

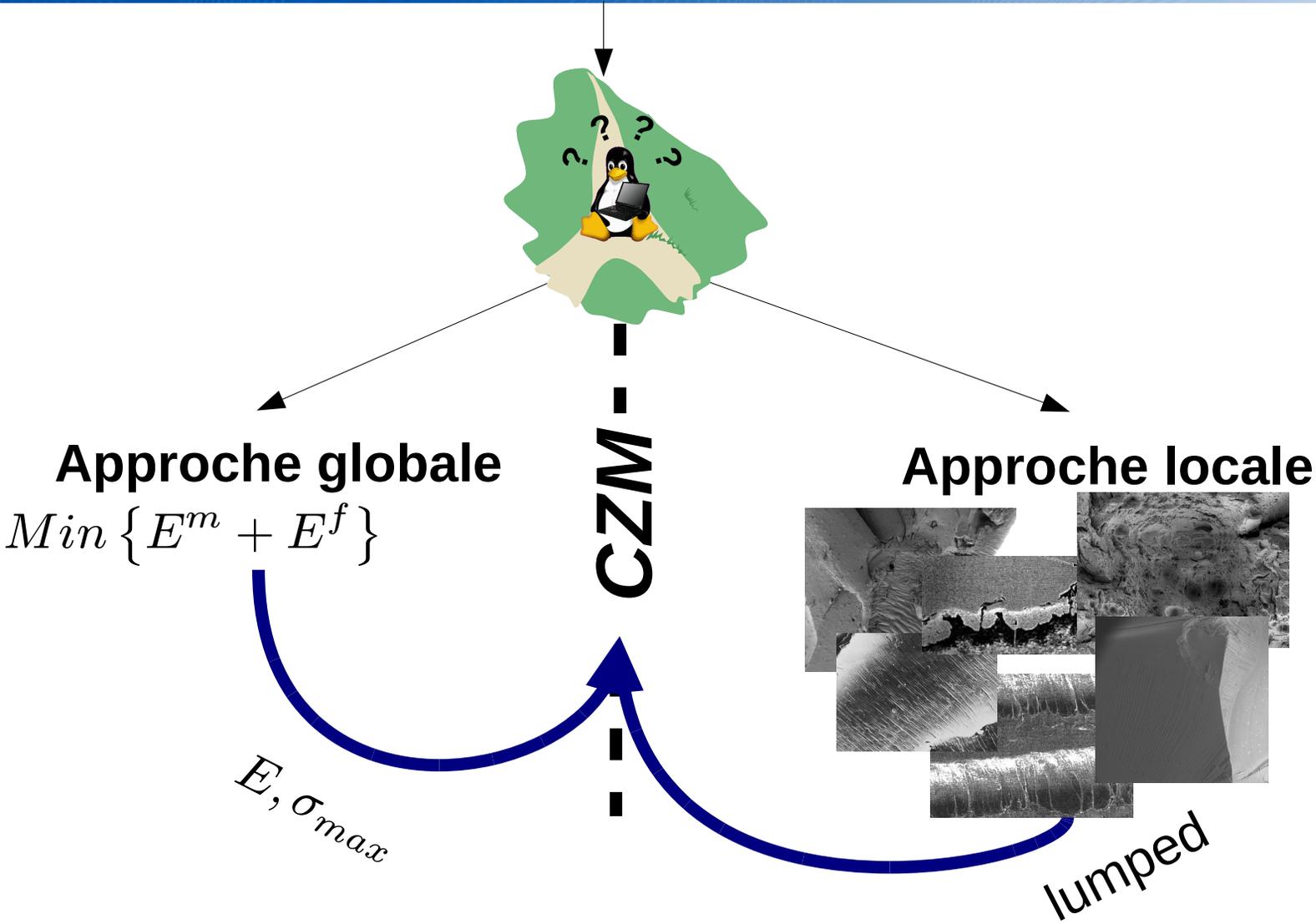
# MZC et comportements volumiques non linéaires

F. Feyel, V. Chiaruttini, S. Payet

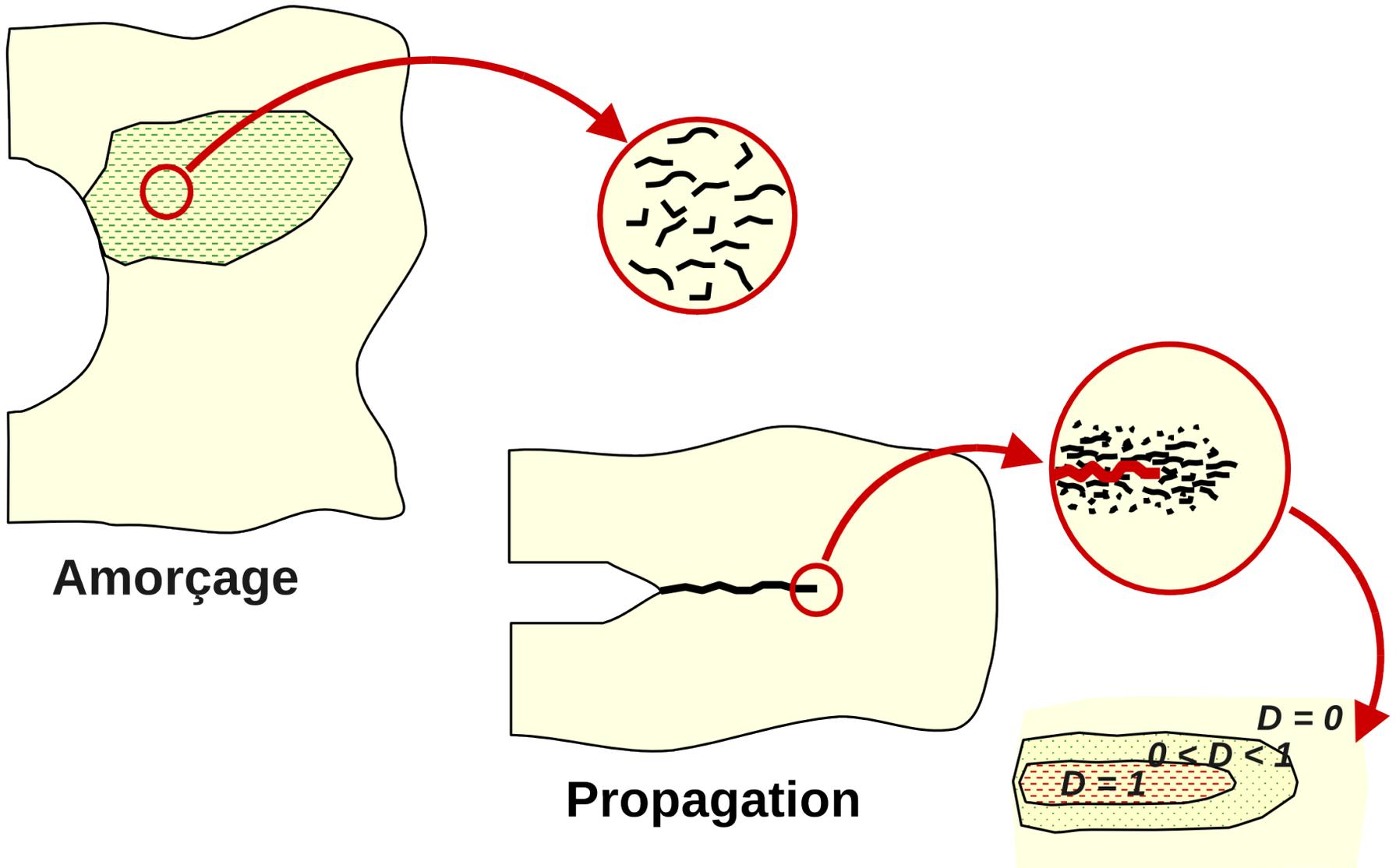


return on innovation

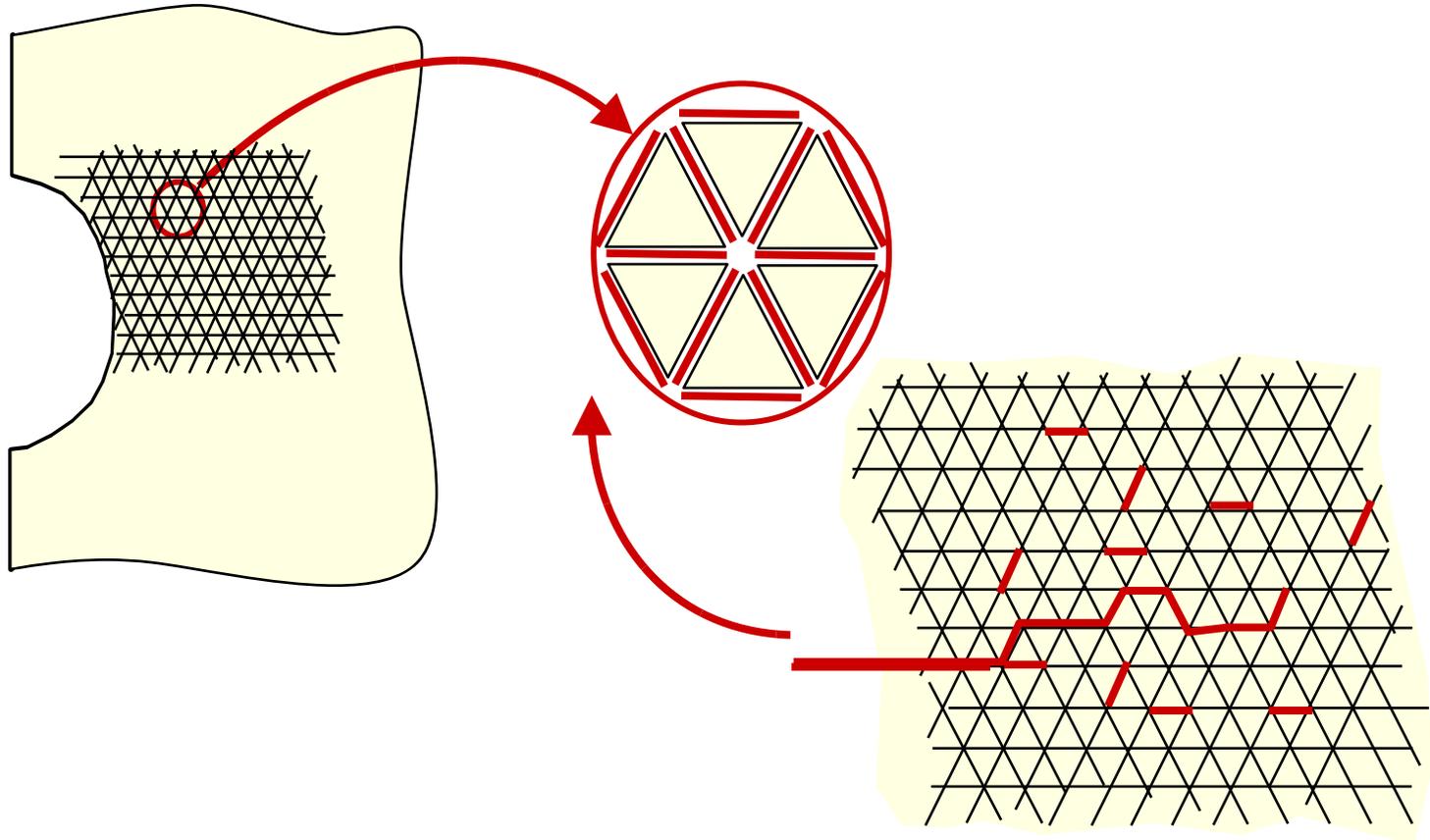
# Modèles numériques pour la fissuration



