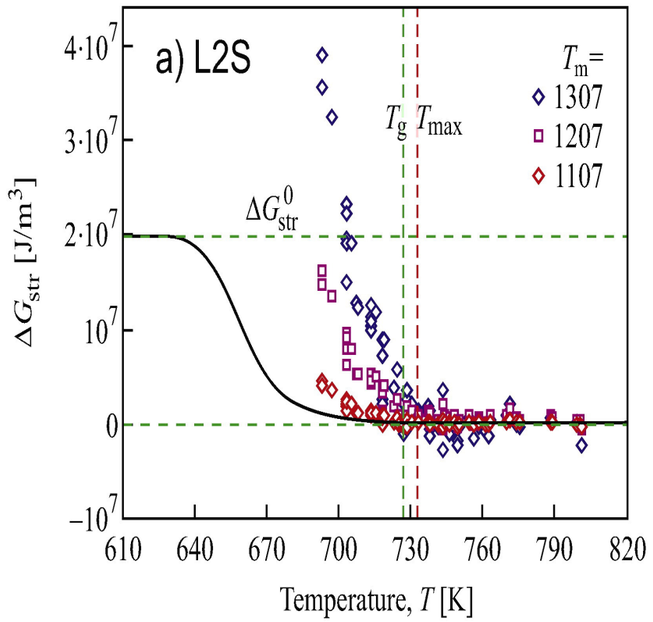
Modify the driving force (most obvious)

Crystallization of glass during cooling

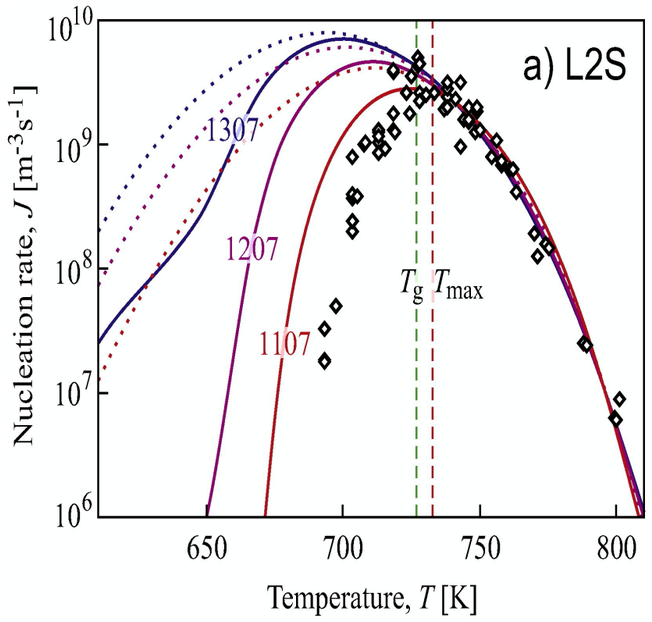
$$\Delta G^{\star} = \frac{16\pi}{3}\frac{\gamma^3}{\Delta G_v + \mathrm{F}_{el}}$$

with $|\Delta G_v + \mathrm{F}_{el}| < |\Delta G_v|$

That's all (almost)



Li_2O-2 SiO_2

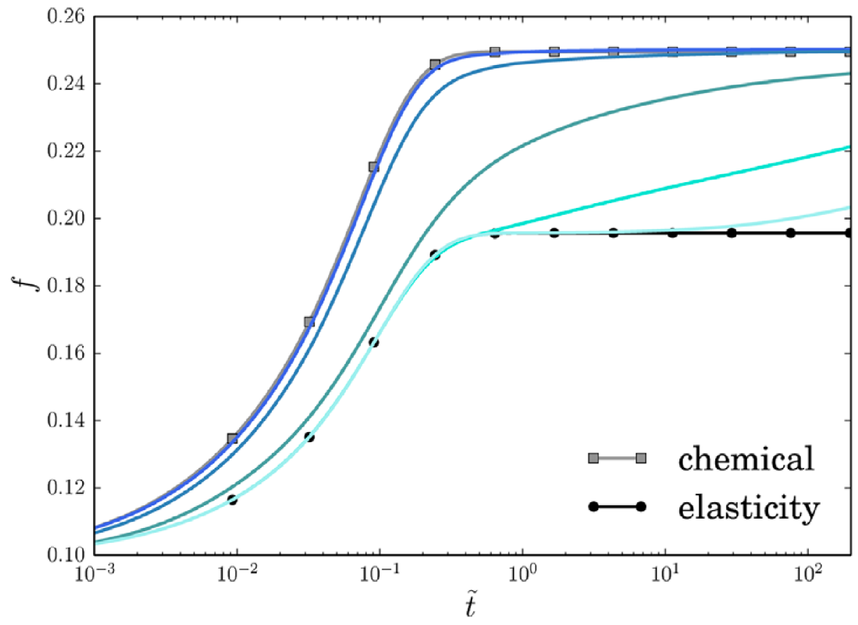


Abyzov et al. (2016)

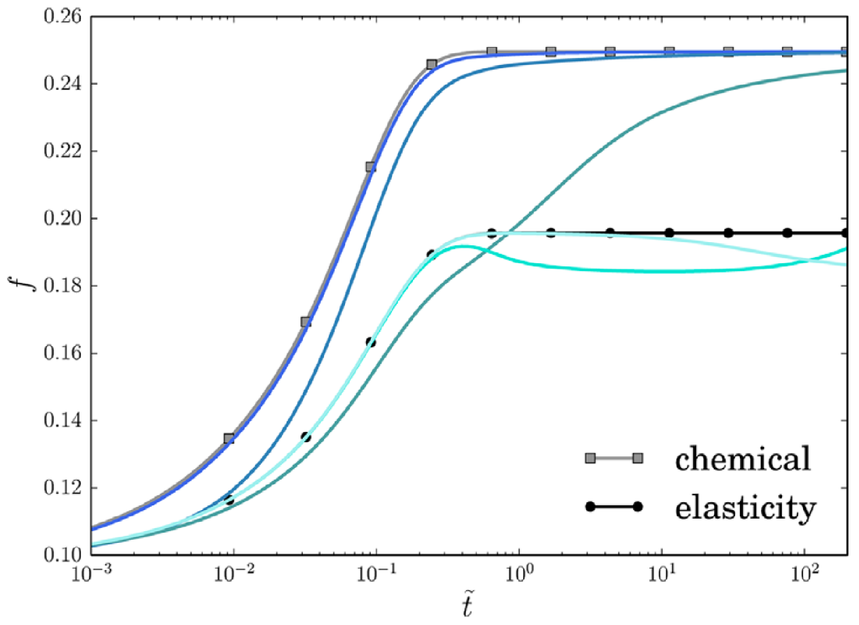
Influence of stress on growth

Change the interfacial concentrations

Planar precipitate growing in a supersaturated matrix



viscoplastic precipitate



viscoplastic matrix de Rancourt et al. (2016)

