# Une alternative aux CZM : la propagation d'endommagement par level set épaisse

N. MOËS
Ecole Centrale de Nantes, FRANCE
GeM Institute / UMR CNRS 6183

Journée MECAMAT, CZM, 21 Juin 2010


ECN
Centrale Nantes

Exposé largement inspiré du papier

**A level set based model for damage growth :**

**The thick level set approach (TLS)**

International Journal for Numerical Methods in engineering

(Accepted)

N. MOËS, C. STOLZ, P.-E. BERNARD, N. CHEVAUGEON

# Introduction

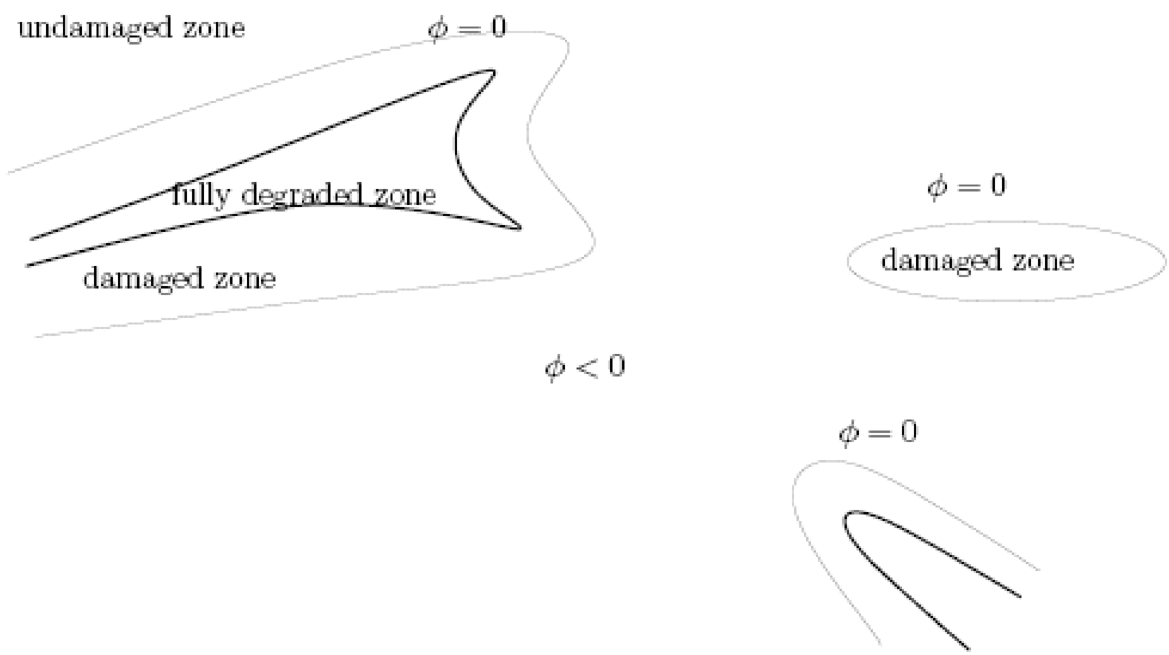
- La modélisation de la dégradation progressive de raideur dans les structures peut se faire par la mécanique de l'endommagement ou l'approche par fissure cohésive.
- Ces deux approches permettent de modéliser les effets d'échelle (pour autant que le modèle d'endommagement soit régularisé).
- Nous proposons une approche dite par level set épaisse (TLS) qui vue des CZM offre une loi pour la direction de propagation (et le branchement) et vu de la mécanique de l'endommagement offre une nouvelle manière de régulariser et un traitement « local ».

# Motivationspour le modèle TLS

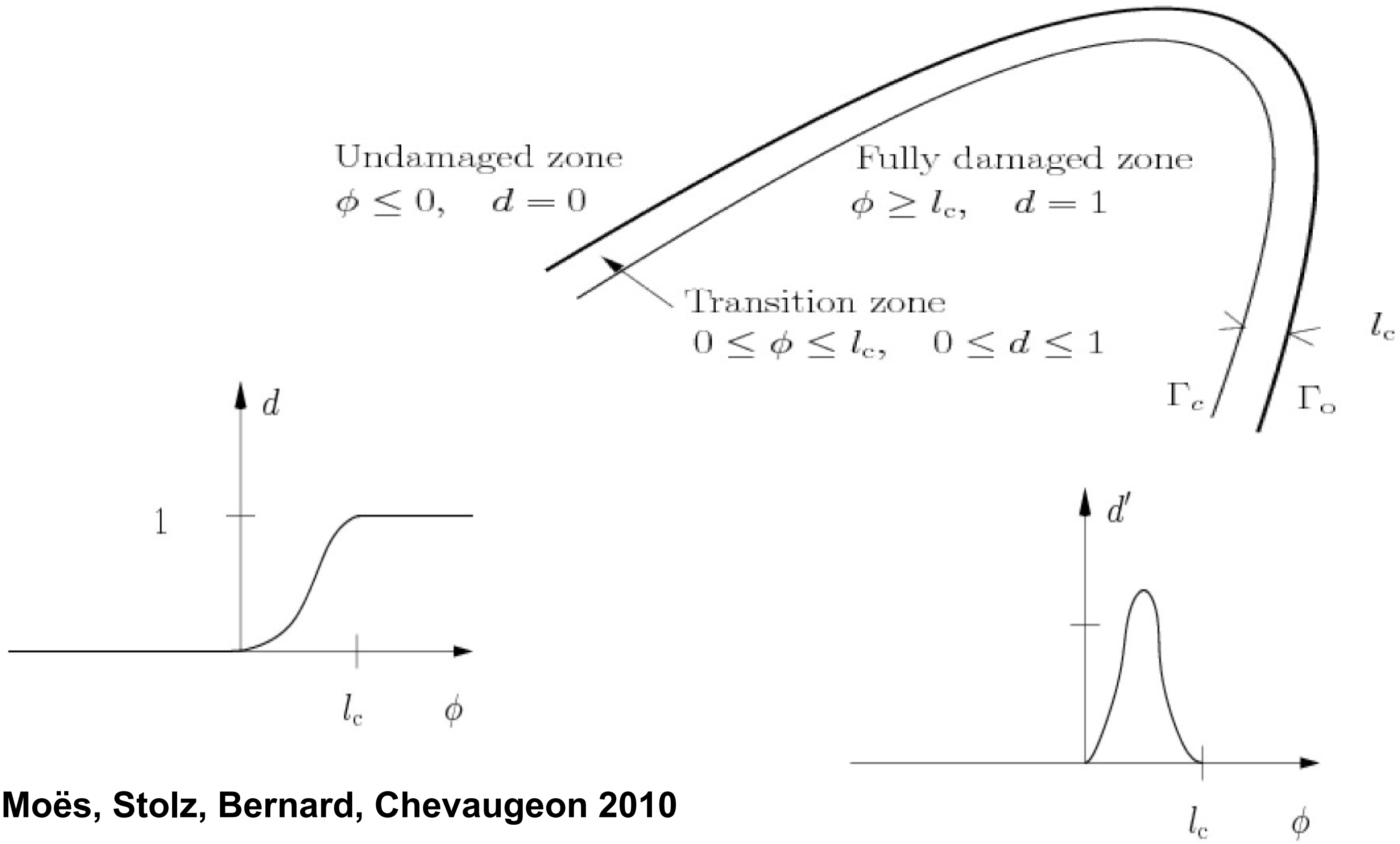
- Gérer le développement de l'endommagement par le biais de la propagation d'un front, ce front étant localisé numériquement par une fonction niveau (level set).
- Obtenir un modèle régularisé mais le plus local possible.
- Etre en mesure d'utiliser « tous » les modèles d'endommagement de la littérature.
- Gérer la transition vers les fissures de manière naturelle.