# Approche locale, modèles d'endommagement

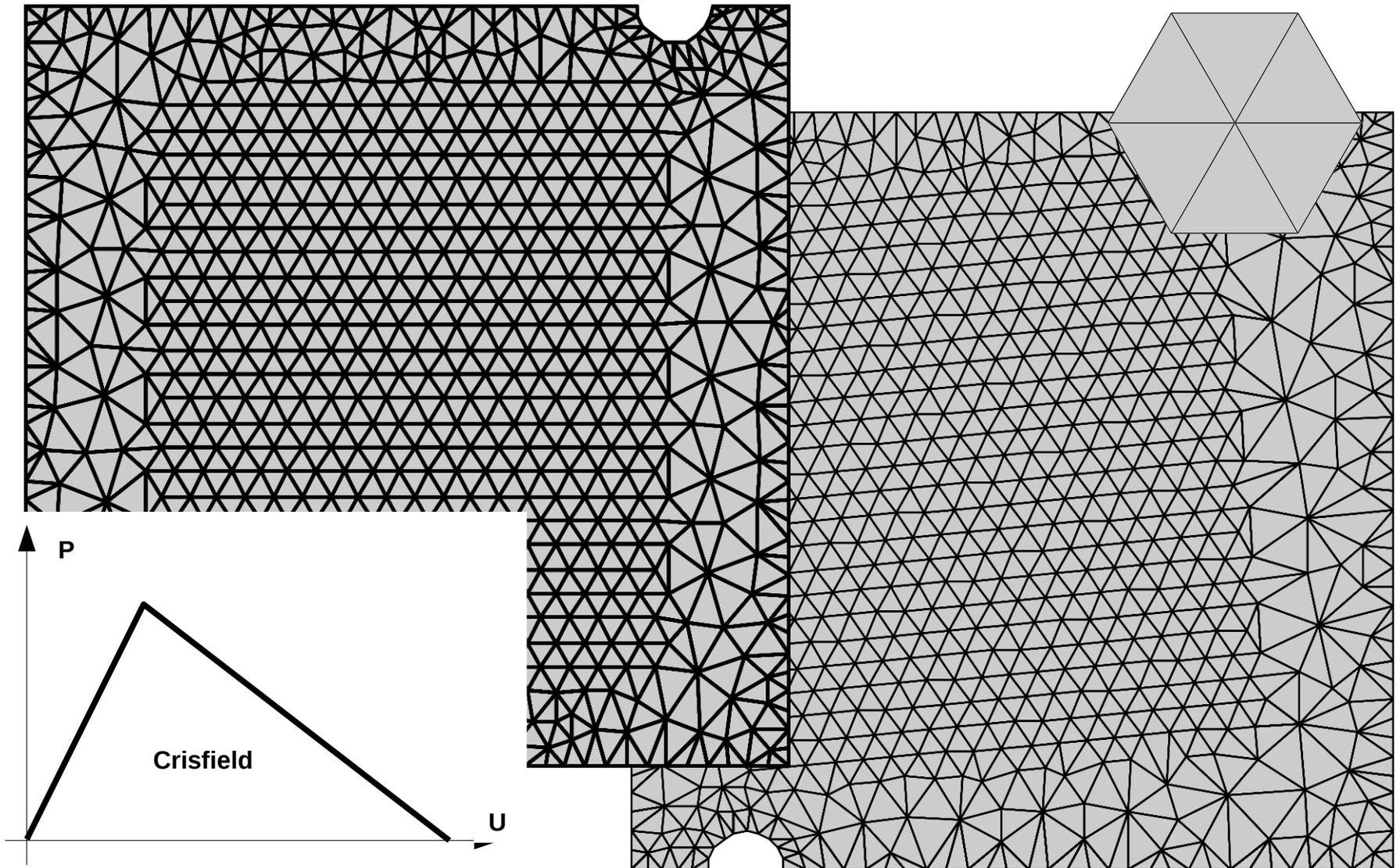


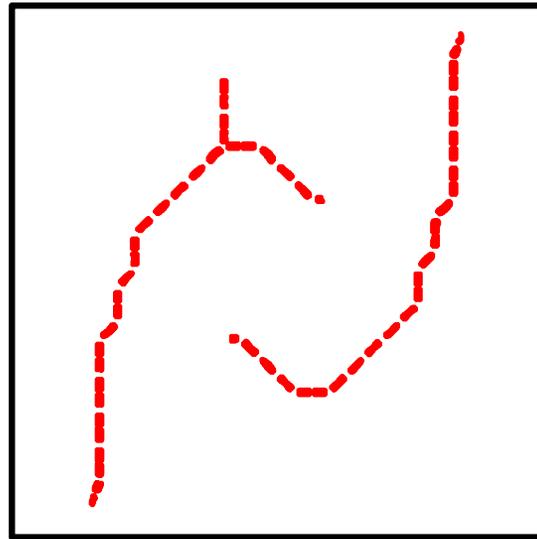
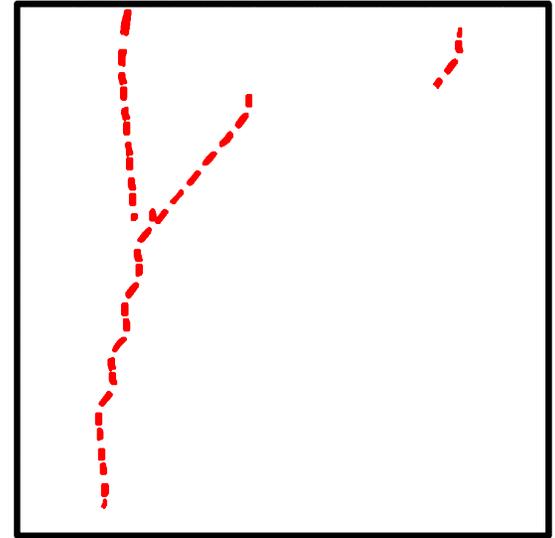
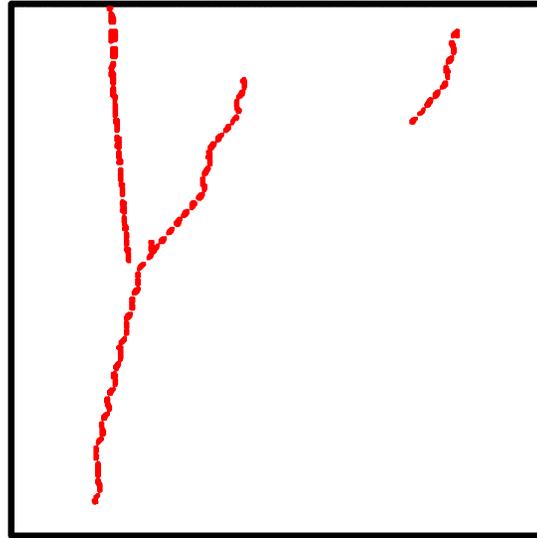
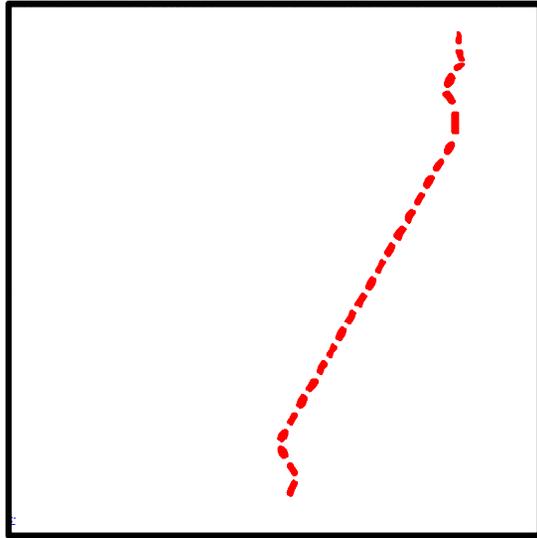
# Equivalence avec des CZM ?



**Fissure localisée**

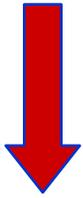
# Influence du maillage



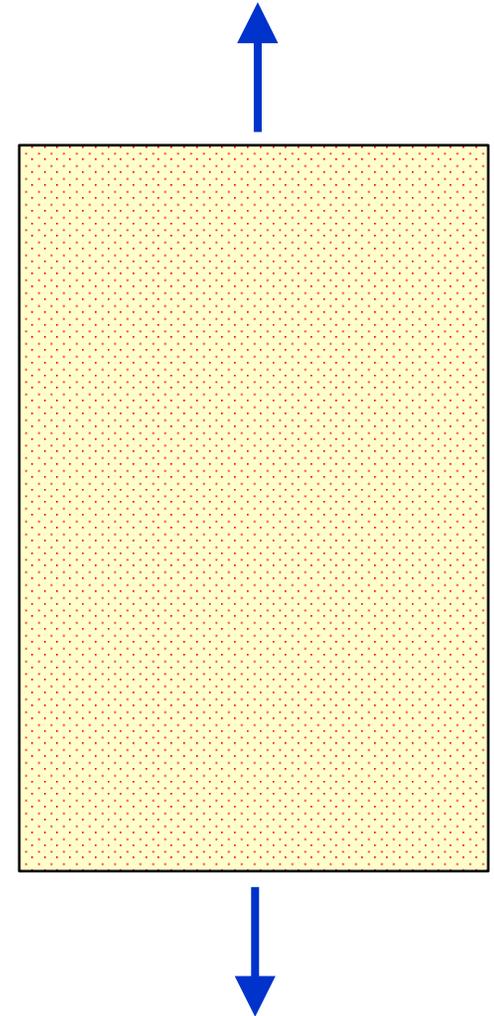
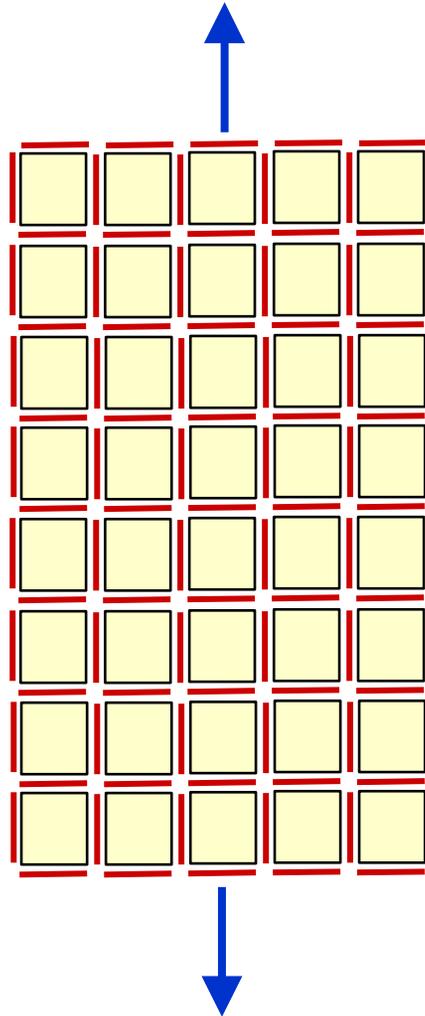


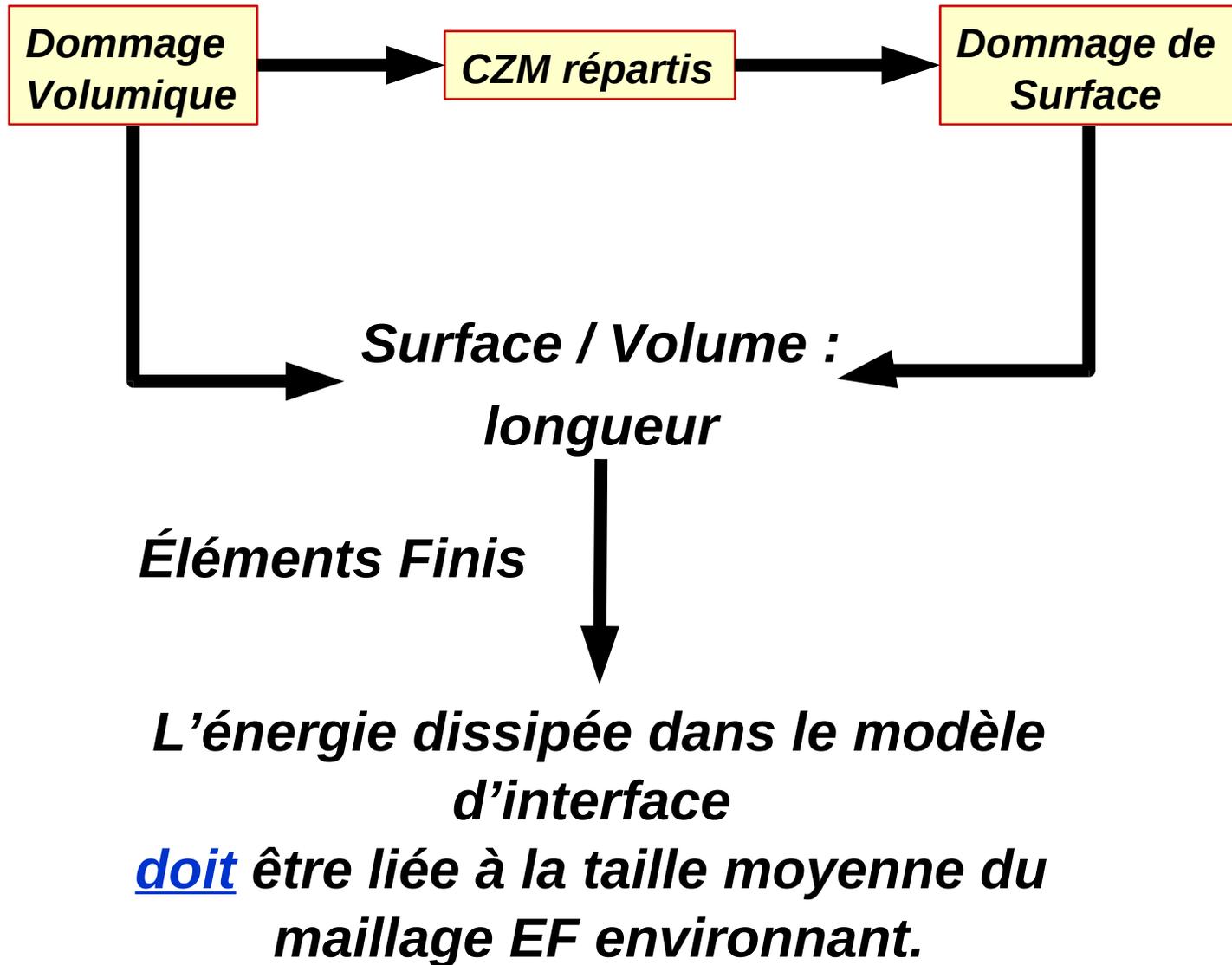
- Tels quels, les CZM
  - Ne **permettent pas** de modéliser la fissuration si le lieu est inconnu
  - Éventuellement, en fragmentation, permettent de construire + **ou** - des statistiques moyennes

**Modèle continu  
Équivalent ?**

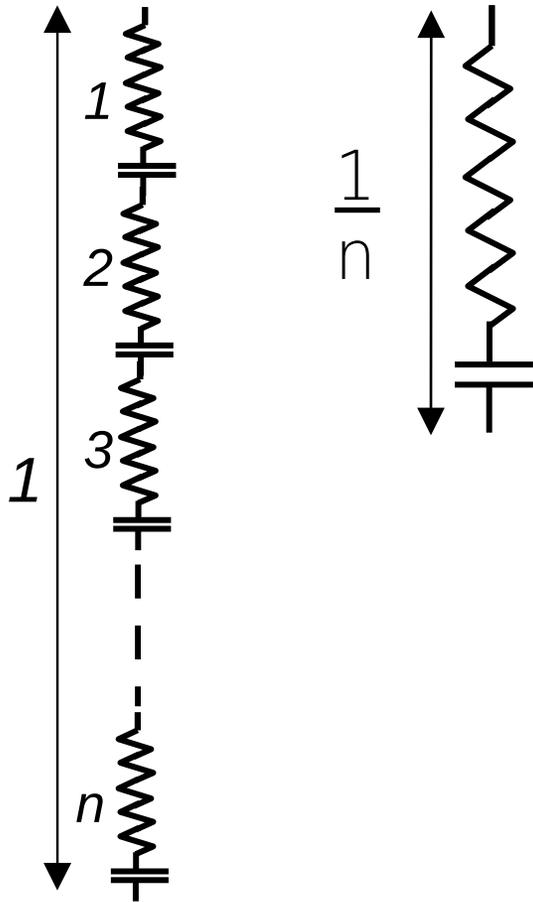


**Appropriation de  
l'énergie de surface à  
la taille du maillage.**





# Vision énergétique



1- Tout dans les barres :