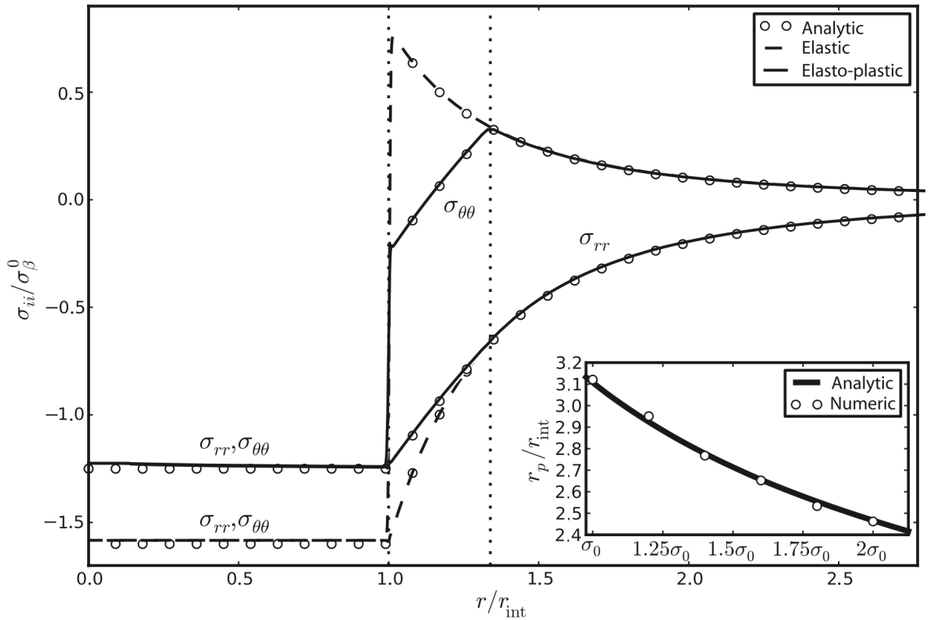
$$c^{\psi} = c^{\psi 0} + [\![f_{el}]\!] - \sigma_{ij}^{\beta} [\![\varepsilon_{ij}]\!] + 2\bar{\kappa}\gamma$$

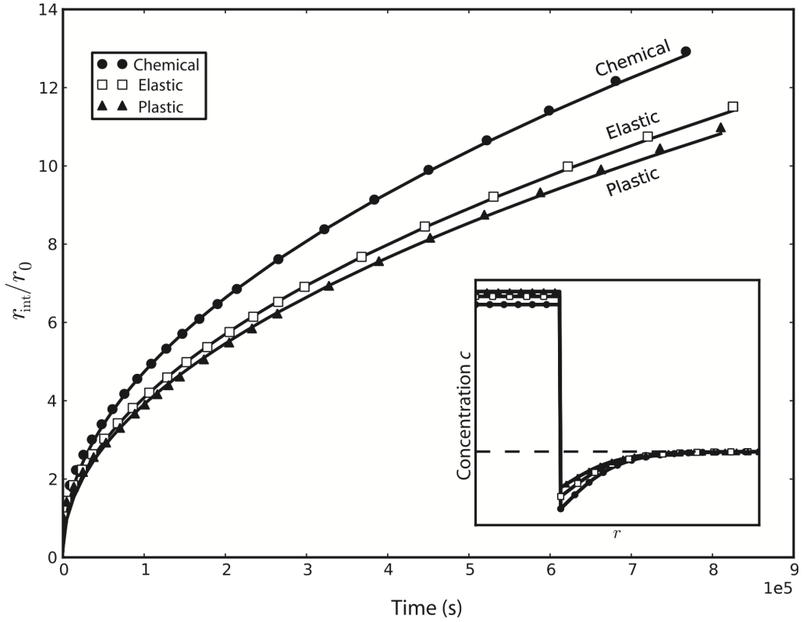
- Two competing time scales

Influence of stress on growth

Change the interfacial concentrations

Spherical precipitate growing in a supersaturated matrix





Ammar et al. (2011)

$$c^{\psi} = c^{\psi 0} + [\![f_{el}]\!] - \sigma_{ij}^{\beta}[\![\varepsilon_{ij}]\!] + 2\bar{\kappa}\gamma$$

- plasticity hinders growth for some morphologies!!
- May also change the bulk diffusion (cf. open elastic constants)

Morphologies

Elasticity selects the shape

The elastic kernel

In Fourier space

- The elastic energy is a simple product

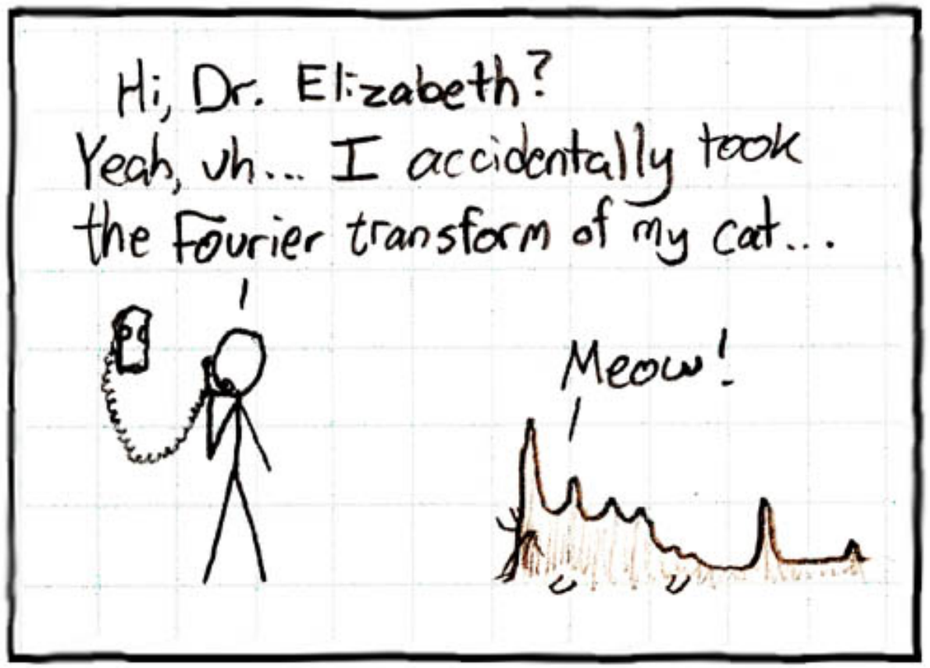
$$\mathrm{E_{el}} = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} B(\vec{n}) \times |\theta(\vec{q})|^2$$

