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Void coalescence modelling : strain hardening, second population and shear effects

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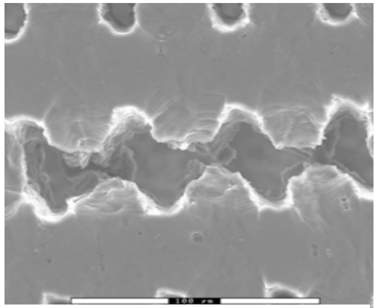

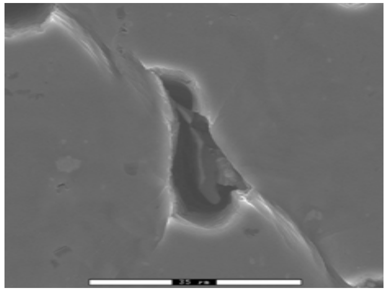

MECAMAT – Journée du GT MECAMAT « Physique et Mécanique de l'Endommagement et de la Rupture »
Modélisation et simulation de la rupture ductile : synthèse des connaissances et questions d'enjeux
Mardi 25 septembre 2012, EDF R&D, Clamart, France

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OUTLINE

1. Experimental observations
2. « Elementary » models for the onset of void coalescence
3. Fine tuning of Thomason model
4. Extension to strain hardening
5. Extension to a second population
6. Extension to combined shear/tension coalescence
7. Models for the coalescence stage