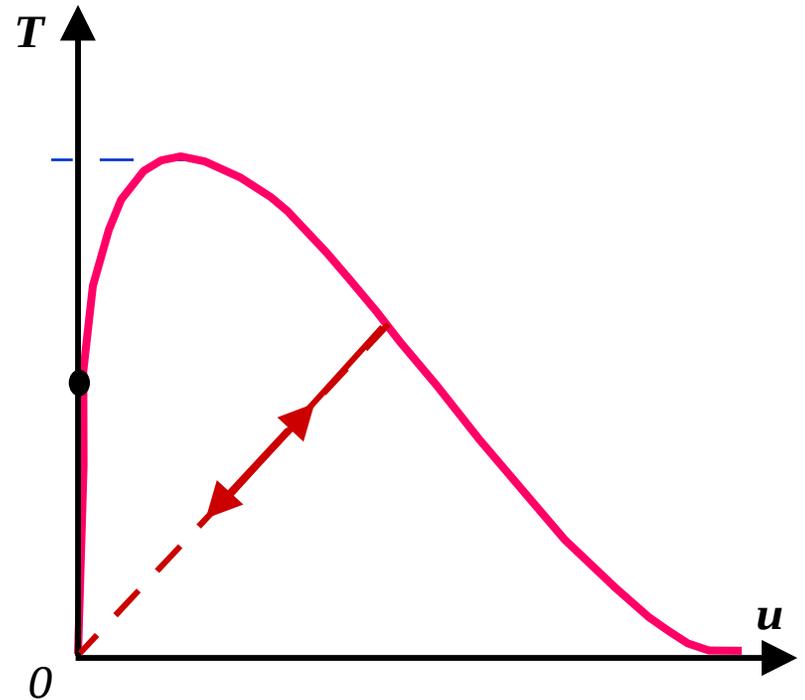
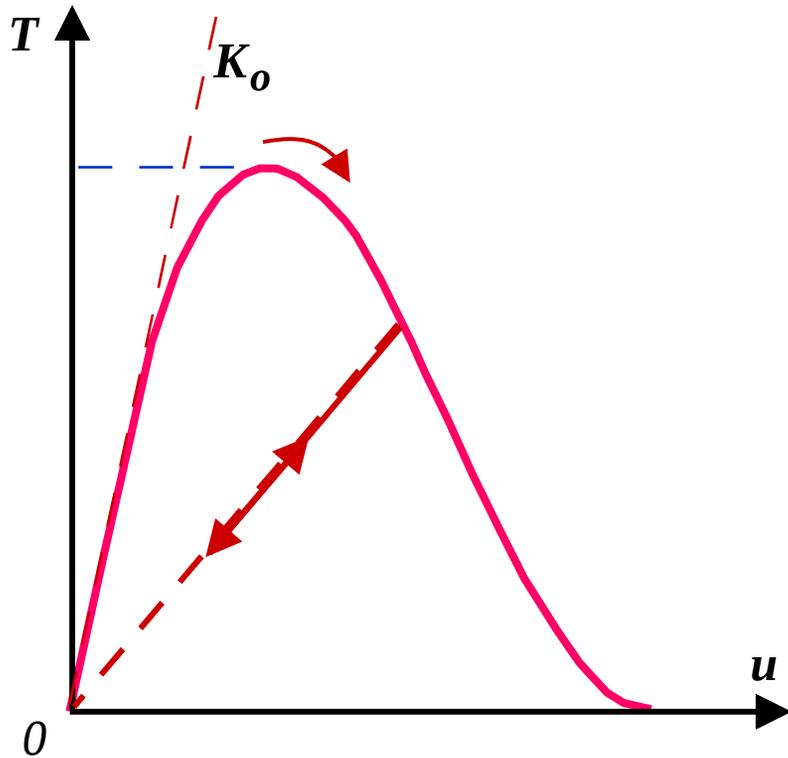
$$\mathcal{E} = \mathcal{E}_{el} = 1/2$$

2- Tout dans les interfaces :

$$\Delta u = 1/n$$
$$\mathcal{E}_{in} = nk(\Delta u)^2 = k/n$$

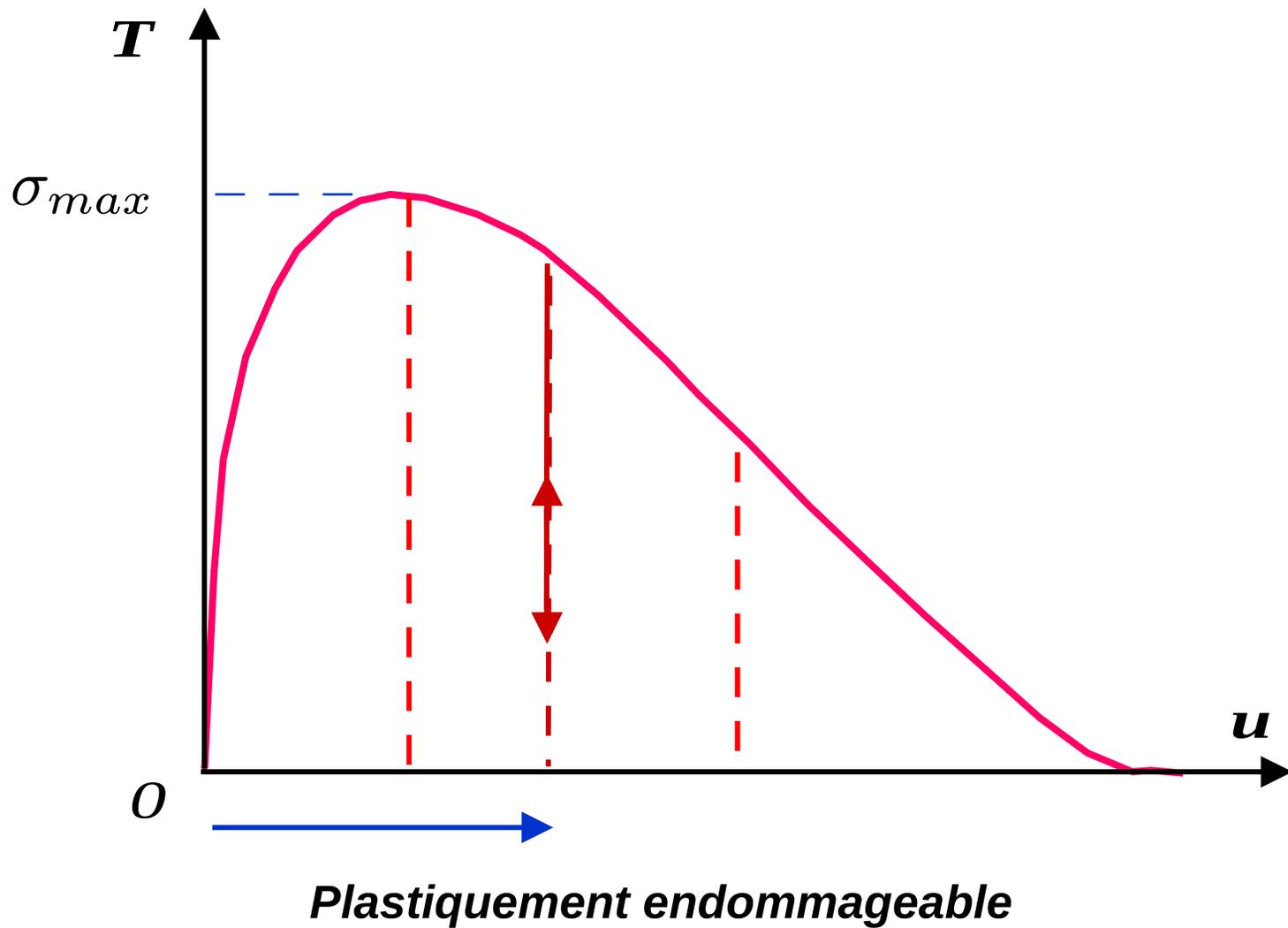
Quand n augmente, 2 est toujours préférable

$$\lim_{n \rightarrow \infty} \mathcal{E}_{in} ?$$



***Une rigidité initiale infinie n'est pas suffisante...***

# Modèle plastiquement endommageable

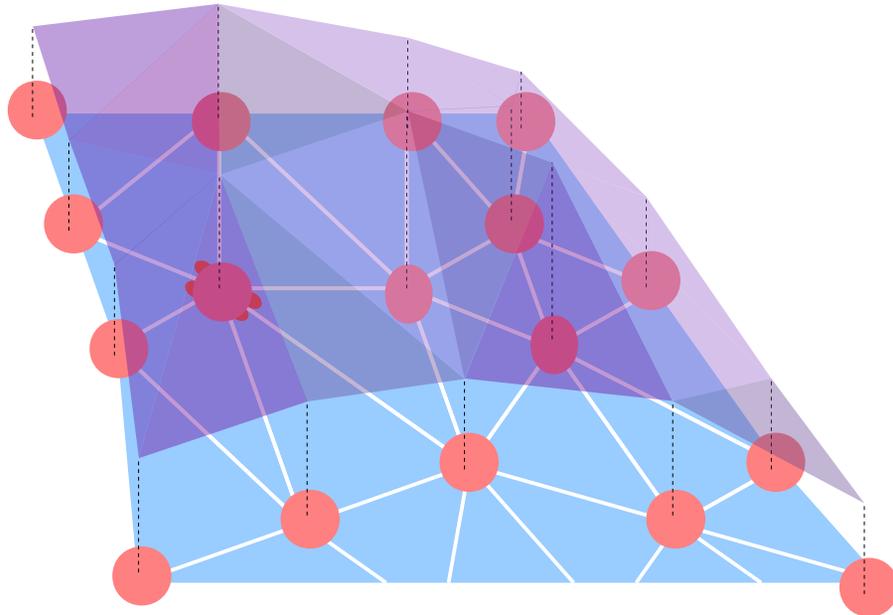


## Ajouter un seuil « propre » à un modèle de ZC...

- Formulation à multiplicateur de Lagrange locale, cf par exemple E. Lorentz
- Utilisation d'approches « Galerkin Discontinu »

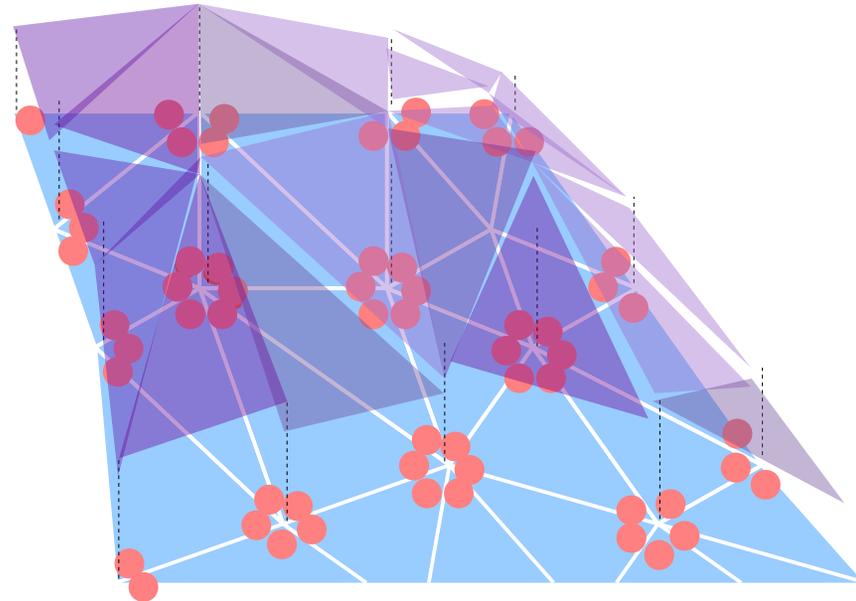
## Galerkin continue

Espace d'approximation continu sur l'ensemble de la structure



## Formulation Galerkin discontinue

Espaces d'approximation continus par morceaux  
Ajout de formulations faibles aux **interfaces inter-éléments**  
continuité au sens faible



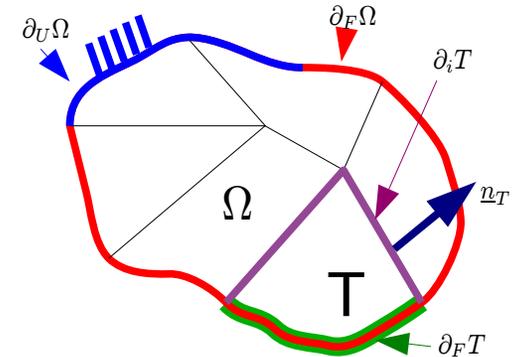
# Approches Galerkin-discontinues

- Pénalisation intérieure pour les problèmes élastiques [Nitsche 1971]  
Application à un problème d'élasticité linéaire

$$\boldsymbol{\sigma} = \mathcal{A} \frac{1}{2} \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) = K \boldsymbol{\varepsilon}(\mathbf{u}), \text{ in } \Omega$$

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{f}_d \text{ in } \Omega$$

$$\mathbf{u} = \mathbf{u}_d \text{ on } \partial_U \Omega, \quad \boldsymbol{\sigma} \mathbf{n} = \mathbf{F}_d \text{ on } \partial_F \Omega$$