→ distribution of the second phase $|\theta(\vec{q})|^2$

→ elastic kernel $B(\vec{n}) = \varepsilon^\star : C : \varepsilon^\star - \vec{n}\cdot\varepsilon^\star : \Omega : \varepsilon^\star \cdot \vec{n}$

$\Omega^{-1}(\vec{n}) = \vec{n}\cdot C\cdot\vec{n}$

Eq. to laminates

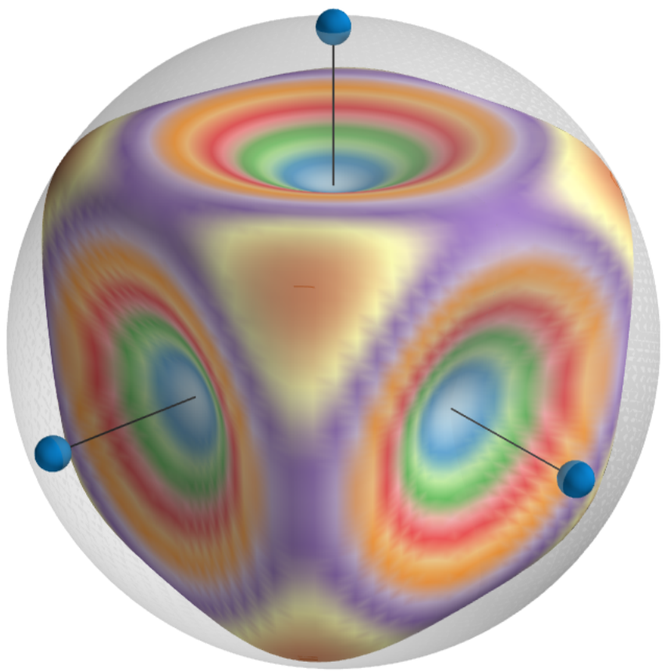


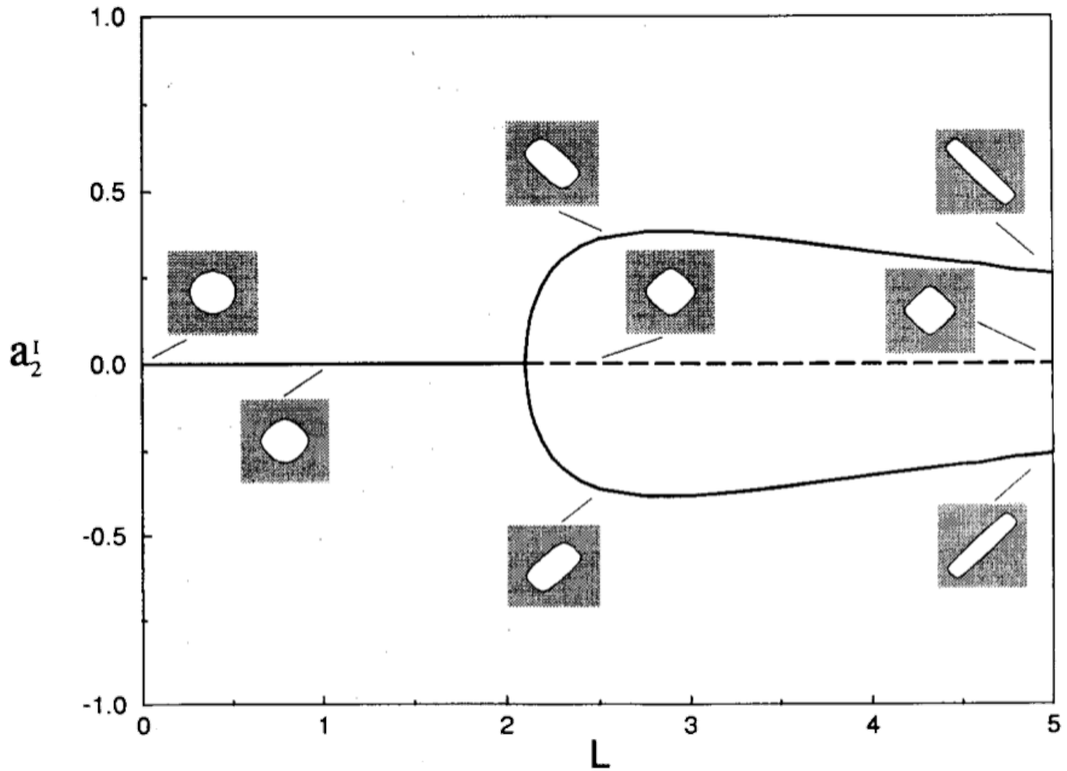
Analyzing $B(\vec{n})$: soft directions (preferred planes) and hard ones

Elasticity selects the shape

Isotropic eigenstrain

- Isotropic elastic constants but inhomogeneous: sphere or plate
- Anisotropic elastic constants: sides normal to soft directions