# A new thick level set approach

- Idea : model a degradation front with a single level set. One and only one level set whatever the complexity of damage.
- At some distance behind the front we consider the material to be completely degraded thus unveiling a crack.



# To provide non-locality we force d=d(ϕ) and give a critical distance lc after which d=1




**Moës, Stolz, Bernard, Chevaugeon 2010**

# Thermodynamical description of the model. PART 1
# The free energy and state laws

Local energy potential and state laws

$$\varphi = \varphi(\epsilon, d) = \frac{1}{2}(1-d)\epsilon : \boldsymbol{E} : \epsilon$$

$$\begin{aligned} \boldsymbol{\sigma} &= \frac{\partial \varphi}{\partial \epsilon} = (1-d)\boldsymbol{E} : \epsilon \\ Y &= -\frac{\partial \varphi}{\partial d} = \frac{1}{2}\epsilon : \boldsymbol{E} : \epsilon \end{aligned}$$

Global potential energy

$$E(\boldsymbol{u}, \phi) = \int_{\Omega(\phi \leq l_c)} \varphi(\epsilon(\boldsymbol{u}), d(\phi))\mathrm{d}\Omega - \int_{\Gamma_T} \boldsymbol{T} \cdot \boldsymbol{u}\mathrm{d}\Gamma$$

# Gobal state law : equilibrium and configurational force associated to the front movement

$$DE(\boldsymbol{u},\phi)[(\delta\boldsymbol{u},\delta\phi)] = \int_{\Omega(\phi\leq l_c)} \boldsymbol{\sigma} : \boldsymbol{\epsilon}(\delta\boldsymbol{u})\mathrm{d}\Omega - \int_{\Gamma_T} \boldsymbol{T}\cdot\delta\boldsymbol{u}\mathrm{d}\Gamma - \int_{\Omega(\phi\leq l_c)} Y d'(\phi)\delta\phi\mathrm{d}\Omega$$

$$\begin{aligned} A_0^u &= \{\delta\boldsymbol{u} : \delta\boldsymbol{u} = 0 \text{ on } \Gamma_u\} \\ A_0^\phi &= \{\delta\phi : \nabla\delta\phi\cdot\nabla\phi = 0 \text{ on } \Omega\} \end{aligned}$$

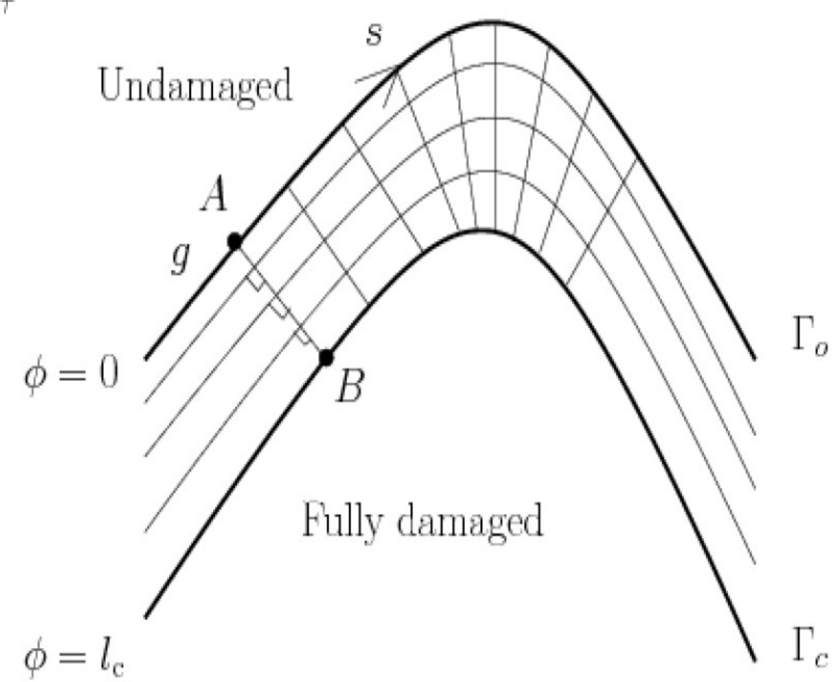
$$DE(\boldsymbol{u},\phi)[(\delta\boldsymbol{u},0)] = \int_{\Omega(\phi\leq l_c)} \boldsymbol{\sigma} : \boldsymbol{\epsilon}(\delta\boldsymbol{u})\mathrm{d}\Omega - \int_{\Gamma_T} \boldsymbol{T}\cdot\delta\boldsymbol{u}\mathrm{d}\Gamma = 0 \quad \forall\delta\boldsymbol{u}\in A_0^u$$

$$
\begin{aligned}
DE(\boldsymbol{u},\phi)[(0,\delta\phi)] &= -\int_{\Omega_{oc}} Y d'(\phi)\delta\phi \mathrm{d}\Omega \\
&= -\int_{\Gamma_0}\int_0^l Y(\phi,s)d'(\phi)\delta\phi(s)(1-\phi/\rho(s))\mathrm{d}\phi\mathrm{d}s \\
&= -\int_{\Gamma_o} g(s)\delta\phi(s)\mathrm{d}s
\end{aligned}
$$