- Formulation Galerkin Discontinue, Opérateurs de moyenne et sauts

$$[[\mathbf{u}]] = \begin{cases} \mathbf{u}^+ - \mathbf{u}^-, & \text{on } \partial_i T \\ \mathbf{u}^+, & \text{on } \partial_F T \cup \partial_U T \cup \end{cases} \quad \langle \mathbf{u} \rangle = \begin{cases} \frac{1}{2} (\mathbf{u}^+ + \mathbf{u}^-), & \text{on } \partial_i T \\ \mathbf{u}^+, & \text{on } \partial_F T \cup \partial_U T \end{cases}$$

- Formulation faible bilinéaire symétrique

$$\left. \begin{aligned} & \int_K \boldsymbol{\varepsilon}(\mathbf{u}_h) : \mathcal{A} : \boldsymbol{\varepsilon}(\mathbf{v}) \, dK - \int_{\partial K} \langle \boldsymbol{\sigma}(\mathbf{v}) \rangle \cdot \mathbf{n} \cdot [[\mathbf{u}_h]] \\ & + \langle \boldsymbol{\sigma}(\mathbf{u}_h) \rangle \cdot \mathbf{n} \cdot [[\mathbf{v}]] \, dS + \int_{\partial K} \frac{\kappa}{h} [[\mathbf{u}_h]] \otimes \mathbf{n} : \mathcal{A} : \mathbf{n} \otimes [[\mathbf{v}]] \, dS \\ & = \int_{\partial_F K} \mathbf{F}_h^D \cdot \mathbf{v} \, dS \end{aligned} \right\}$$

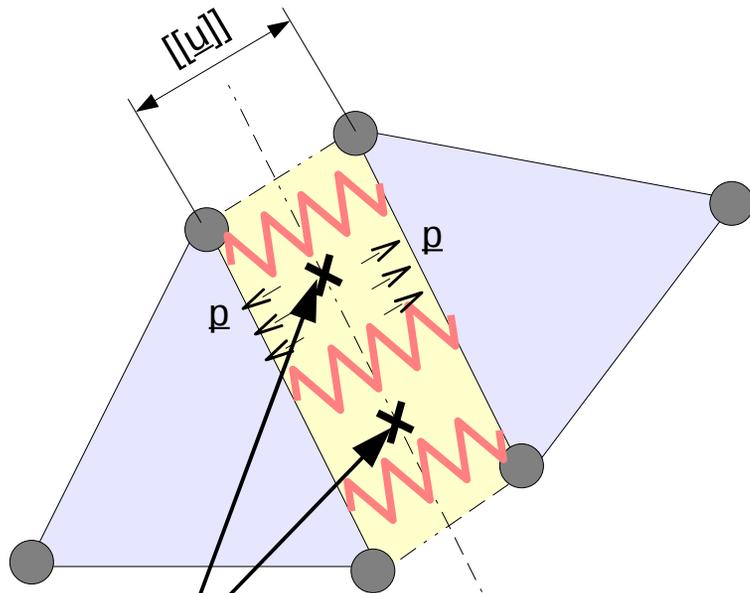
$\forall v$  régulier et continu par morceaux sur la triangulation  $T$

Classique

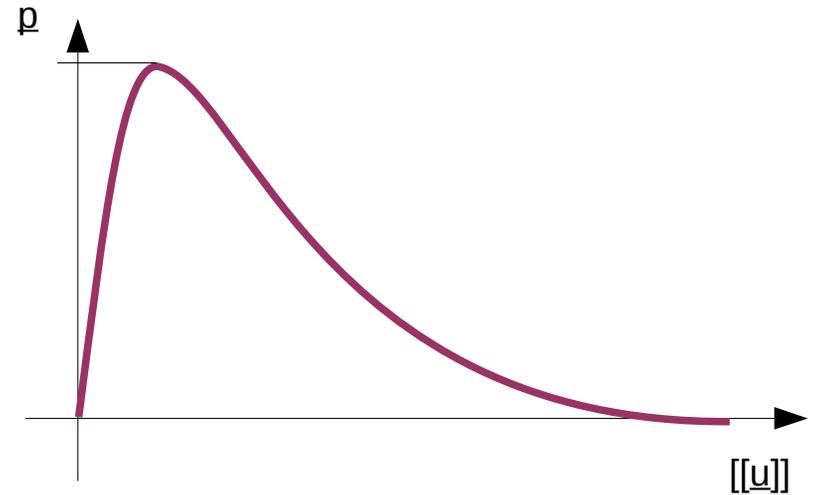
$$\int_K \boldsymbol{\varepsilon}(\mathbf{u}_h) : \mathbf{A} : \boldsymbol{\varepsilon}(\mathbf{v}) \, dK - \int_{\partial K} \langle \boldsymbol{\sigma}(\mathbf{v}) \rangle \cdot \mathbf{n} \cdot [[\mathbf{u}_h]]$$
$$+ \langle \boldsymbol{\sigma}(\mathbf{u}_h) \rangle \cdot \mathbf{n} \cdot [[\mathbf{v}]] \, dS + \int_{\partial K} \frac{\kappa}{h} [[\mathbf{u}_h]] \otimes \mathbf{n} : \mathbf{A} : \mathbf{n} \otimes [[\mathbf{v}]] \, dS$$
$$= \int_{\partial_F K} \mathbf{F}_h^D \cdot \mathbf{v} \, dS$$

Classique

# Analogie CZM



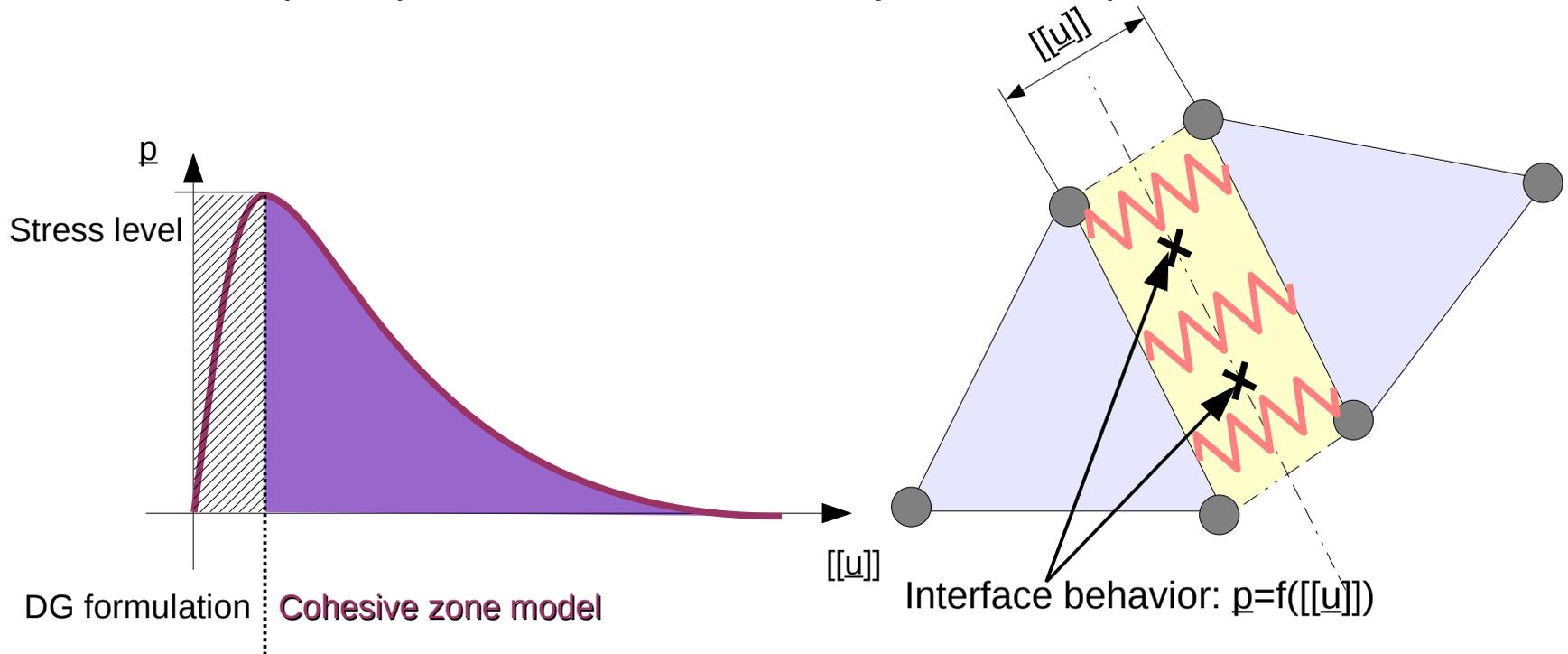
Comportement d'interface :  
 $p=f([[u]])$



Relation force/déplacement d'interface

# Couplage DG - CZM

- Formulation d'interface GD pendant la phase élastique du modèle cohésif
- Critère de transition en contrainte maximale
- Activation d'un modèle dissipatif
- Idée identique à (*Steinman et al 2004, Levy et al. 2009*)

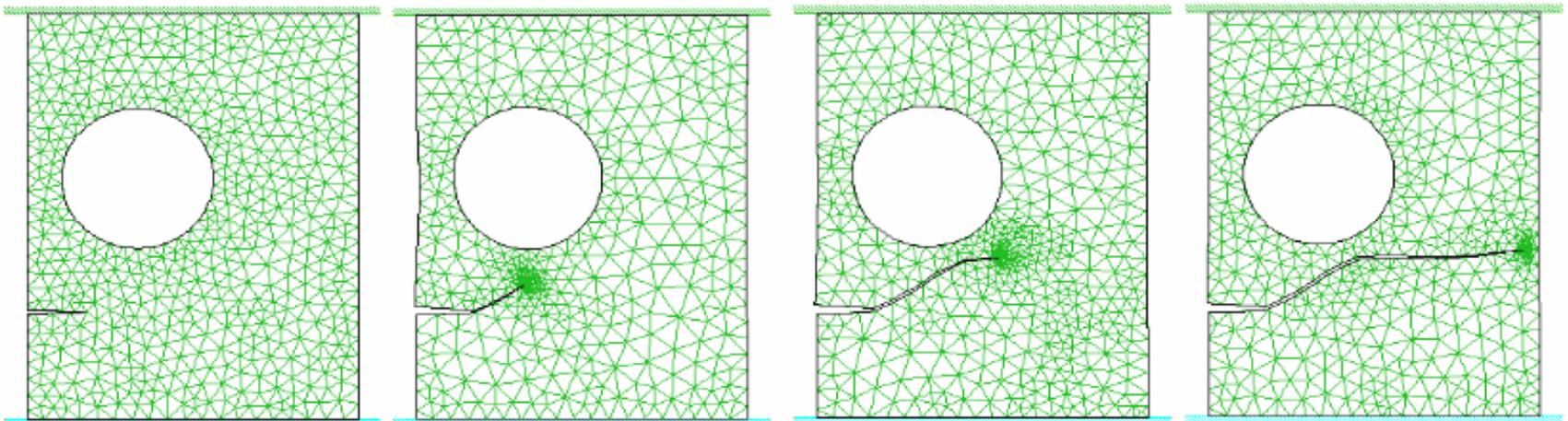


$$\begin{aligned}
& \int_K \boldsymbol{\varepsilon}(\mathbf{u}_h) : \mathbf{A} : \boldsymbol{\varepsilon}(\mathbf{v}) \, dK - \int_{\partial K} \langle \boldsymbol{\sigma}(\mathbf{v}) \rangle \cdot \mathbf{n} \cdot [[\mathbf{u}_h]] \\
& + \langle \boldsymbol{\sigma}(\mathbf{u}_h) \rangle \cdot \mathbf{n} \cdot [[\mathbf{v}]] \, dS + \int_{\partial K} \frac{\kappa}{h} [[\mathbf{u}_h]] \otimes \mathbf{n} : \mathbf{A} : \mathbf{n} \otimes [[\mathbf{v}]] \, dS \\
& = \int_{\partial_F K} \mathbf{F}_h^D \cdot \mathbf{v} \, dS
\end{aligned}$$