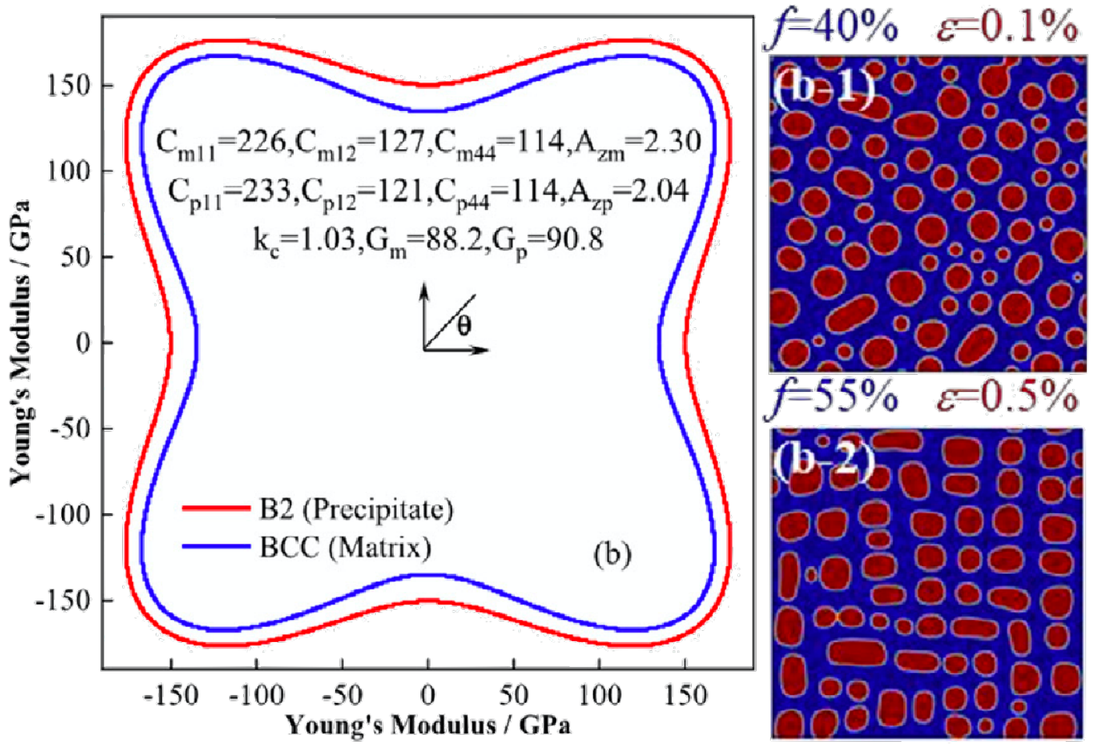


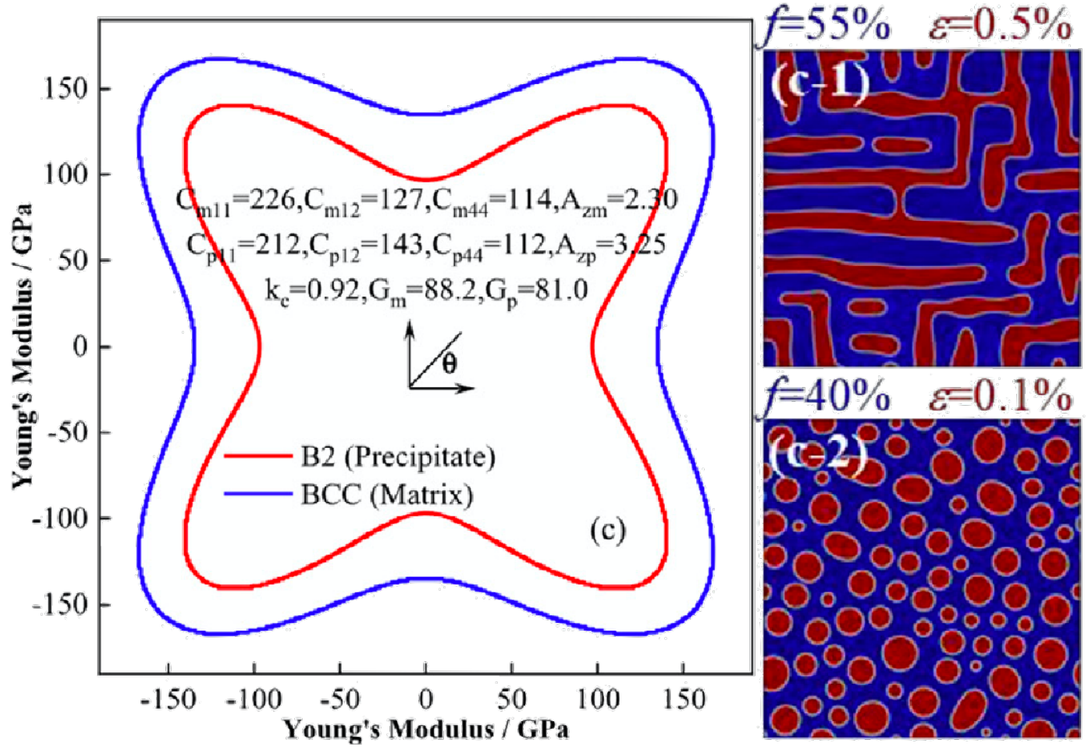
Thompson, Su, and Voorhees (1994)

Elasticity selects the shape

Isotropic eigenstrain

Al-Co-Cr-Fe-Ni J. L. Li et al. (2020)