$$
g(s)=\int_0^l Y(\phi,s)d'(\phi)(1-\frac{\phi}{\rho(s)})\mathrm{d}\phi
$$

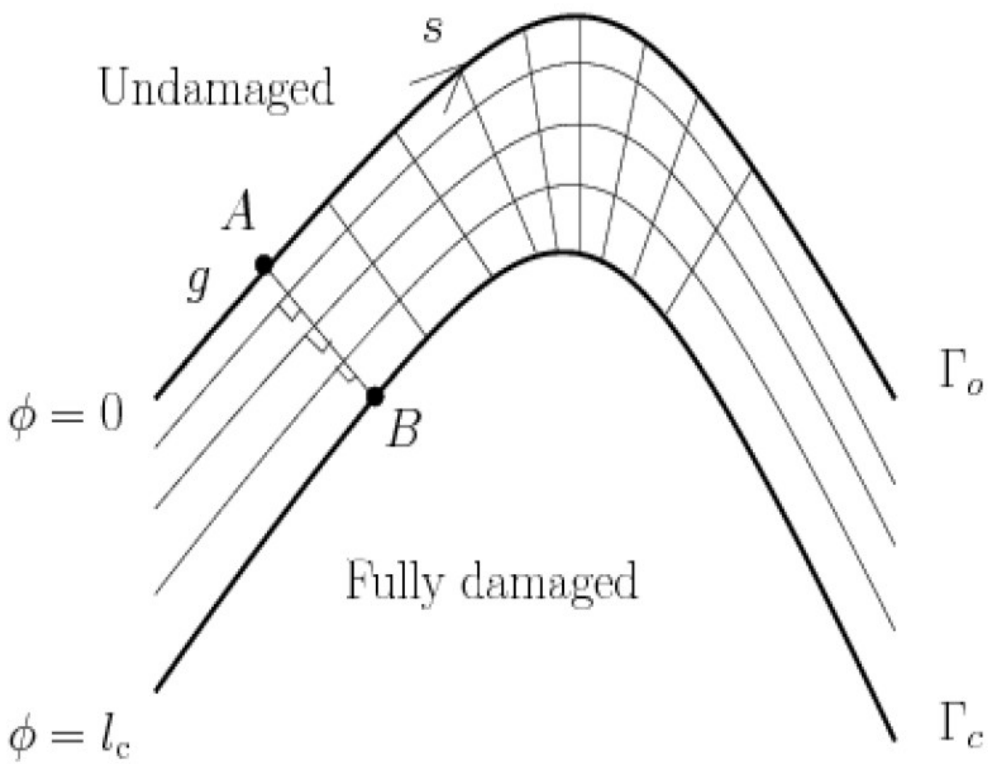
$$
\Omega_{oc}=\{\boldsymbol{x}\in\Omega : 0\leq\phi(\boldsymbol{x})\leq l_{\mathrm{c}}\}
$$

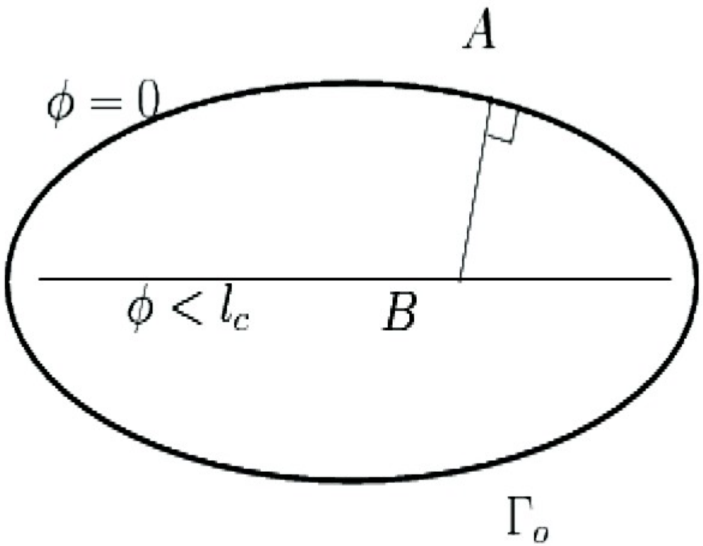
$$
\mathrm{d}\Omega=\frac{\rho(\phi,s)}{\rho(0,s)}\mathrm{d}\phi\mathrm{d}s=(1-\frac{\phi}{\rho(s)})\mathrm{d}\phi\mathrm{d}s
$$



# The front driving force is a weighted average of the local driving force

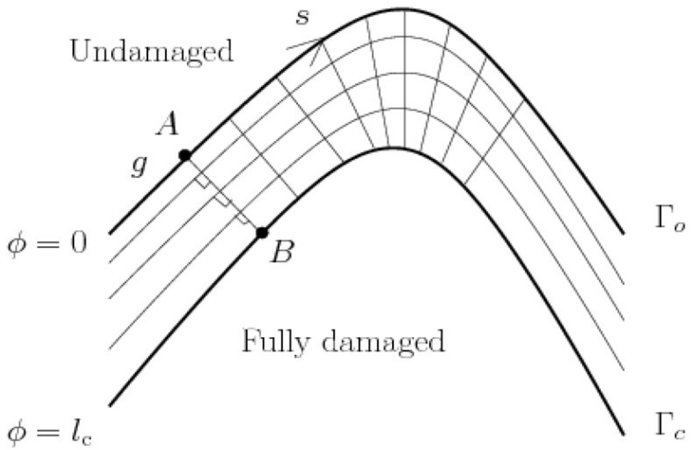
$$g(A) = \int_{AB} Y d'(\phi) \frac{\rho(\phi)}{\rho(A)} \, d\phi$$