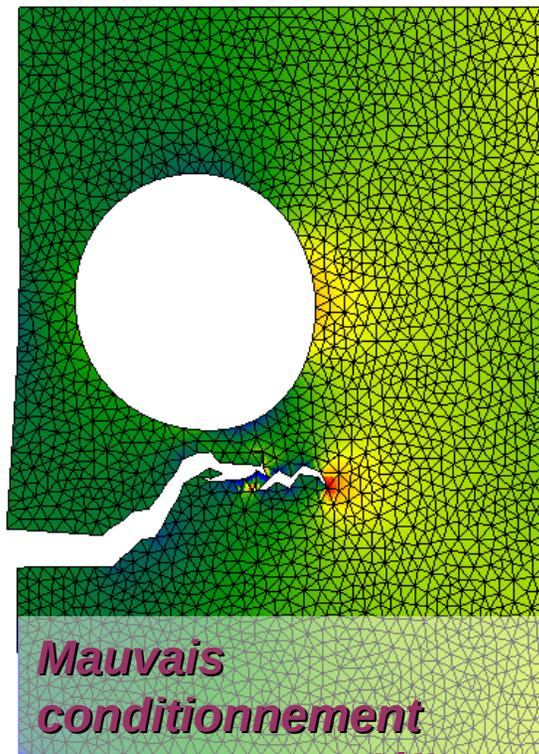
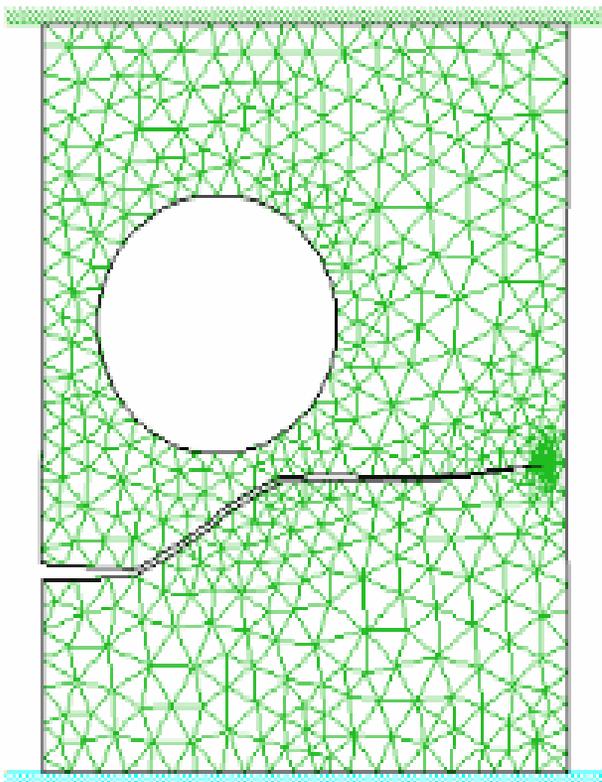
$$\begin{aligned}
& \int_K \boldsymbol{\varepsilon}(\mathbf{u}_h) : \mathbf{A} : \boldsymbol{\varepsilon}(\mathbf{v}) \, dK - (1 - \alpha) \int_{\partial K} \langle \boldsymbol{\sigma}(\mathbf{v}) \rangle \cdot \mathbf{n} \cdot [[\mathbf{u}_h]] \\
& - \langle \boldsymbol{\sigma}(\mathbf{u}_h) \rangle \cdot \mathbf{n} \cdot [[\mathbf{v}]] \, dS + \int_{\partial K} \frac{\kappa_s}{h} [[\mathbf{u}_h]] \otimes \mathbf{n} : \mathbf{A} : \mathbf{n} \otimes [[\mathbf{v}]] \, dS \\
& = \int_{\partial_F K} \mathbf{F}_h^D \cdot \mathbf{v} \, dS + \alpha \int_{\partial K} \mathbf{P} ([[ \mathbf{u}_h ]]) \cdot [[\mathbf{v}]] \, dS
\end{aligned}$$

- Simulation de fissuration sur une éprouvette plane perforée en présence d'une fissure initiale sous un chargement de traction verticale

*Solution de référence obtenue à l'aide d'une approche énergétique  $G-q$ , avec un critère de bifurcation en taux de restitution d'énergie maximale et une avancée de pointe de fissure par remaillage  
(thèse de P. O. Bouchard, 2000)*

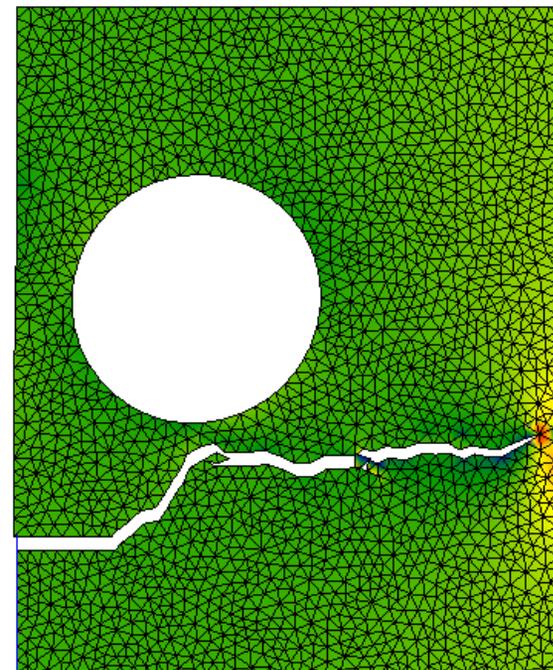


CZM généralisé



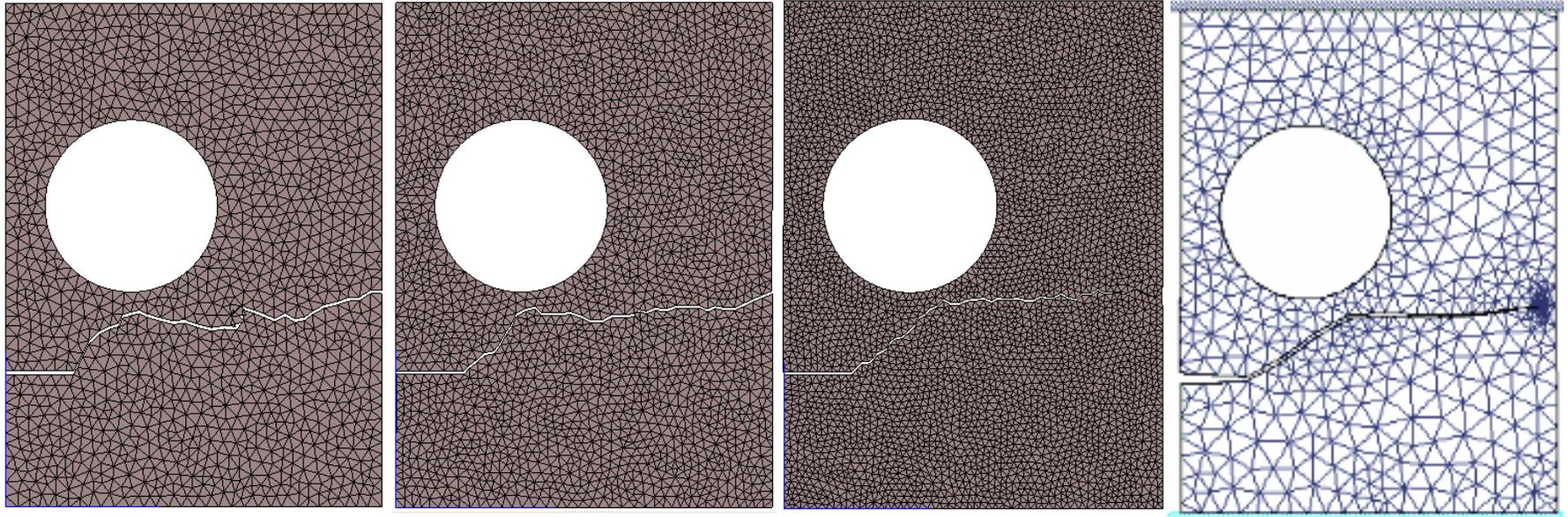
*Mauvais  
conditionnement  
Convergence délicate*

Approche hybride  
DG/CZM



*Convergence assurée*

# Sensibilité à la taille de maille

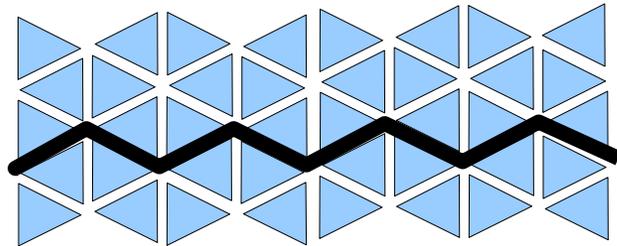




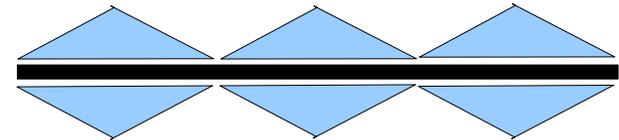
# Limitations intrinsèques

- Nécessité d'un maillage très fin (très couteux en 3D)
- La dissipation d'énergie est nécessairement surestimée

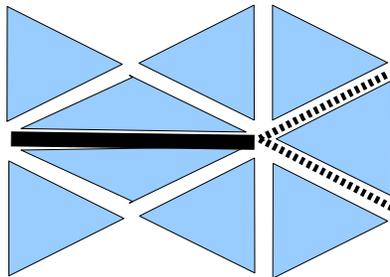
- A cause d'un trajet de fissuration trop long



≠



- A cause des nombreux branchements non fructueux



?

## Au final...

- ... Rien à faire : les CZM seuls ne peuvent pas décrire proprement un trajet de fissuration dont le lieu est inconnu
- Il faut (avis personnel !) les coupler à un modèle continu
- Soit de d'endommagement
- Soit énergétique global
- Eventuellement les deux

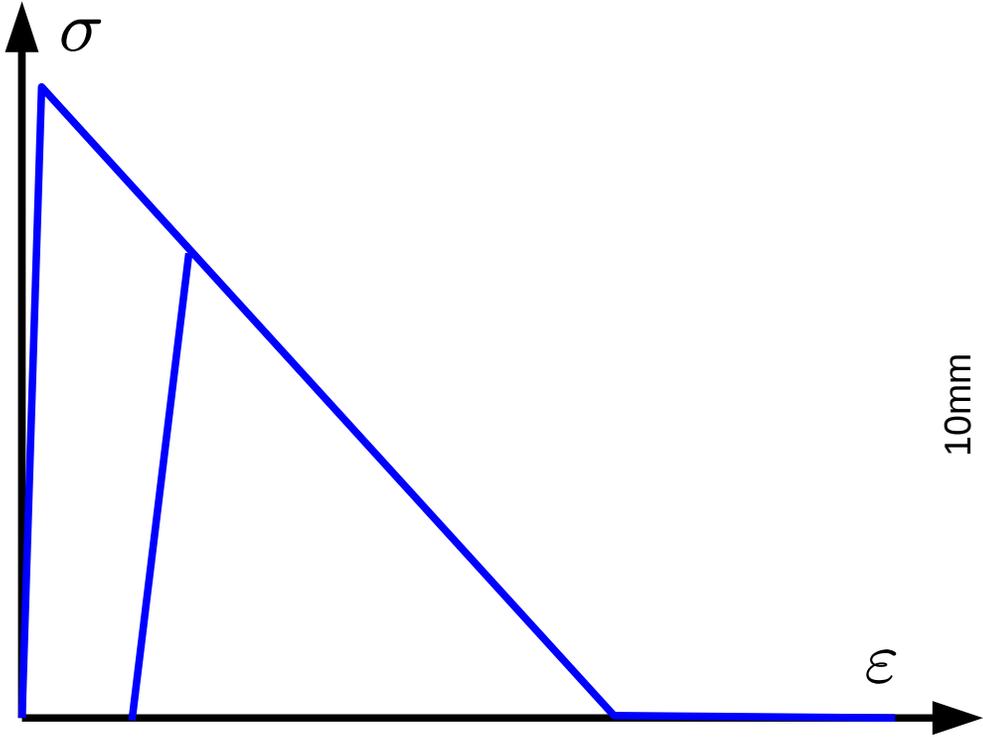
# Exemple de couplage avec un modèle de dommage

- Thèse de S. Payet
  - Transition à  $D \approx 1$
  - Adaptativité de maillage
  - Insertion d'une fissure
- Thèse de S. Cuviliez
  - Transition à  $D < 1$
  - Conservation de l'énergie
  - Aspects théoriques

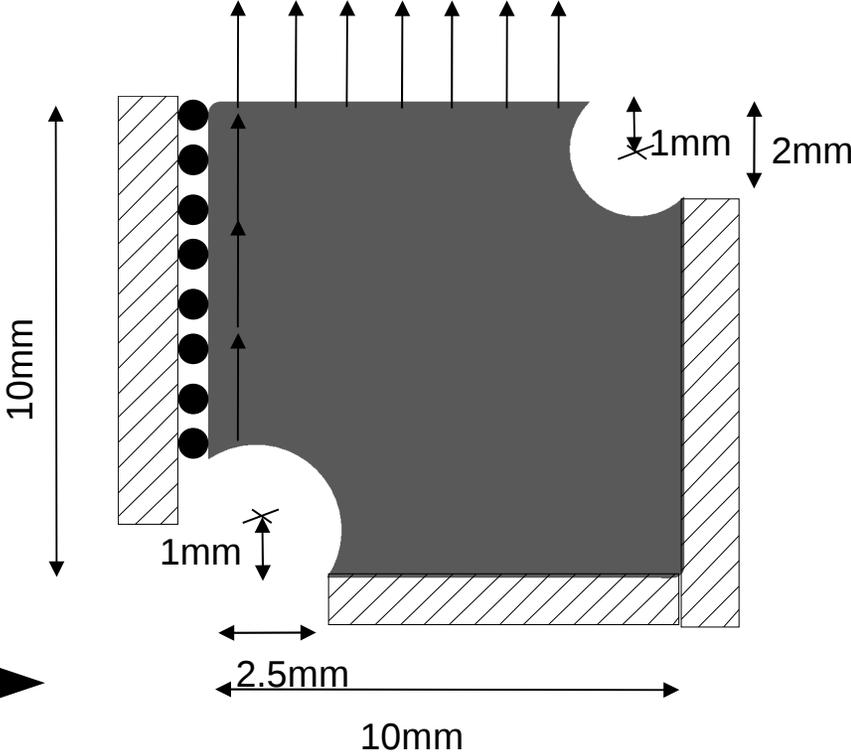
J. Besson  
E. Lorentz  
S. Michel-Ponnelle  
J. Laverne