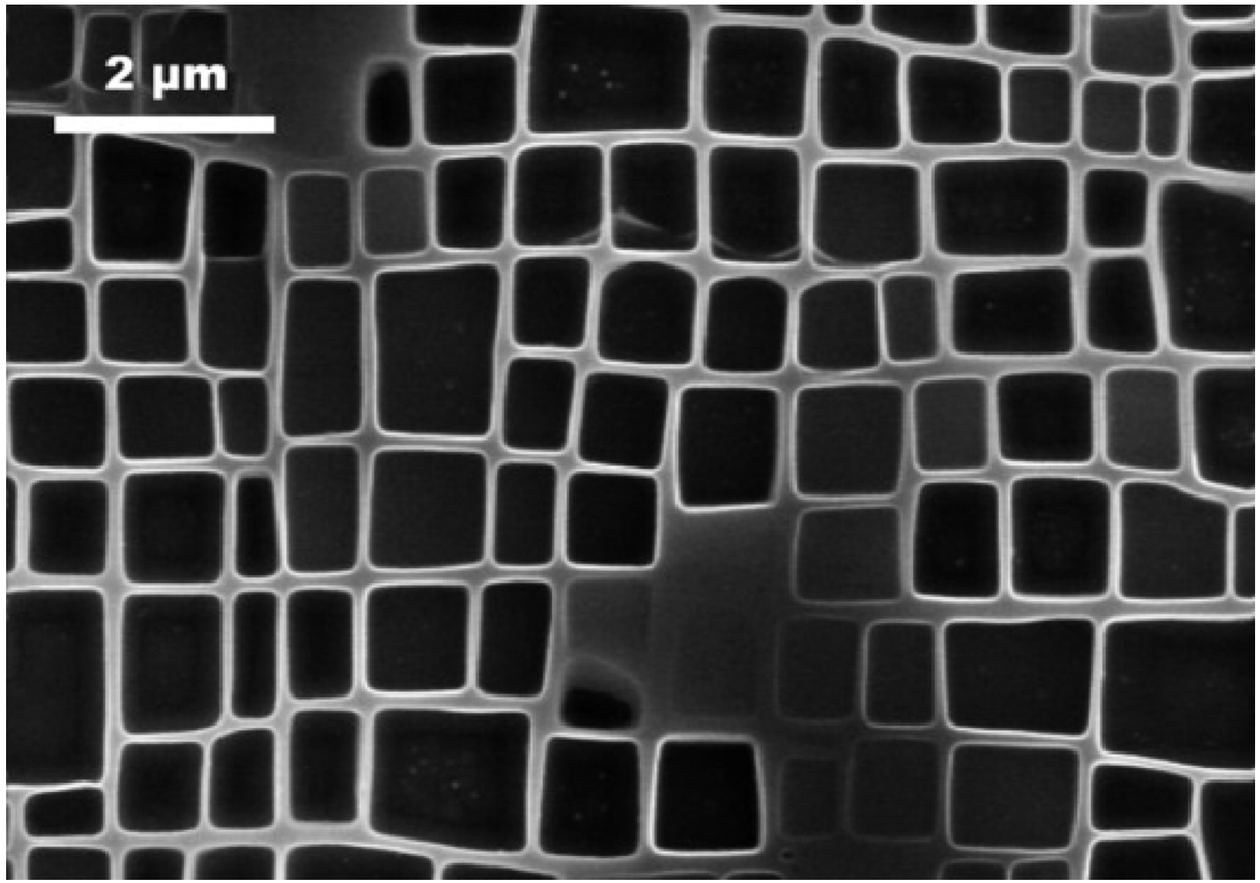


- For small $\mathrm{F}_{el} \implies$ spheres

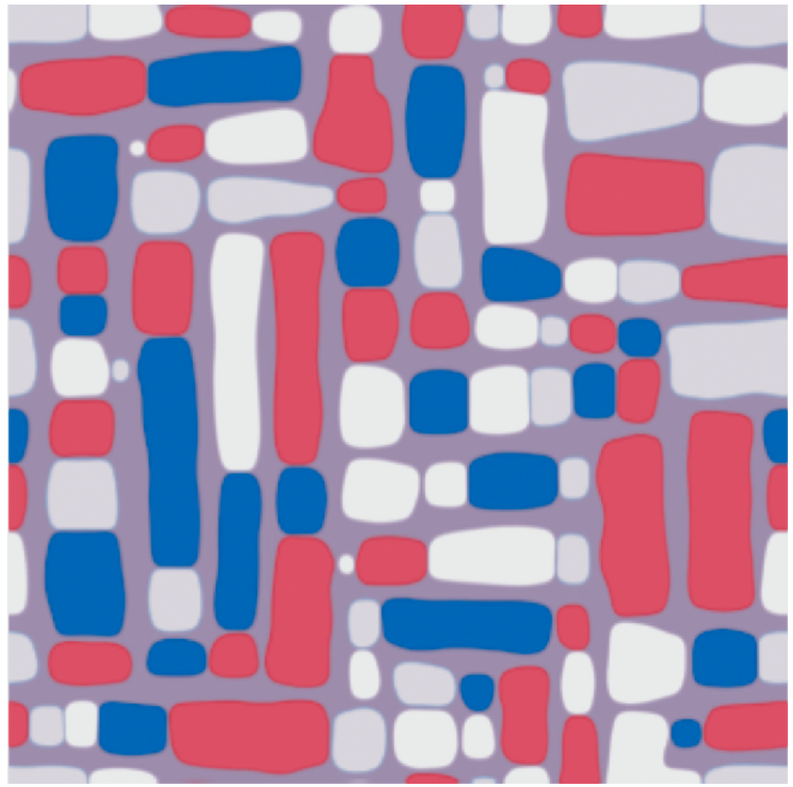
- For large F_{el}
  - → hard in soft $\implies$ cubes
  - → soft in hard $\implies$ plates

Elasticity selects the shape

Elastic constants are not always easy to know



AM1 Diologent and Caron (2004)



Cottura et al. (2015)

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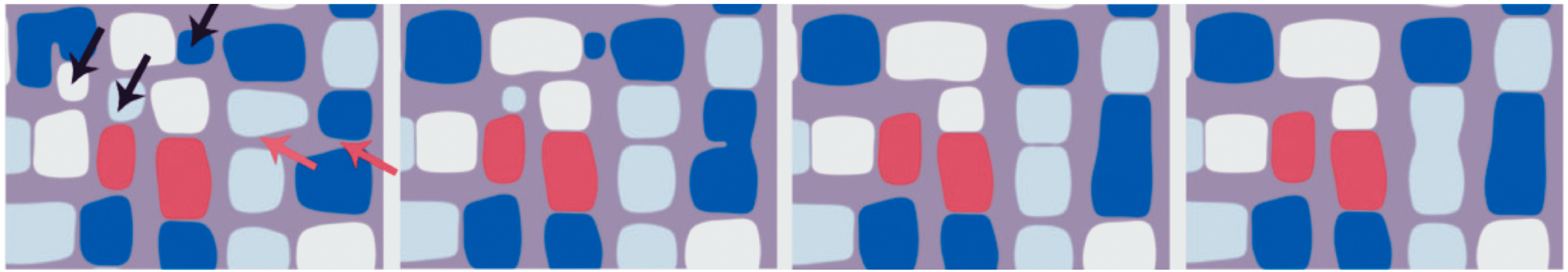
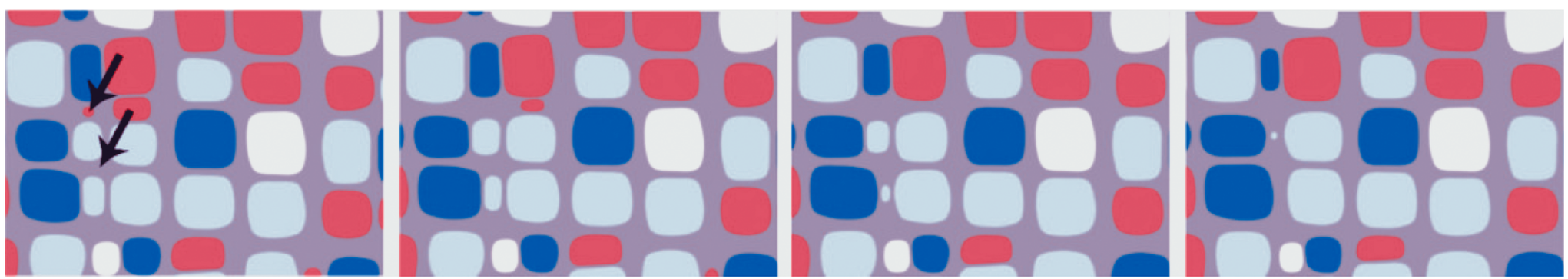
Elastic constants are not always easy to know

Importance of $\Delta C' = (C_{11} - C_{12})/2$ to get cubes

$\Delta C' = 17\%$ (reverse engineering)