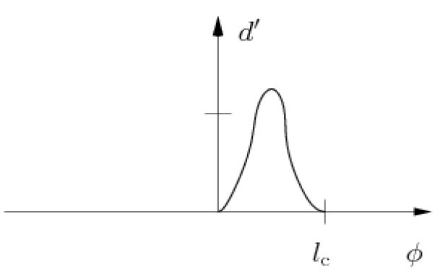


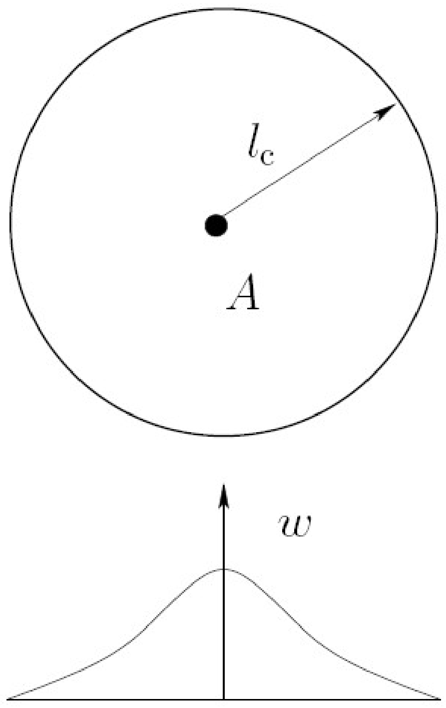
# Comparison with non-local integral formulation of damage

$$g(A) = \int_{AB} Y d'(\phi) \frac{\rho(\phi)}{\rho(A)} \, d\phi$$




$$\overline{Y}(A) = \int_{B_A} Y w \, d\Omega$$




Pijaudier-Cabot, Bazant 1987

# Other non-local approaches

When it comes to modeling damage, special care needs to be taken to avoid so called spurious localization. Several damage models were proposed in the literature to avoid spurious localization as detailed in (Peerlings 1994):

- non local integral damage model: the damage evolution is governed by a driving force which is non-local i.e. it is the average of the local driving force over some region (Bazant, Belytschko, and Chang 1984), (Pijaudier-Cabot and Bazant 1987);

- higher order, kinematically based, gradient models through the inclusion of higher-order deformation gradient (Aifantis 1984; Triantafyllidis and Aifantis 1986; Schreyer and Chen 1986) or additional rotational degrees of freedom (Mühlhaus and Vardoulakis 1987);

- higher order, damage based, gradient models: the gradient of the damage is a variable as well as the damage itself. This leads to a second order operator acting on the damage: (Fremond and Nedjar 1996; Pijaudier-Cabot and Burlion 1996; Nguyen and Andrieux 2005).

# Other non-local approaches (cont'd)

Non-local and higher order gradient approaches were compared in (Peerlings, Geers, de Borst, and Brekelmans 2001).

More recently two strategies to avoid spurious localization also appeared. The phase-field approach emanating from the physics community (Hakim and Karma ; Hakim and Karma 2005; Karma, Kessler, and Levine 2001) and the so-called variational approach (Francfort and Marigo 1998; Bourdin, Francfort, and Marigo 2000; Bourdin, Francfort, and Marigo 2008).

# Thermodynamical description of the model. PART II Dissipation processes and evolution laws

- The dissipation is given by

$$D = -\frac{dE(\boldsymbol{u},\phi)}{dt} + \int_{\Gamma_T} \boldsymbol{T}\cdot\dot{\boldsymbol{u}}\,\mathrm{d}\Gamma = \int_{\Omega_{oc}} Y\dot{d}\,\mathrm{d}\Omega = \int_{\Gamma_o} g v_{\mathrm{n}}\,\mathrm{d}\Gamma$$

- The relation above is similar to the Hill-Mandel micro-macro compatibility in homogenization theory. Thus why not build the (v_n,g) relationship by homogenizaing in some sense a given ($\dot{d}$,Y) relationship.

Local dissipation potentials and evolution laws

$$\psi^*(Y) + \psi(\dot{d}) - Y\dot{d} = 0 \Leftrightarrow \dot{d} = \frac{\partial \psi^*(Y)}{\partial Y} \Leftrightarrow Y = \frac{\partial \psi(\dot{d})}{\partial \dot{d}}$$

$$\psi(\dot{d}) \text{ convex}, \quad \psi(\dot{d}) \geq 0, \quad \psi(0) = 0$$

Due to the non-locality of d, we have

$$d = d(\phi) \longrightarrow \dot{d} = d'(\phi) v_{\mathrm{n}}$$

Global potential energy is defined by

$$\int_{\Gamma_o} \overline{\psi}(v_{\mathrm{n}}, s)\, d\Gamma = \int_{\Omega_{oc}} \psi(d'(\phi) v_{\mathrm{n}})\, d\Omega$$

We can prove that

$$v_{\mathrm{n}} = \frac{\partial \overline{\psi}^*(g, s)}{\partial g}, \quad \Leftrightarrow g = \frac{\partial \overline{\psi}(v_{\mathrm{n}}, s)}{\partial v_{\mathrm{n}}}, \quad \Leftrightarrow \overline{\psi}^*(g, s) + \overline{\psi}(v_{\mathrm{n}}, s) - g v_{\mathrm{n}} = 0$$

# Front evolution law, an example

Local model

$$\dot{d} = k < \frac{Y}{Y_c} - 1 >_+^n, \quad n > 0, \quad k > 0, \quad Y_c \geq 0$$

$$\begin{aligned} \psi^*(Y) &= \frac{kY_c}{n+1} < \frac{Y}{Y_c} - 1 >_+^{n+1} \\ \psi(\dot{d}) &= Y_c \dot{d} + \frac{kY_c}{n'+1} (\frac{\dot{d}}{k})^{n'+1} + I_+(\dot{d}) \end{aligned}$$

Corresponding front model

$$v_{\mathrm{n}} = \overline{k}(s) < \frac{g}{\overline{Y_c}(s)} - 1 >_+^n$$

$$\begin{aligned} \overline{\psi}(v_{\mathrm{n}}, s) &= \overline{Y}_c(s) v_{\mathrm{n}} + \frac{\overline{k}(s)\overline{Y_c}(s)}{n'+1} (\frac{v_{\mathrm{n}}}{\overline{k}(s)})^{n'+1} + I_+(v_{\mathrm{n}}) \\ &= \overline{\psi}_r(v_{\mathrm{n}}, s) + I_+(v_{\mathrm{n}}) \\ \overline{\psi}^*(g, s) &= \frac{\overline{k}(s)\overline{Y_c}(s)}{n+1} < \frac{g}{\overline{Y}_c(s)} - 1 >_+^{n+1} \end{aligned}$$

$$v_{\mathrm{n}} = \overline{k}(s) < \frac{g}{\overline{Y_c}(s)} - 1 >_+^n$$

$$\begin{aligned}
\overline{Y}_c(s) &= Y_c \int_0^l d'(\phi)(1 - \frac{\phi}{\rho(s)})\, d\phi \\
\overline{k}(s) &= k \left( \frac{\int_0^l d'(\phi)(1 - \frac{\phi}{\rho(s)})\, d\phi}{\int_0^l (d'(\phi))^{n'+1}(1 - \frac{\phi}{\rho(s)})\, d\phi} \right)^n
\end{aligned}$$