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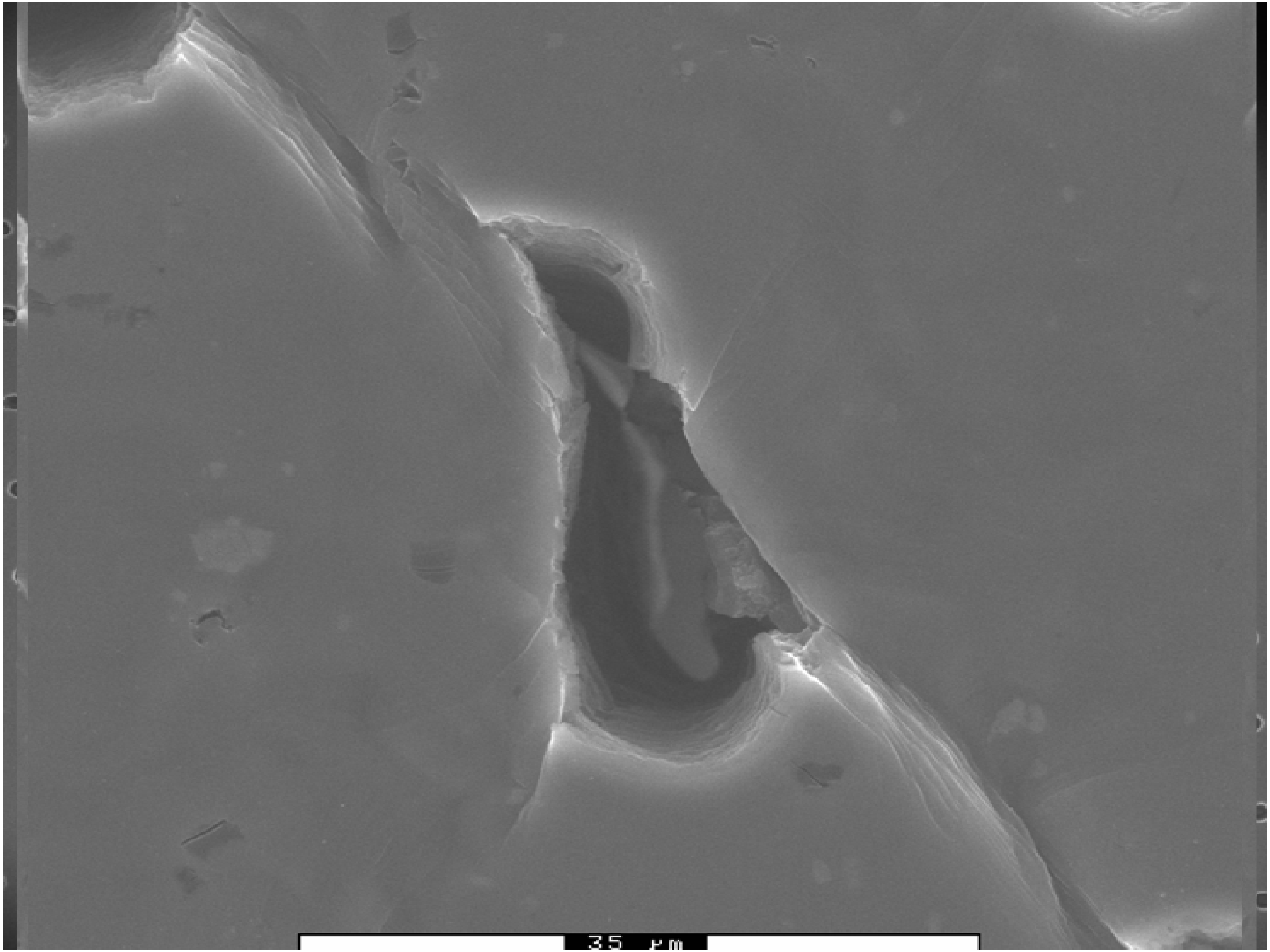
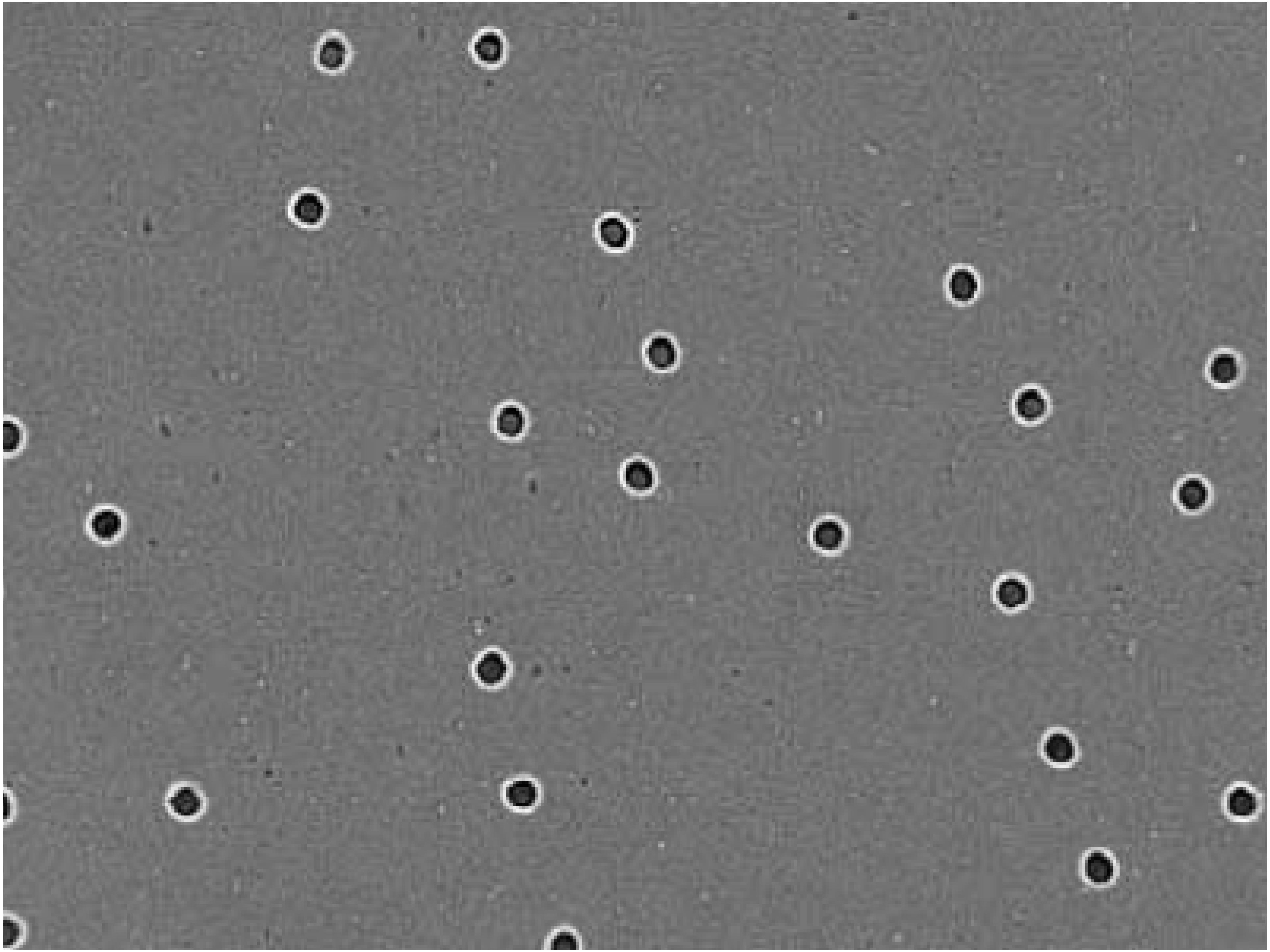
Void coalescence – experiments