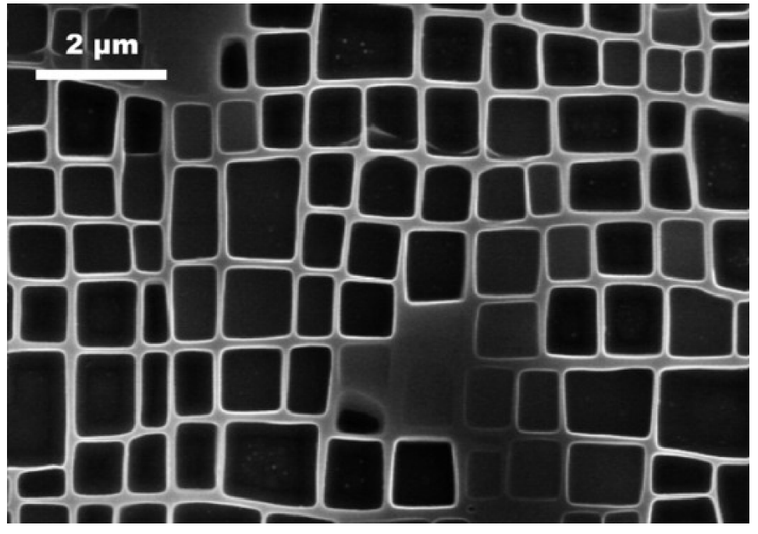
$\Delta C' = 50\%$



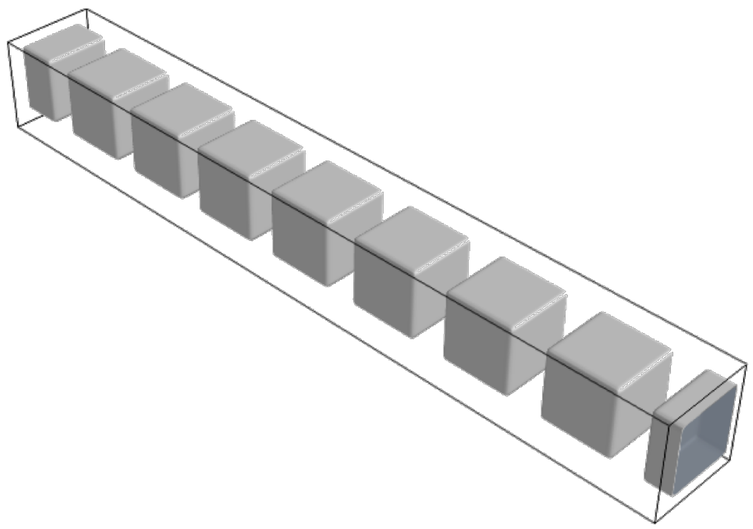
Elasticity selects the shape

Why no perfect rows and columns? [Degeiter et al. (2020)]

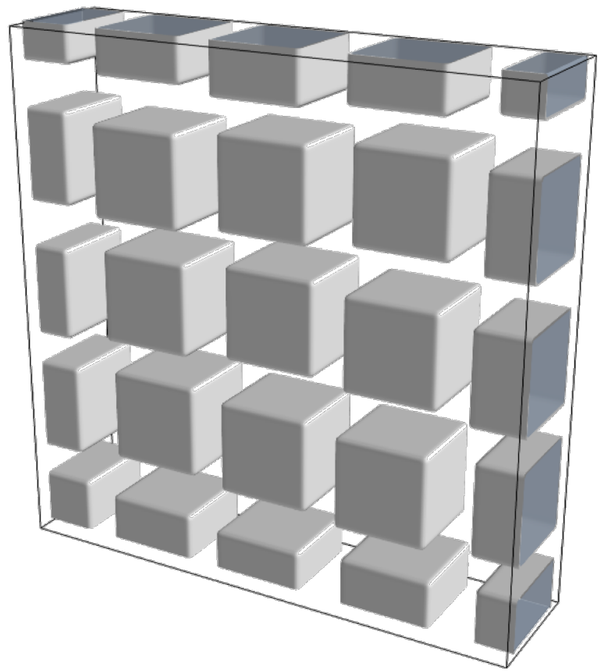
- perfect array of spheres = stable
- perfect array of cubes = unstable



Squeezed mode



Zigzag mode



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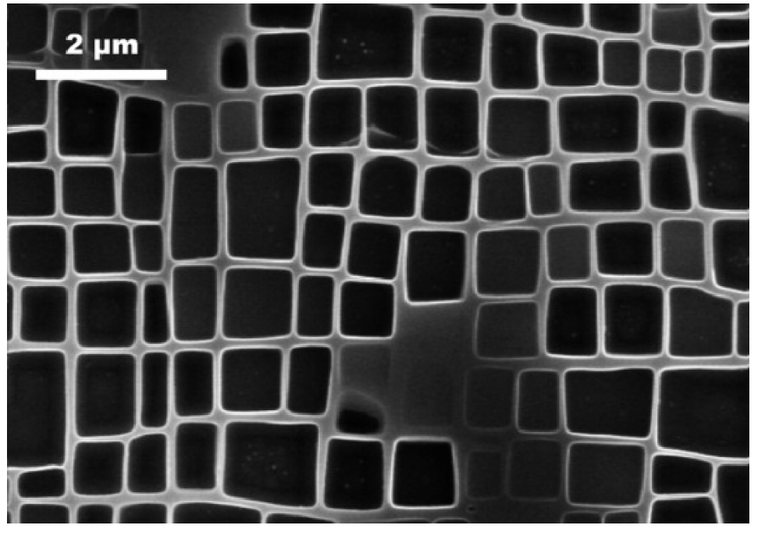

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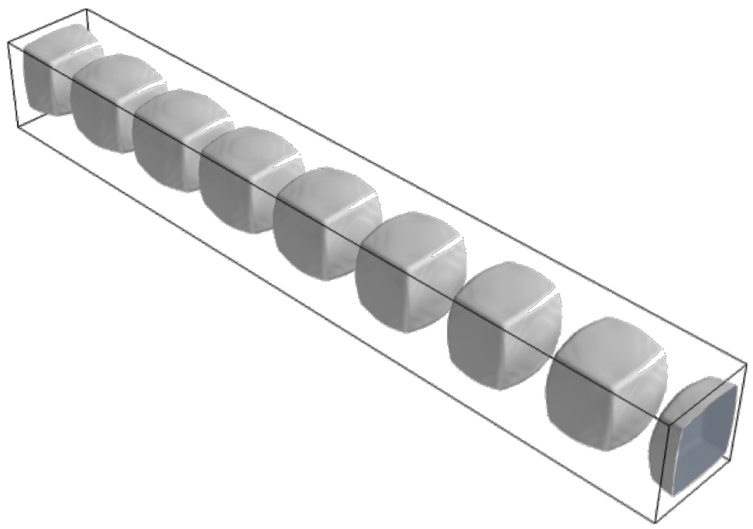