Note that the ratio between the configurational force g and the critical force Yc bar is well defined even when l goes to zero. In this case the ratio tends to Y/Y_c, i.e. we recover the local criteria for initiation. Indeed

$$g(s) = \int_0^l Y(\phi, s) d'(\phi)(1 - \frac{\phi}{\rho(s)}) \mathrm{d}\phi \qquad \overline{Y}_c(s) = Y_c \int_0^l d'(\phi)(1 - \frac{\phi}{\rho(s)})\, d\phi$$

# Computation of the configurational force using a domain integral and the Eshelby tensor

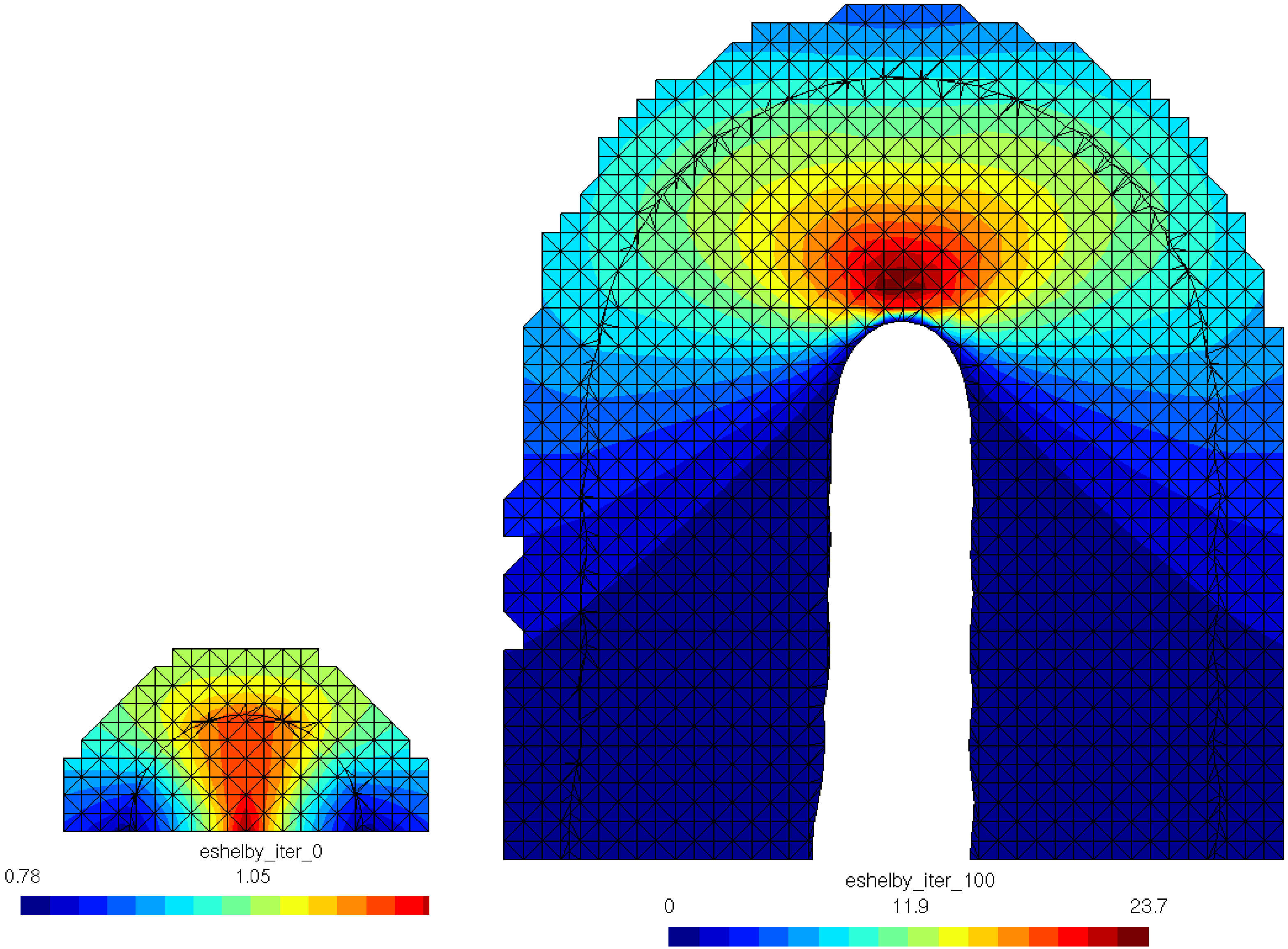
$$\int_{\Omega_{oc}} Y d'(\phi)\delta\phi \, d\Omega = \int_{\Gamma_o} g\delta\phi \, d\Gamma \quad \forall\delta\phi$$

$$P_{ij} = \varphi(\boldsymbol{\epsilon}, d)\delta_{ij} - u_{k,i}\sigma_{kj}$$

$$Y d'(\phi)\delta\phi = Y d'(\phi)\vec{\nabla}\phi \cdot \vec{\nabla}\phi\delta\phi = Y\vec{\nabla}d \cdot \vec{\nabla}\phi\delta\phi = -\vec{div}\boldsymbol{P} \cdot \vec{\nabla}\phi\delta\phi$$

$$\begin{aligned} \int_{\Omega_{oc}} -\vec{div}\boldsymbol{P} \cdot \vec{\nabla}\phi\delta a \, d\Omega &= \int_{\Omega_P} -\vec{div}\boldsymbol{P} \cdot \vec{\nabla}\phi\alpha\delta a \, d\Omega \\ &= \int_{\Omega_P} \boldsymbol{P} : \nabla(\alpha\delta a\vec{\nabla}\phi) \, d\Omega \\ &- \int_{\partial\Omega_P} \boldsymbol{n} \cdot \boldsymbol{P} \cdot (\alpha\delta a\vec{\nabla}\phi) \, d\Gamma \end{aligned}$$