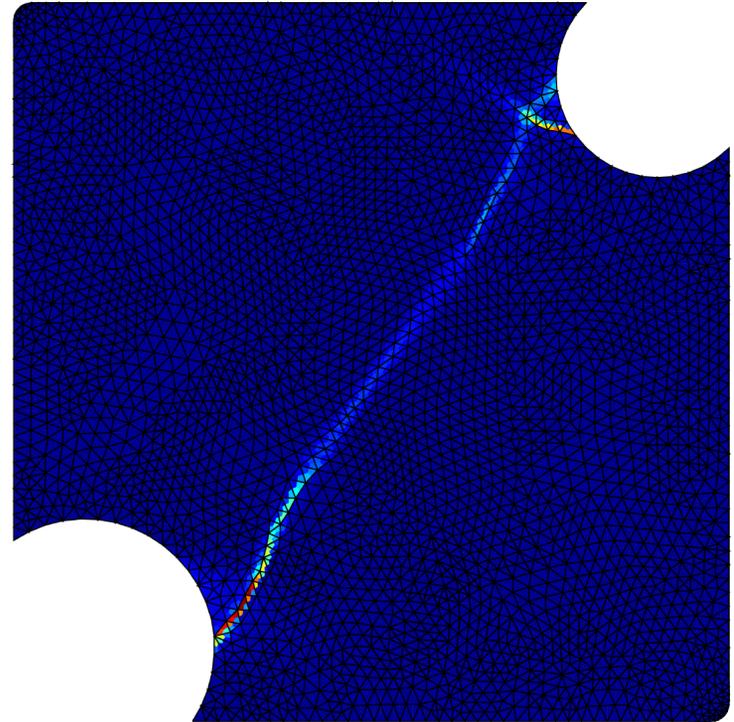
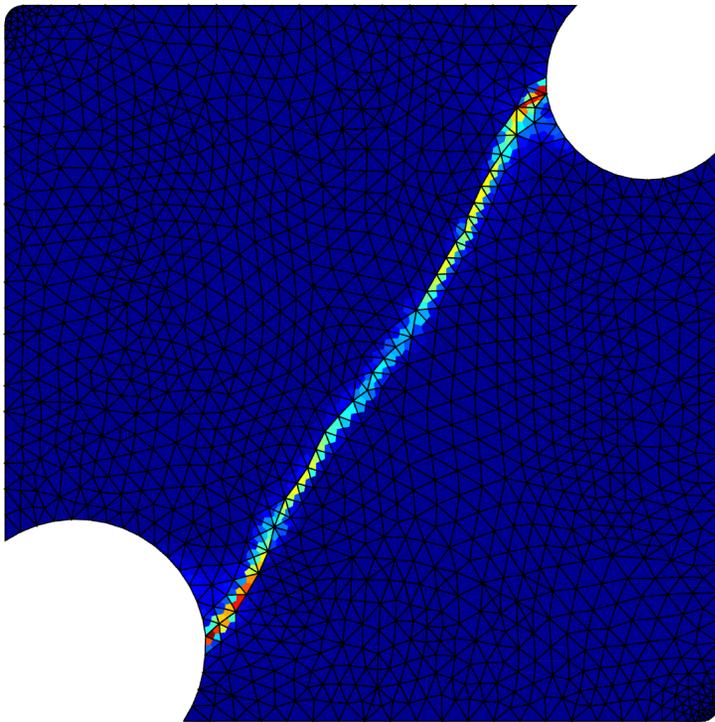


# Mesh dependency



Coupled elasto-plastic-damage model





Weird damage path  
Structure breaks without dissipation

# Scale coupling using non local models

Kinematic enrichment

$$E(u, a) + c \|\nabla \chi\|^2 + \dots$$

Gradient models

$$E(u, a) + c \|\nabla a\|^2$$

Regularized models

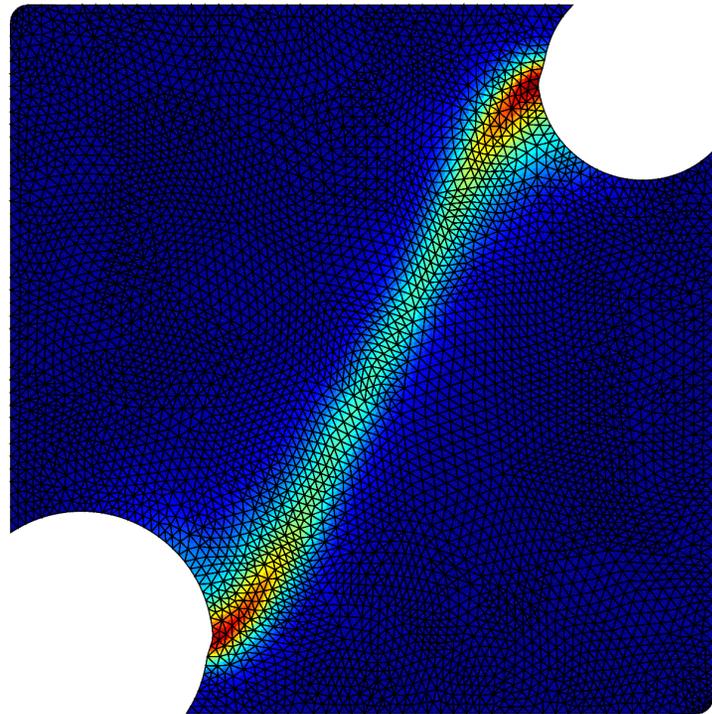
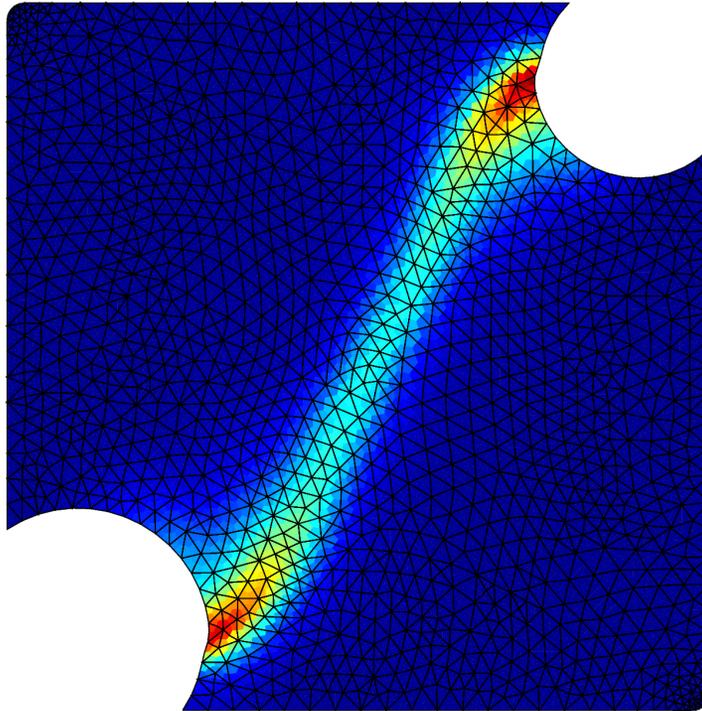
$$\bar{a} = \mathcal{F}(a)$$

$$\begin{cases} E(u, \bar{a}) \\ \bar{a} + c \Delta \bar{a} = a \end{cases}$$

$$\begin{cases} E(u, \bar{a}) \\ \bar{a} = \alpha \int a d\Omega \end{cases}$$



[Forest, Lorentz, Peerlings]



Regularized  $(u, \theta, p, \bar{P})$  elements

Works well, but ... *expensive* !  $\rightarrow$  mesh adaptivity

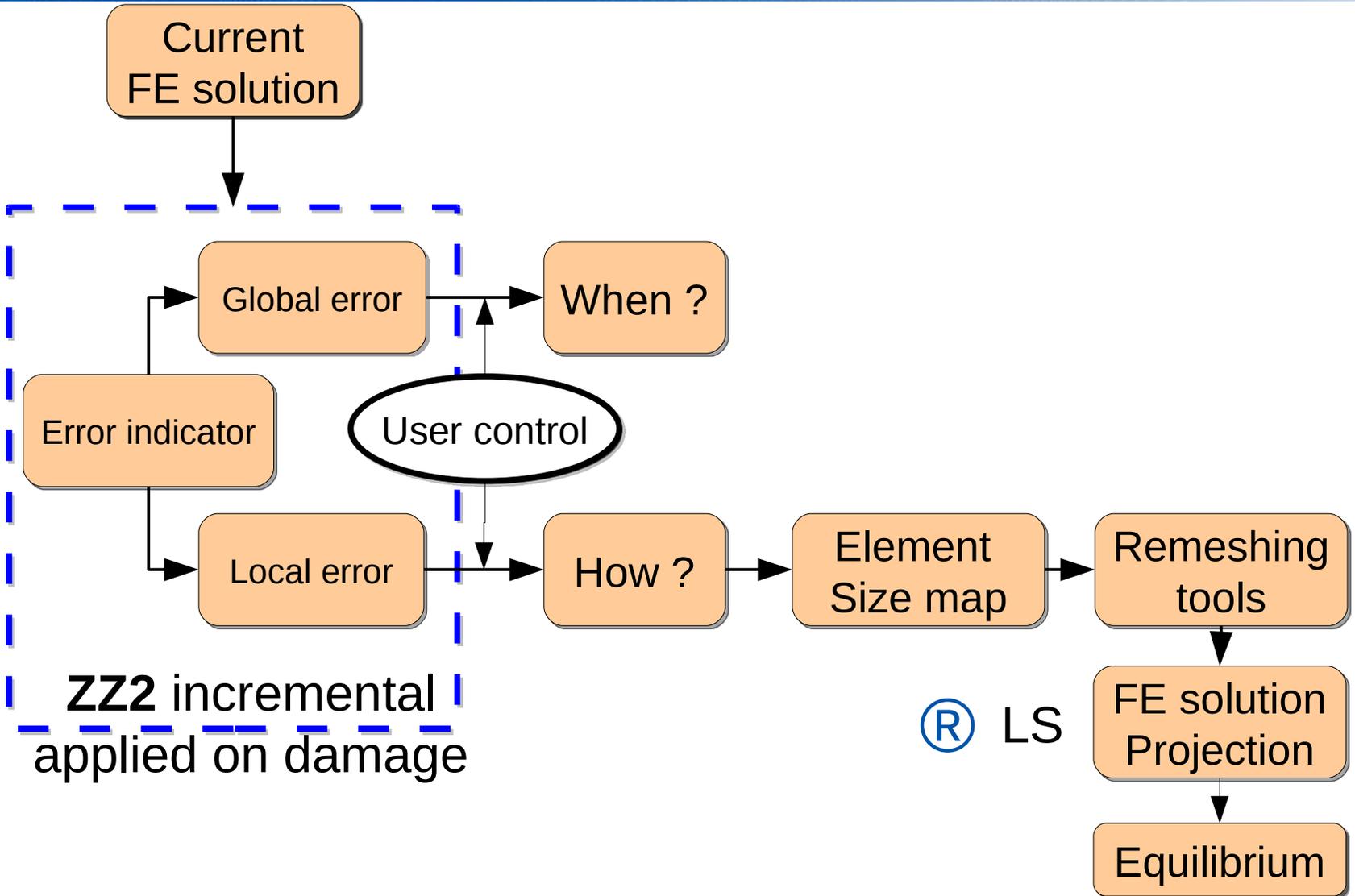


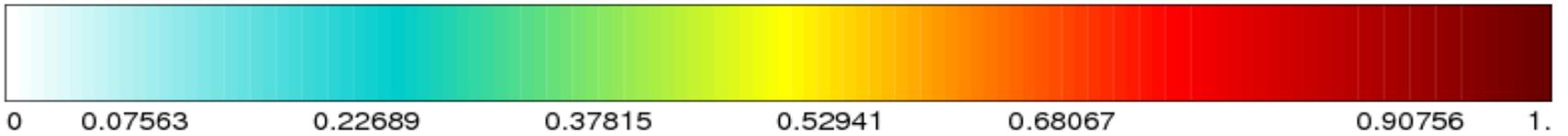
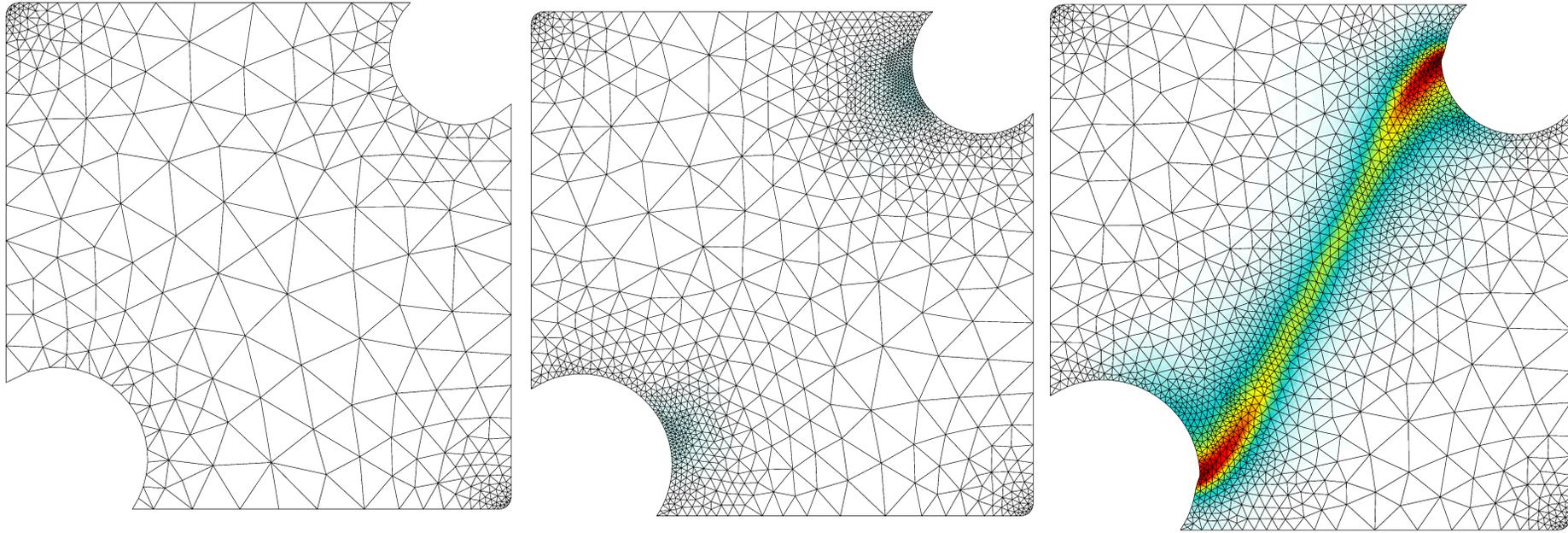
[Engelen, Geers, Baaijens, 2003]