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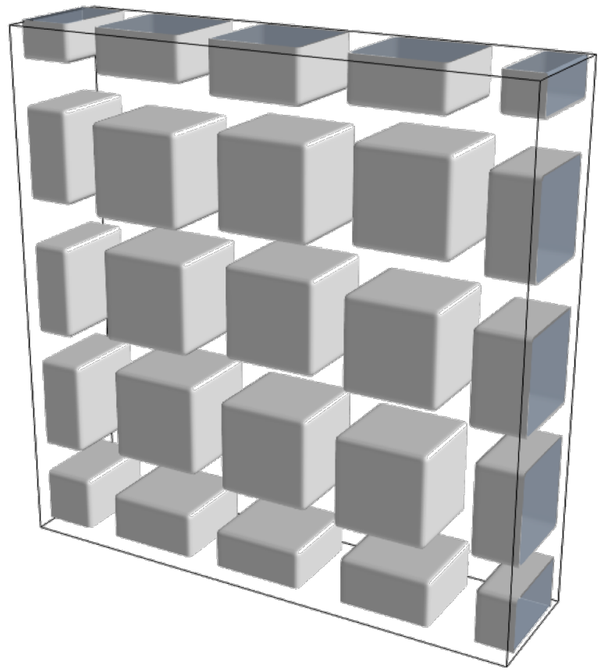

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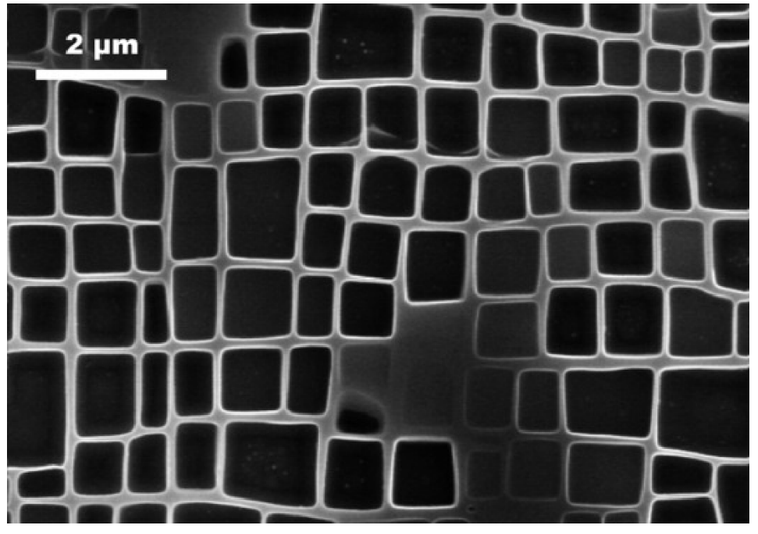

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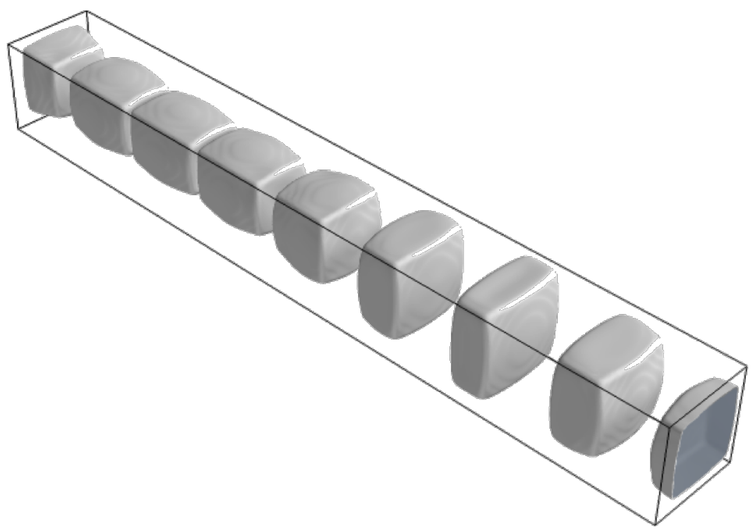

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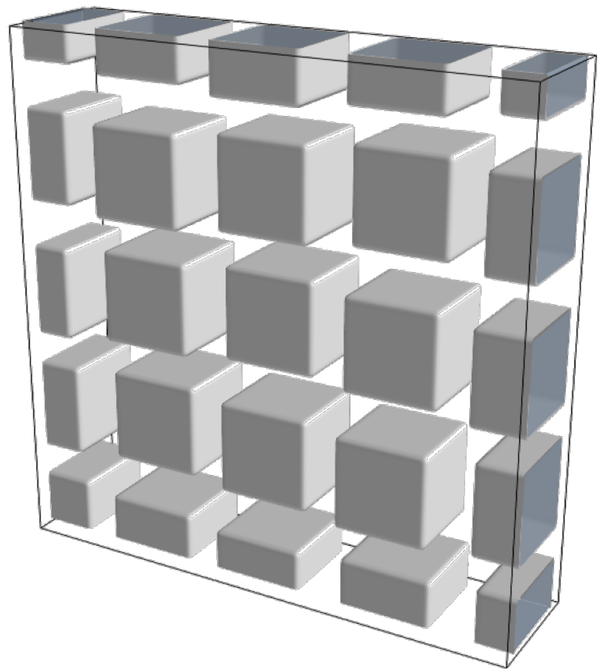

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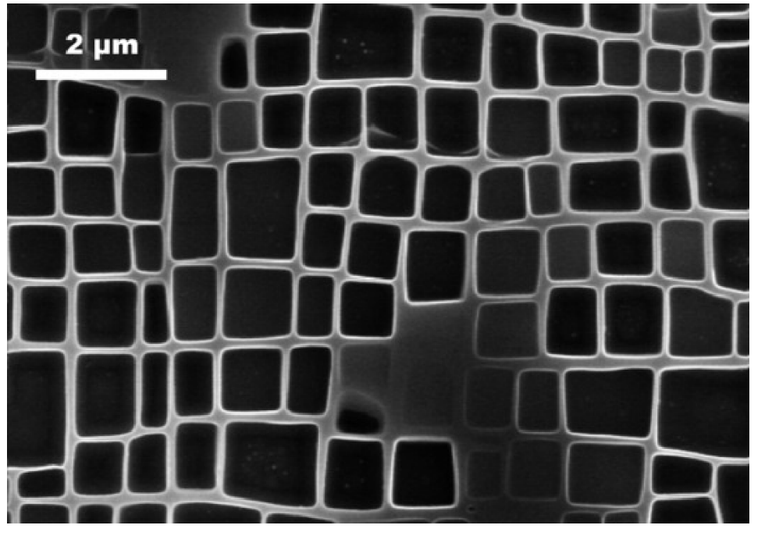