# Distribution in the damage band and second invariant of the Eshelby tensor

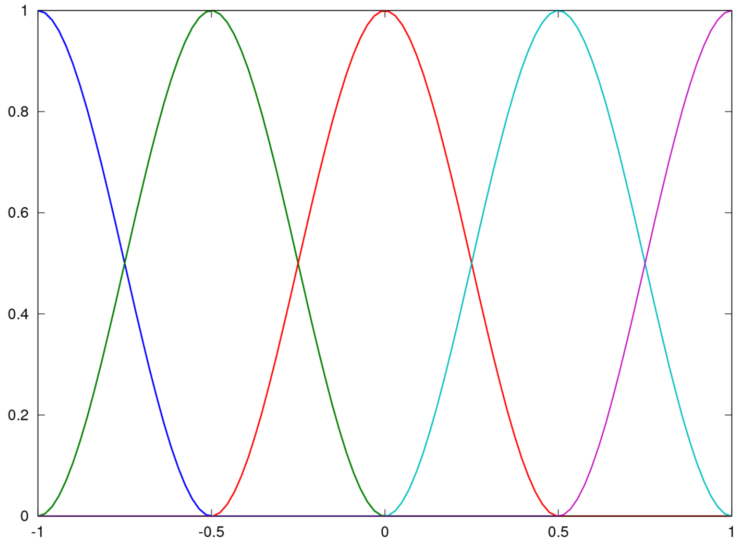


# Discretization of the config. force

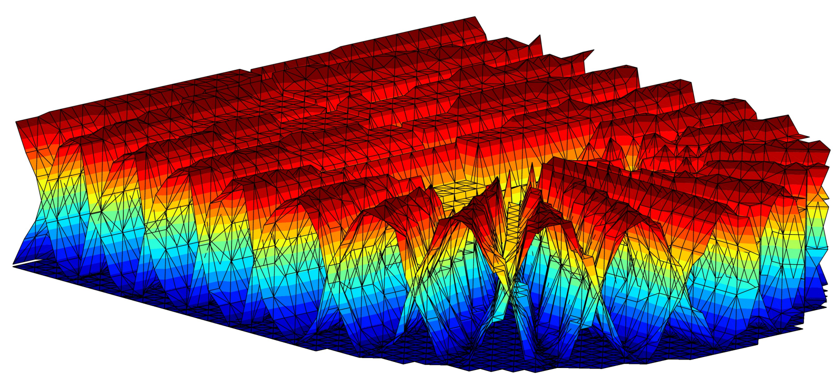
$$g = \sum_{I=1}^{N} g_I F_I, \quad \delta a = \sum_{I=1}^{N} \delta a_I F_I$$

$$\sum_{J=1}^{N} M_{IJ} g_J = \int_{\Omega_P} \boldsymbol{P} : \nabla(\alpha F_I \vec{\nabla}\phi)\, d\Omega - \int_{\partial\Omega_P} \boldsymbol{n}\cdot\boldsymbol{P}\cdot(\alpha F_I \vec{\nabla}\phi)\, d\Gamma \quad \forall I = 1, \ldots, N.$$

Hermite polynomials provides a partition of unity



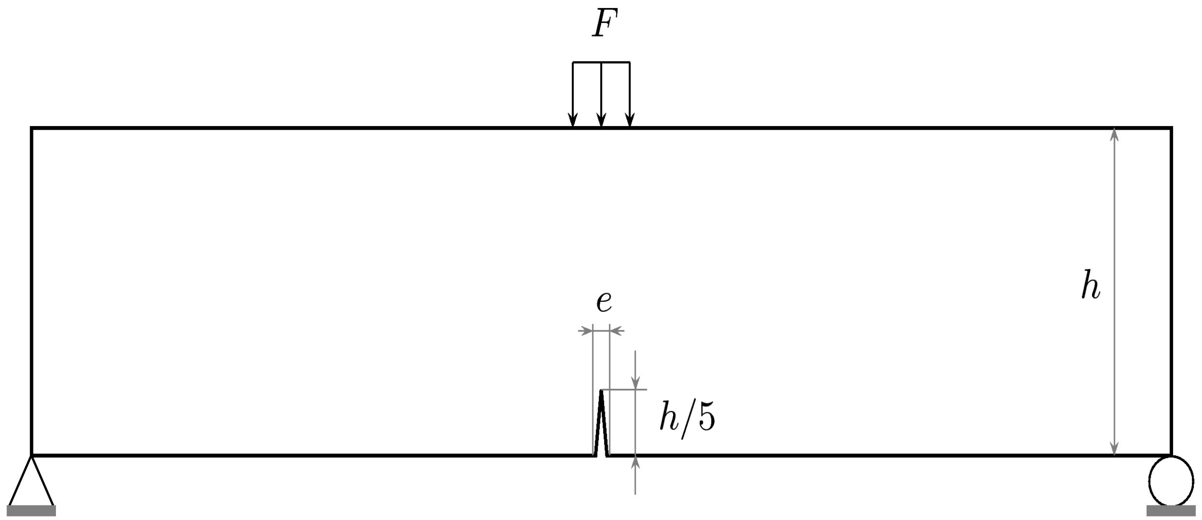

2D extension on the damage band



# Numerical implementation (2D so far)

- Fully degraded zone are taken into account by the X-FEM (treatment of voids, Daux et al. 2000) or a dissymetric tension/compression model to take into account contact.
- Octree adaptive grid to follow the front.
- Second order elements.
- The configurational force along the front is computed using a domain type integral involving the Eshelby tensor and more recently by a velocity variational principle.