[Feld-Payet, Besson, Feyel, 2010]

# Mesh adaptativity



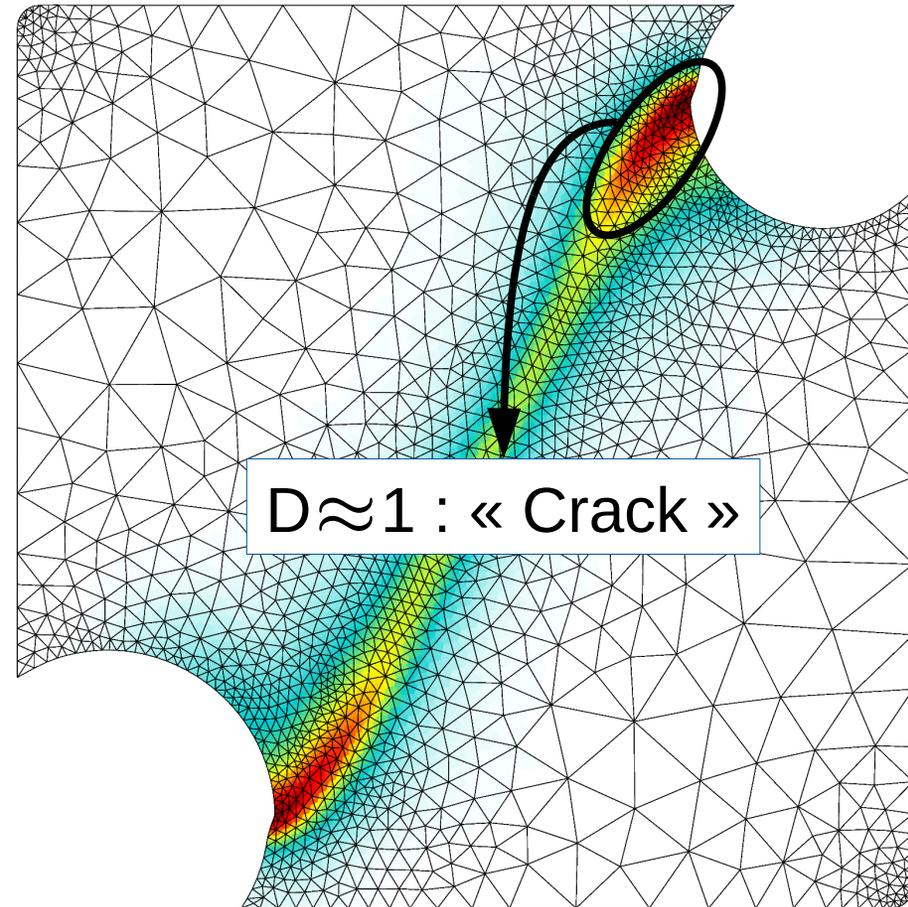


## Meshes vs. damage evolution

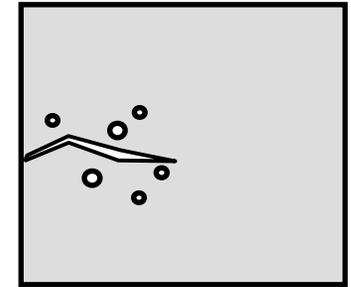
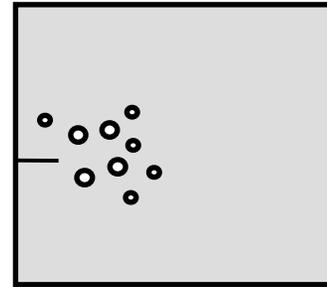
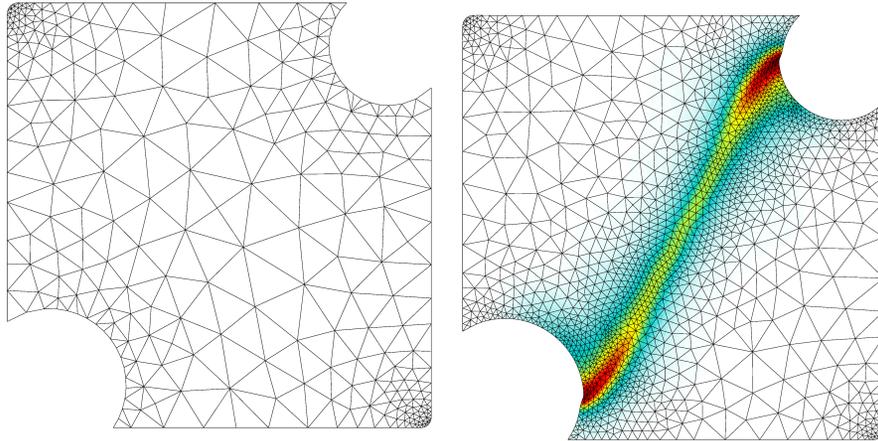
## Crack insertion ?

- Reduce comp. Costs
- Reduce num. problems
- The end-user wants a crack

TU / Eindhoven collaboration



# Crack insertion and propagation



## ■ Continuous description

- Localization band / volume
  - Remeshing techniques (accuracy, cost)

## ■ Dis-continuous description

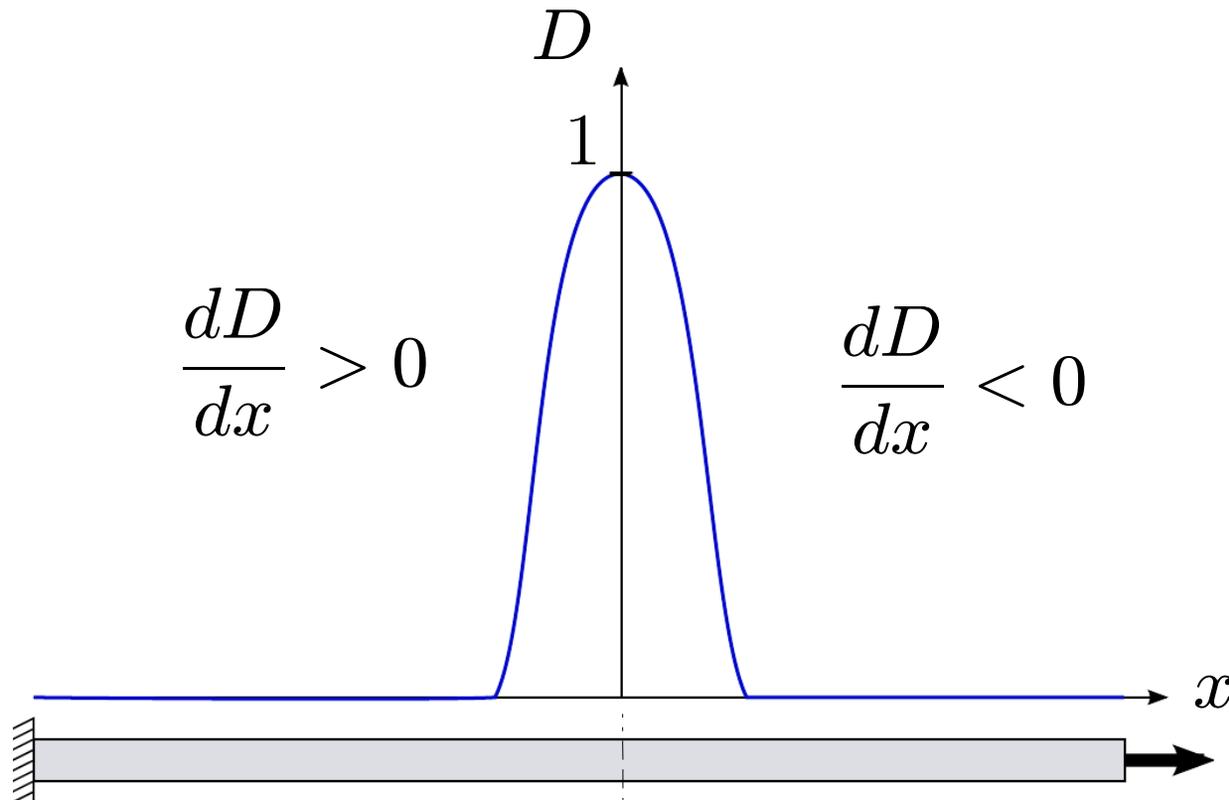
- Localization line / surface
  - Crack orientation criterion
  - Crack insertion
- Energy balance ? ( $D \approx 1$ )



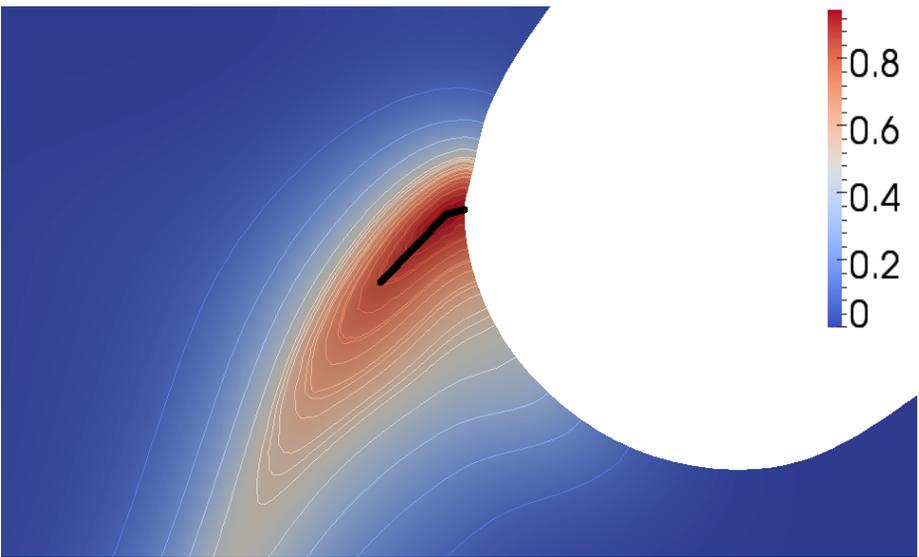
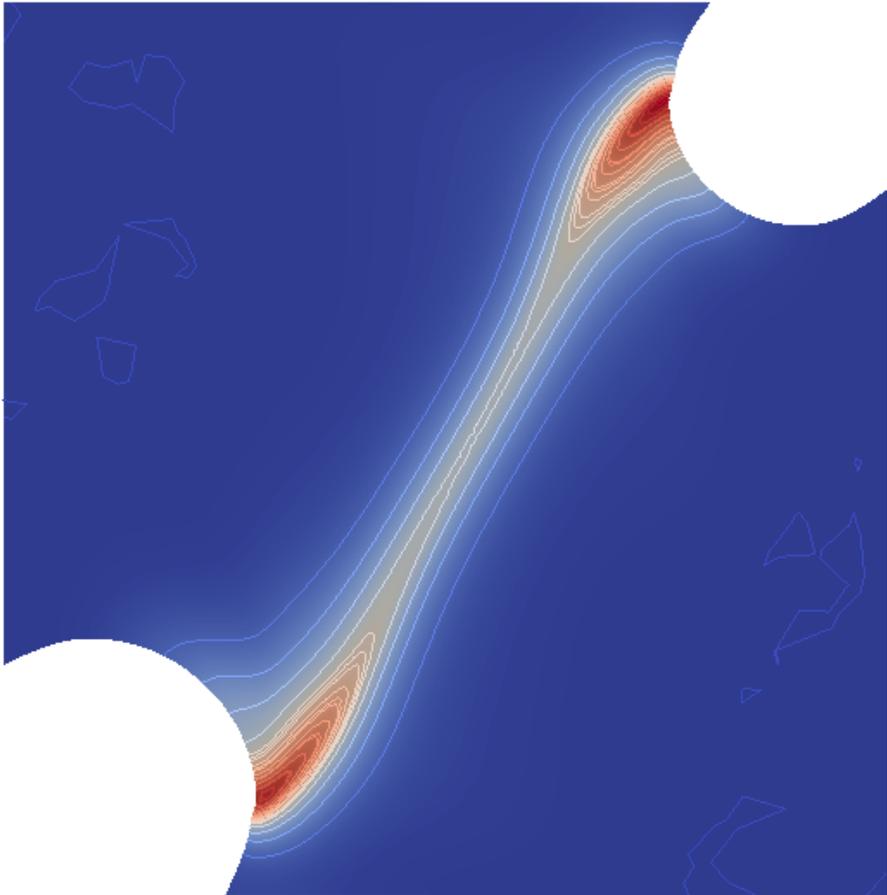
[Hambli 01, Simone et al 03, Mediavilla 05]