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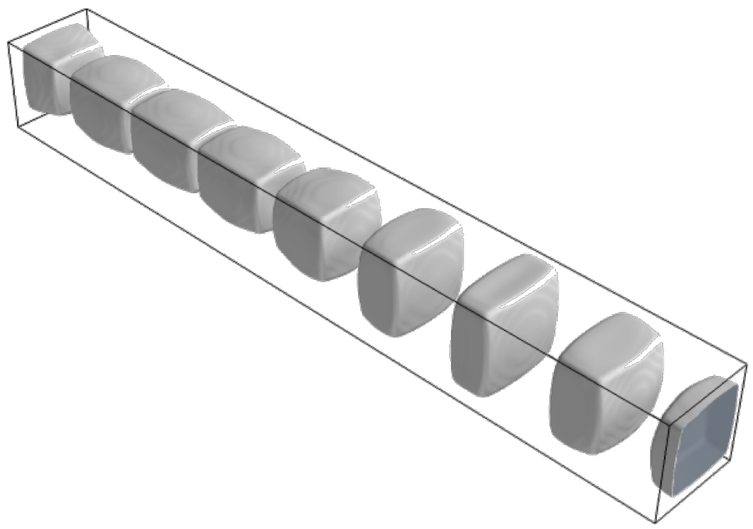
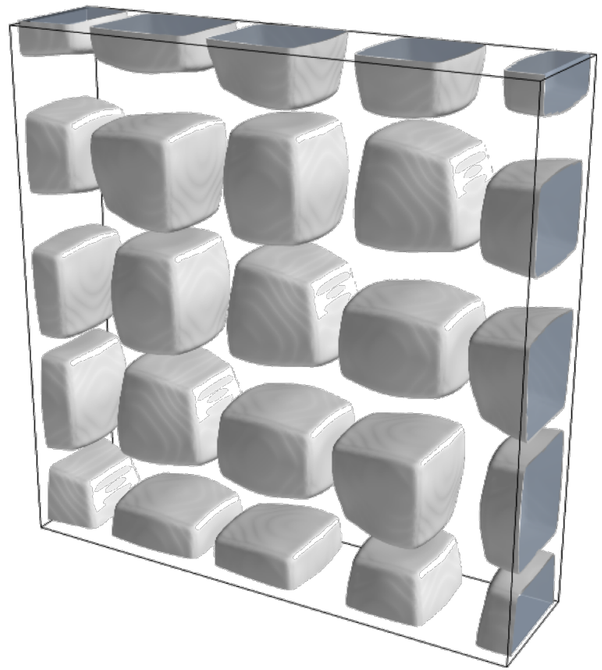


Zigzag mode



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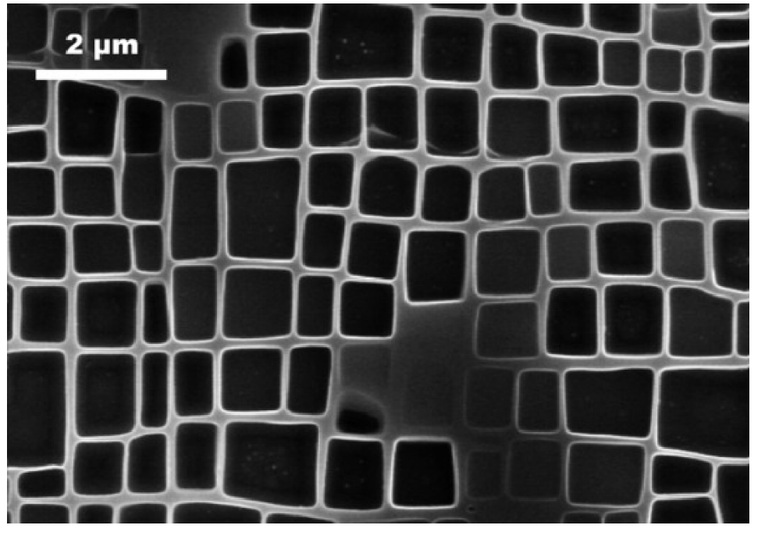

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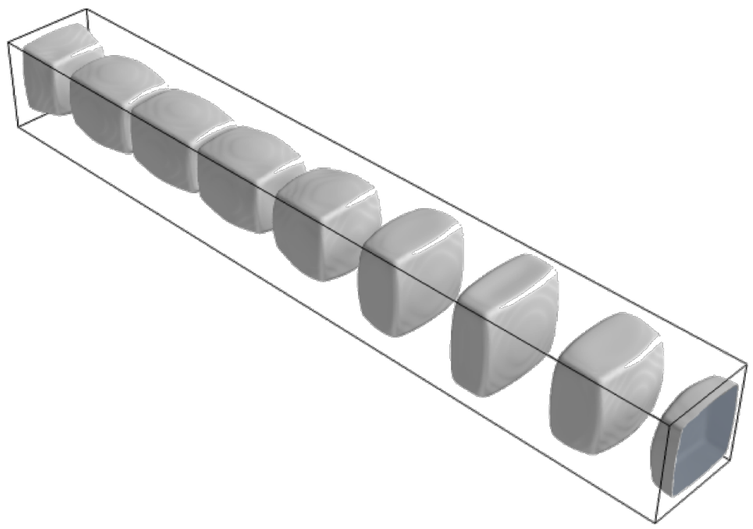

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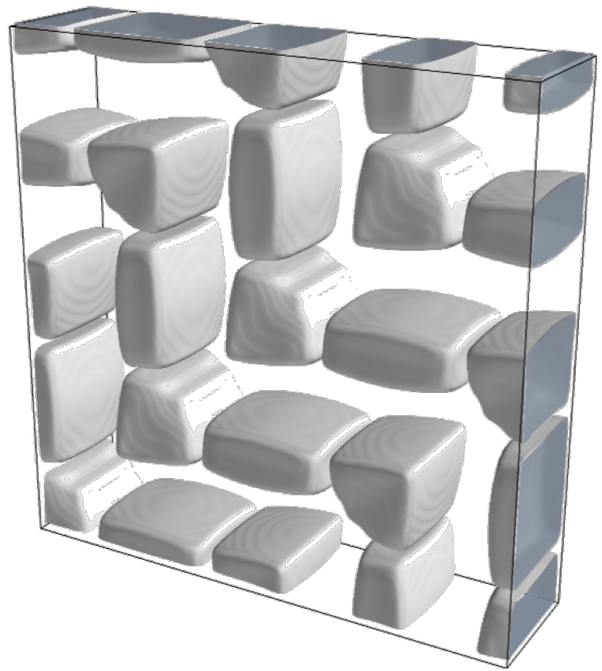

Conclusion

No more time (read good papers)

A few points

- Kinetics and shape of acicular precipitates (Widmanstätten and beyond) Struggling against solidification like solution
- Plasticity may promote shape change
  - → isotropic: competition between time scales of loading and diffusion
  - → anisotropic: impact for moderate
- Kinetics is related to morphology

Is there anything to do at the process scale?

- Coupling mechanics and phase transformation (except for SMA) has been the realm of full-field computations because of the morphological evolutions
- But it is maybe time to upscale these calculations, although there is surely not a universal strategy

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