# Three point bending



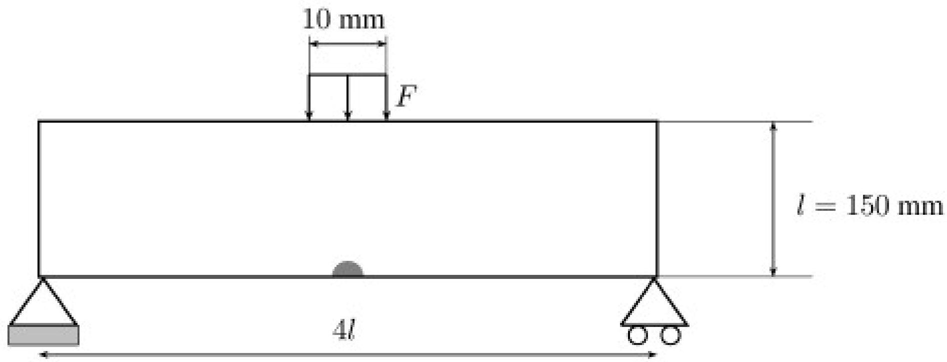
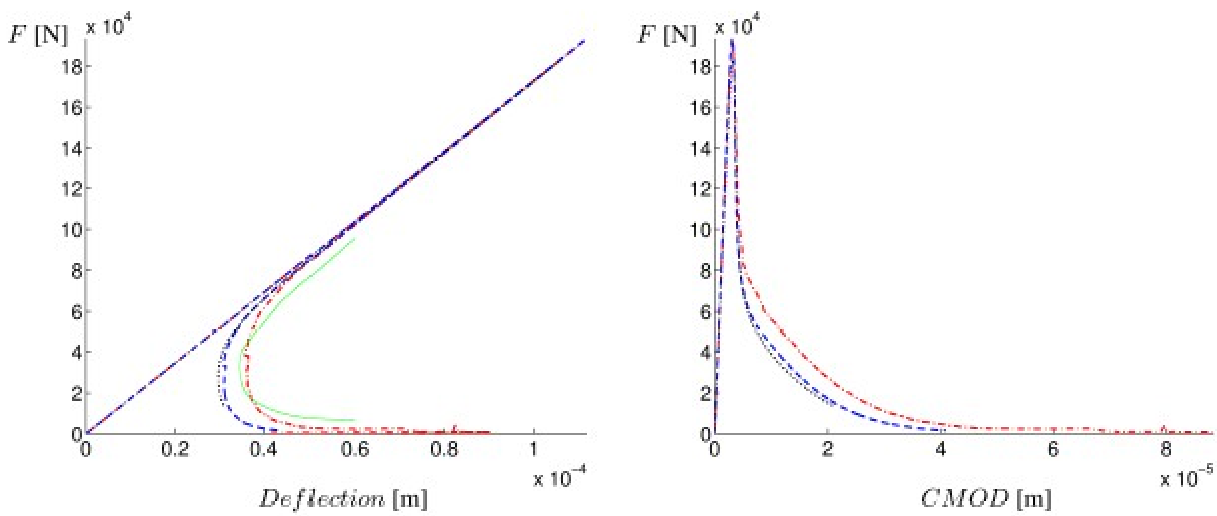
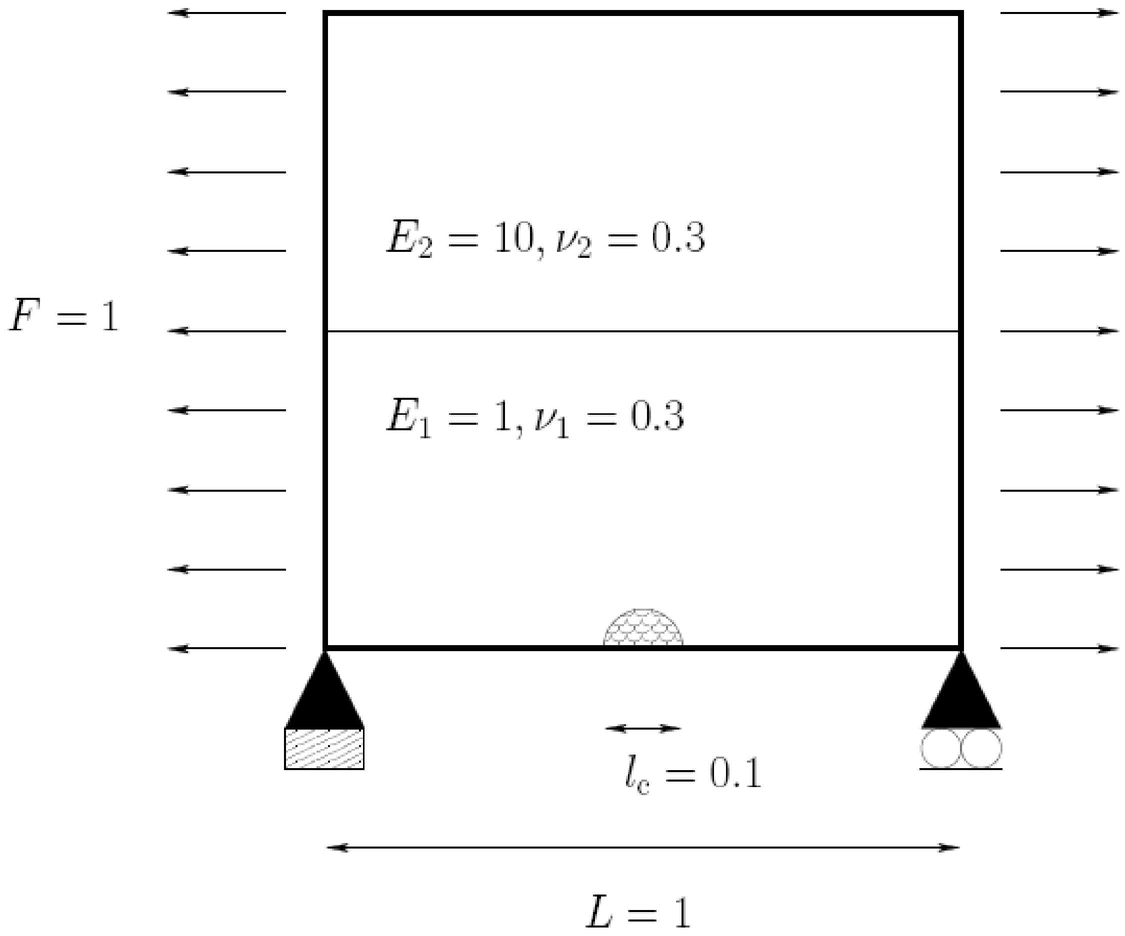



Figure 8. Three points bending test case configuration.