# 1D: main idea

Crack location = maximum of damage



2D: Crack initiation = maximum of damage + crack direction



$$\underline{\nabla} D \cdot \underline{e}_\theta$$

Look at each damage zone

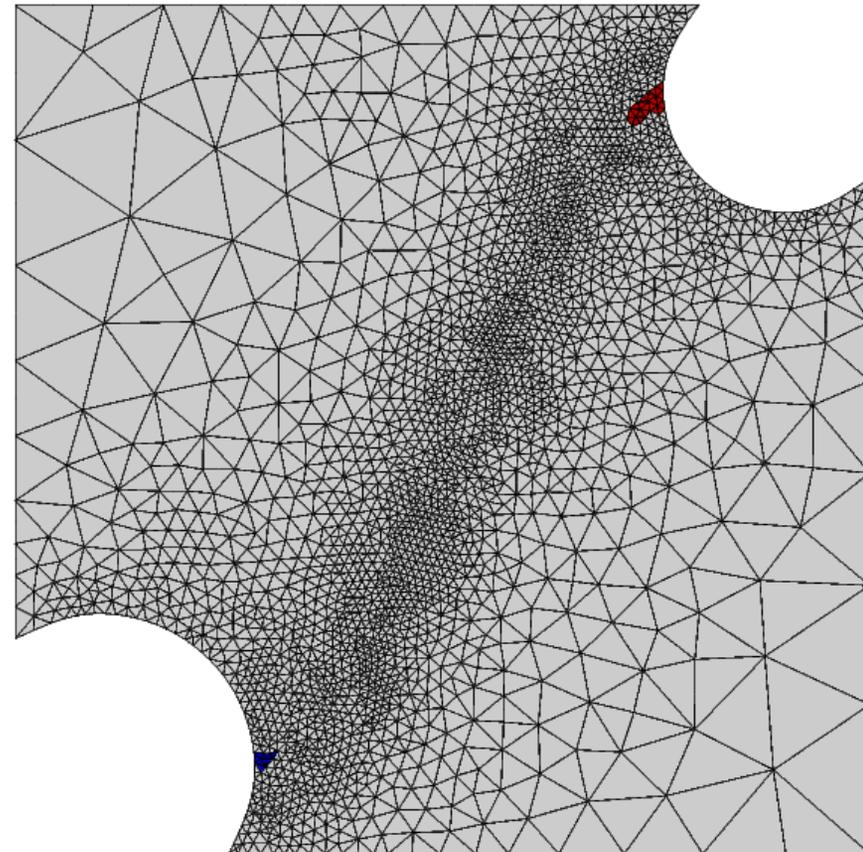
Find the maximum damage point

Find the crack directions

Project to find  $L_{crack}$

If  $L_{crack} > L_{min}$  store crack geometry

If  $D^* > 1 - \epsilon$  ahead of next front: insertion



$$D^* > D_{min}^*$$

Look at each damage zone

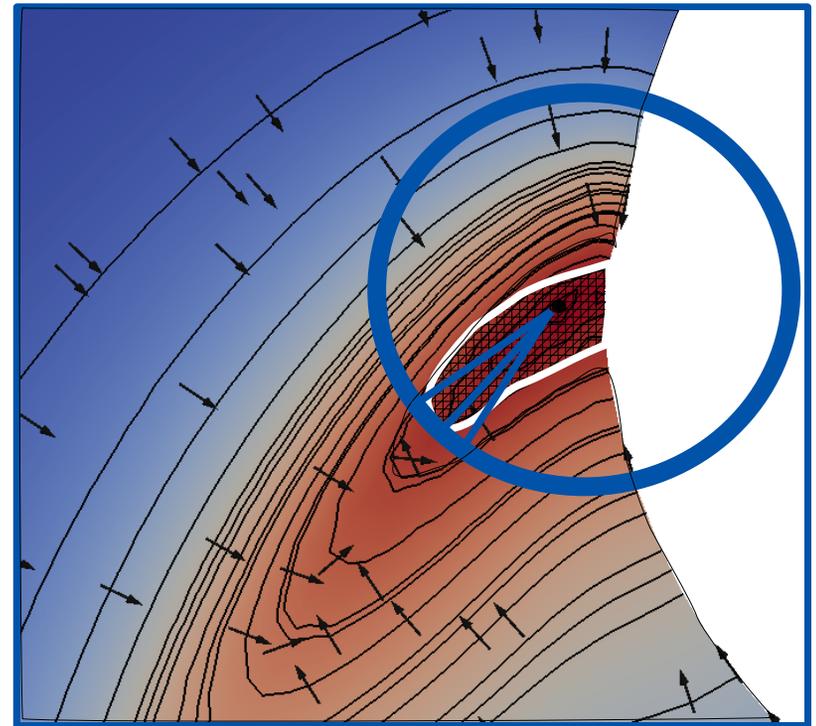
Find the maximum damage point

Find the crack directions

Project to find  $L_{\text{crack}}$

If  $L_{\text{crack}} > L_{\text{min}}$  store crack geometry

If  $D^* > 1 - \varepsilon$  ahead of next front: insertion



Look at each damage zone

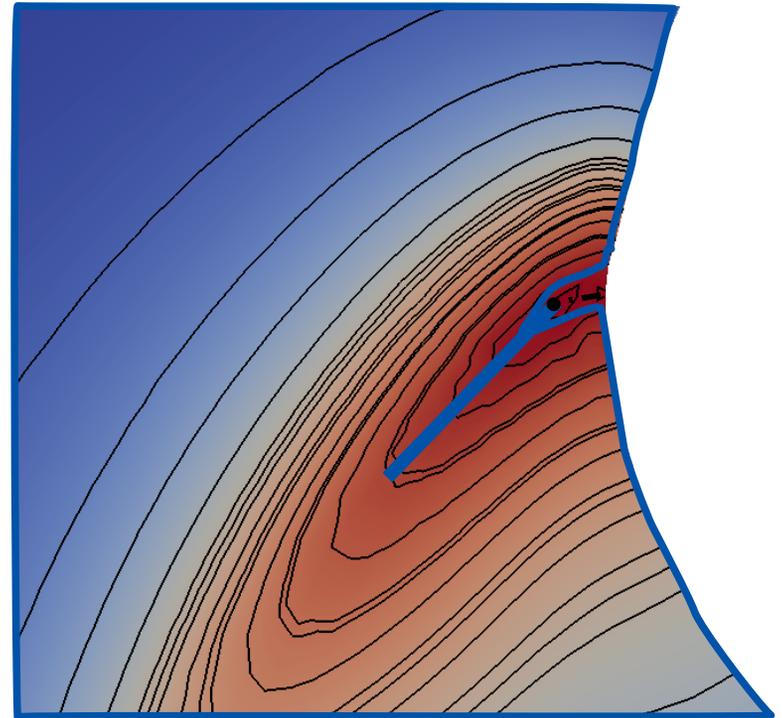
Find the maximum damage point

Find the crack directions

Project to find  $L_{\text{crack}}$

If  $L_{\text{crack}} > L_{\text{min}}$  store crack geometry

If  $D^* > 1 - \varepsilon$  ahead of next front: insertion



$$D^* > 1 - \varepsilon$$

Look at each damage zone

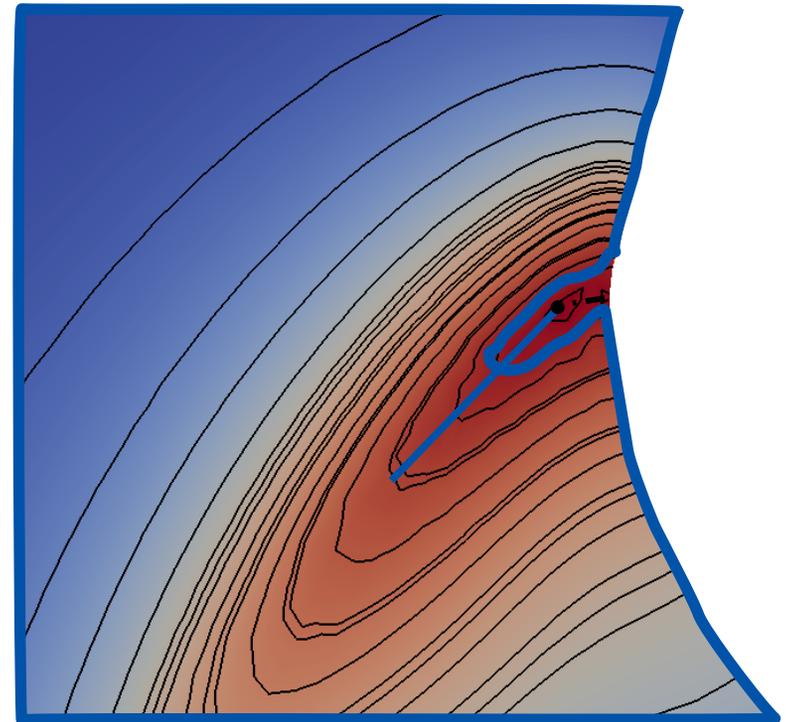
Find the maximum damage point

Find the crack directions

Project to find  $L_{\text{crack}}$

If  $\Delta L_{\text{crack}} > \Delta L_{\text{min}}$ : new crack geom

If  $D^* > 1 - \varepsilon$  ahead of next front: insertion



$$D^* > 1 - \varepsilon$$

Look at each damage zone

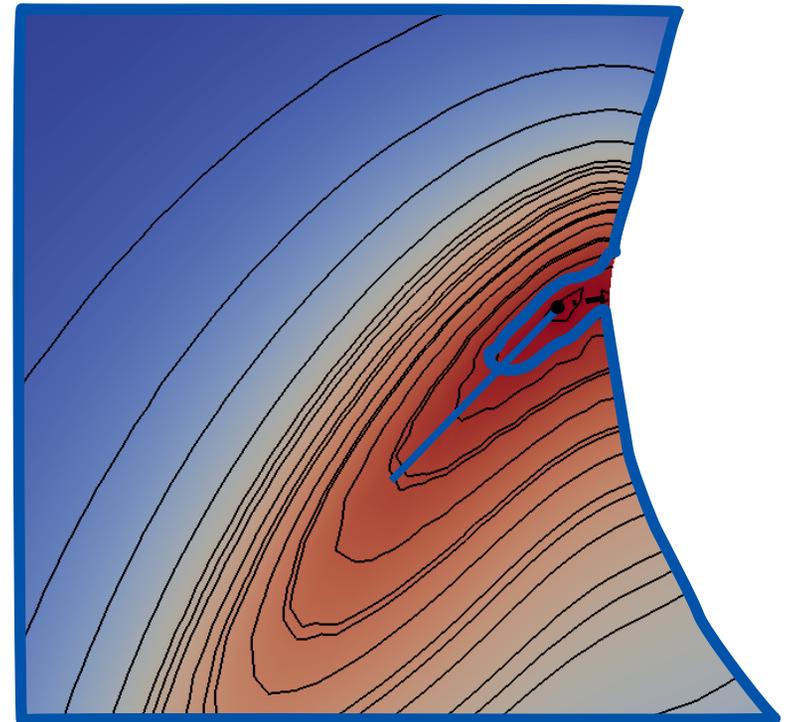
Find the maximum damage point

Find the crack directions

Project to find  $L_{\text{crack}}$

If  $\Delta L_{\text{crack}} > \Delta L_{\text{min}}$ : new crack geom

If  $D > 1 - \varepsilon$  ahead of next front: insertion



$$D^* > 1 - \varepsilon$$

Look at each damage zone

Find the maximum damage point

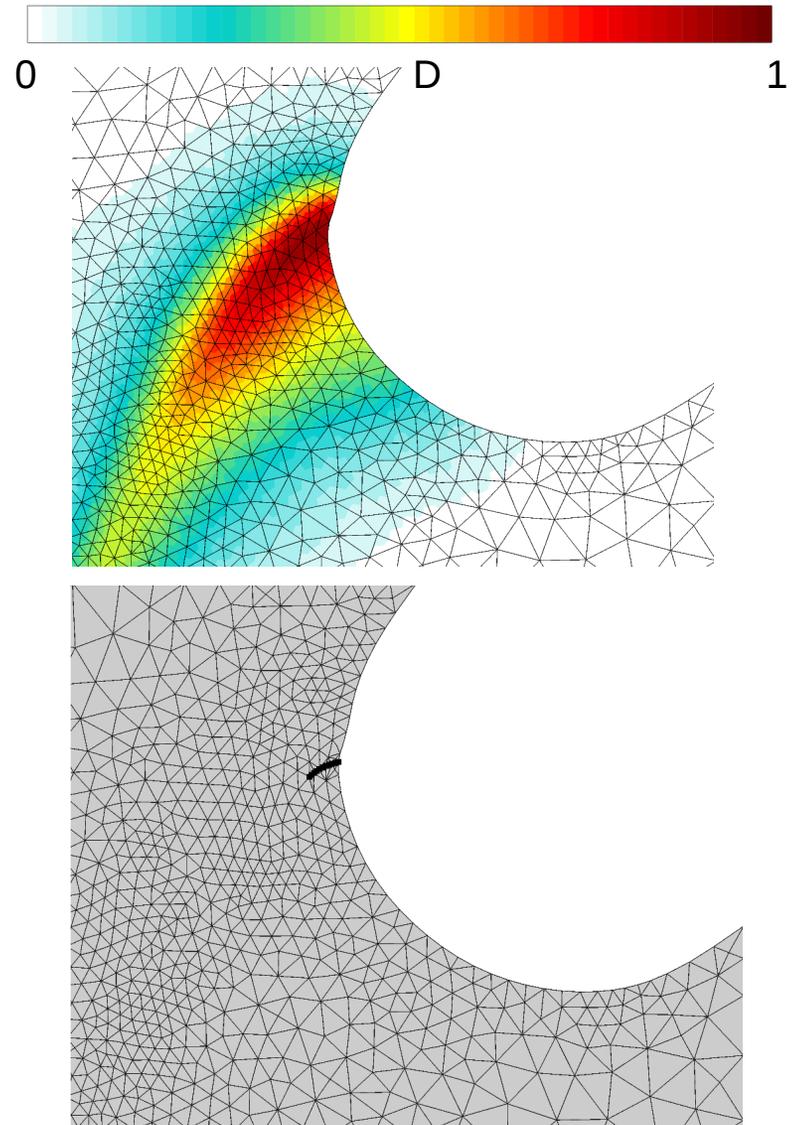
Find the crack directions

Project to find  $L_{\text{crack}}$

If  $L_{\text{crack}} > L_{\text{min}}$  store crack geometry

If  $D^* > 1 - \epsilon$  ahead of next front: insertion

Use of surface / volume  
intersection algorithm



Look at each damage zone

Find the crack tip

Find the crack directions

Project to find  $L_{\text{crack}}$

If  $L_{\text{crack}} > L_{\text{min}}$  store crack geometry

If  $D^* > 1 - \epsilon$  ahead of next front: insertion

Field transfer