# Crack approaching to a stiffer media



## Strength of this new model

- Level set based model of damage.
- The model embeds the case d=1 and a “crack” appears.
- The “non-locality is treated locally”, numerical treament is in the thickness of the front.
- Initiation is based on a local criteria (basically Y=Yc) and thus if there is an initial crack, damage starts at the tip and not further away as in non-local formulation (Simone, Askes, Sluys, 2004).
- The model is thermodynamically sound and the fulfillment of second law of thermodynamics is easy. Duality is not corrupted as in most other non-local models (Lorentz et Andrieux 2003).
- “Any” local model may be used as an input.
- Damage model may be homogenized to fracture model

# Relationships between damage and fracture based crack growth law.

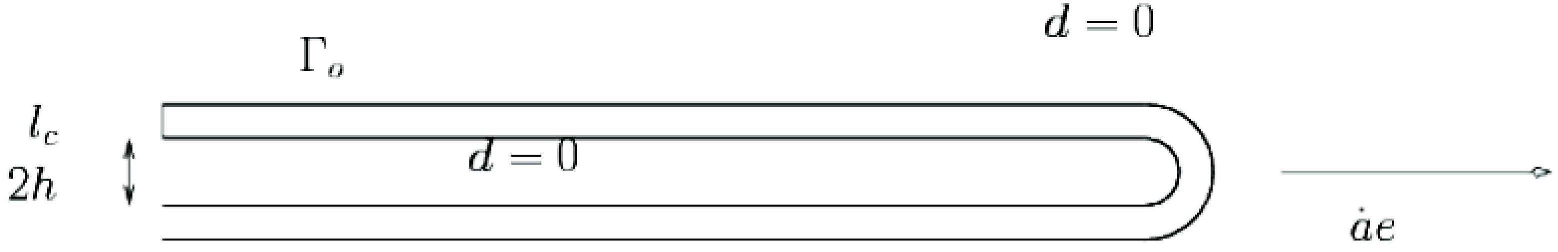
$$\dot{d} = k < \frac{Y}{Y_c} - 1 >_+^n \qquad \dot{d} = d'(\phi) v_{\rm n}$$

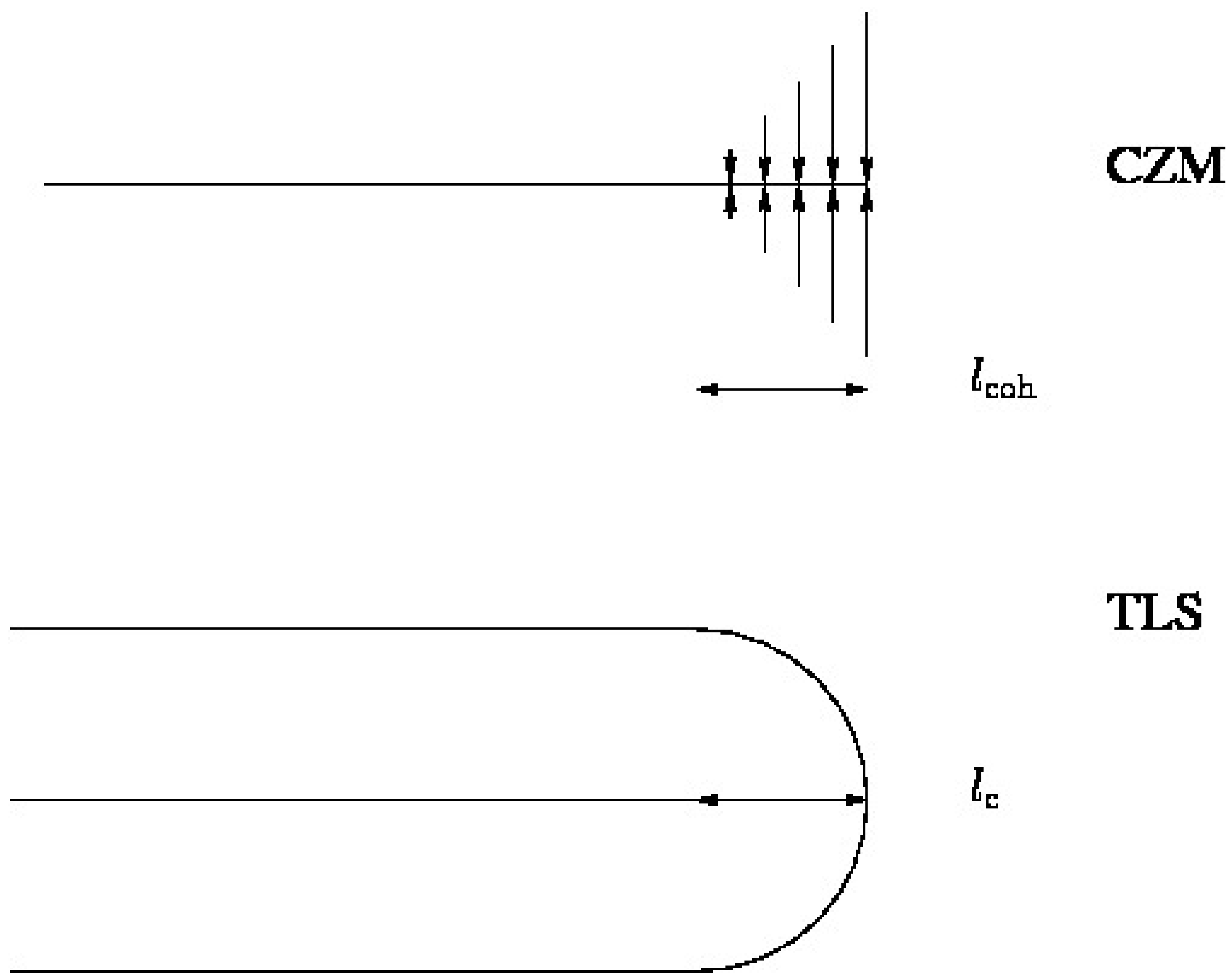
$$v_n = \bar{k} < \frac{g}{\bar{Y}_c} - 1 >^n$$

$$v_n = \vec{n} \cdot \vec{e_x} \dot{a}$$

$$\dot{a} = K < \frac{G}{G_c} - 1 >_+$$

$$G = \int_\Gamma g \; \vec{n} \cdot \vec{e_x} d\Gamma$$





CZM
$l_{coh}$
TLS
$l_c$

# Relationship

## CZM

## TLS

$\sigma_{\rm coh}, G_{\rm coh}$ $\quad \sigma = \sigma(w)$

$l_{\rm c}, Y_{\rm c}$ $\quad d = d(\phi)$

$$\frac{\sigma}{\sigma_{\rm coh}} = (1 - (\frac{w}{w_{\rm coh}})^{2/3})^{1/2}$$

$$G_c \simeq Y_{\rm c} l_{\rm c}$$

Smith 1975

$$l_{\rm coh} = \frac{3 E w_{\rm coh}}{4(1-\nu^2)\sigma_{\rm coh}}$$

$$G_c = G_{\rm coh} \qquad Y_{\rm c} = \frac{(1-\nu^2)}{2E}\sigma_{\rm coh}^2$$

$$\rightarrow \qquad l_{\rm coh} = \frac{2}{\pi} l_{\rm c} = 0.63 l_{\rm c}$$