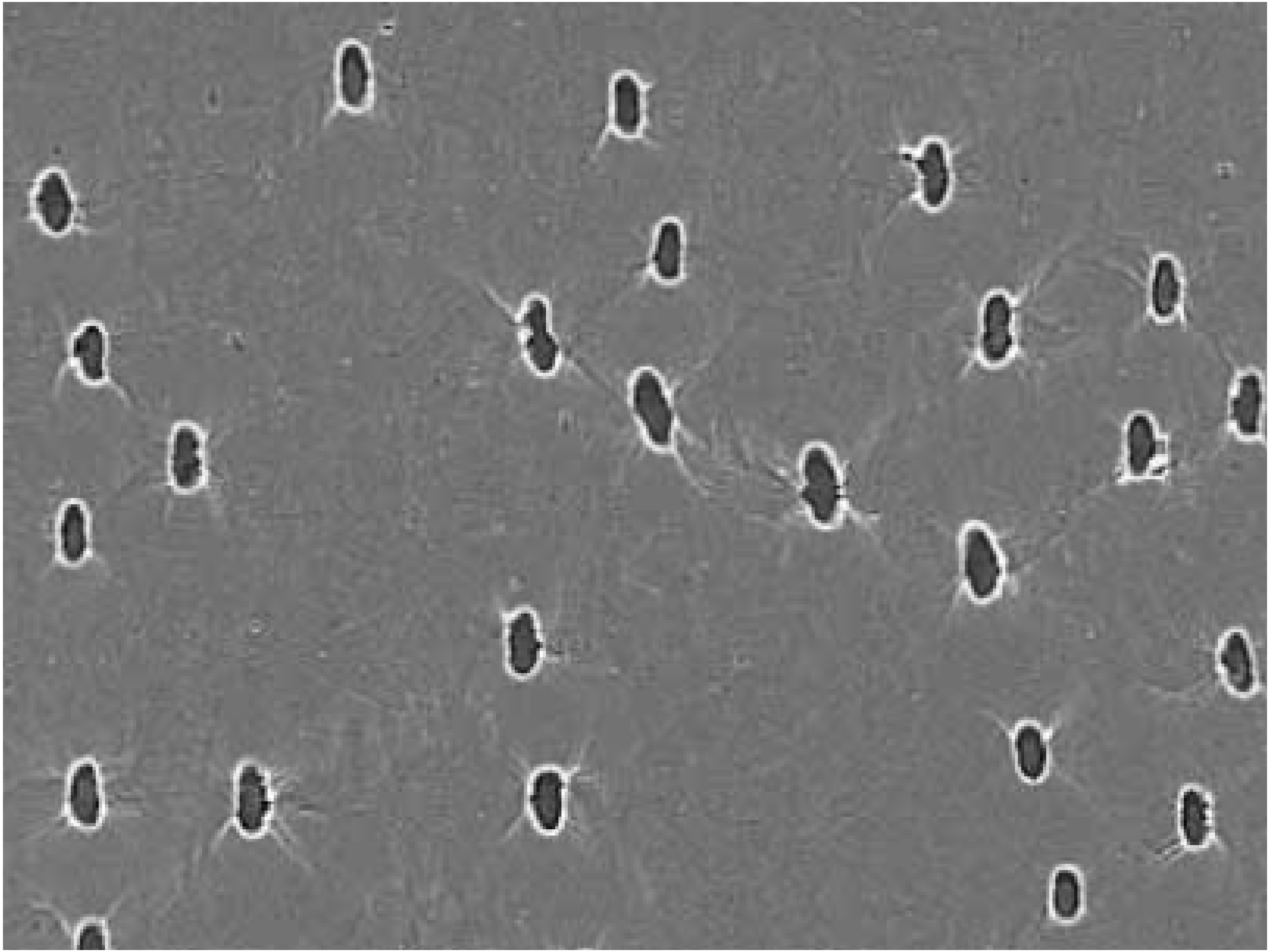
Deformation sequence of a 2D model material:

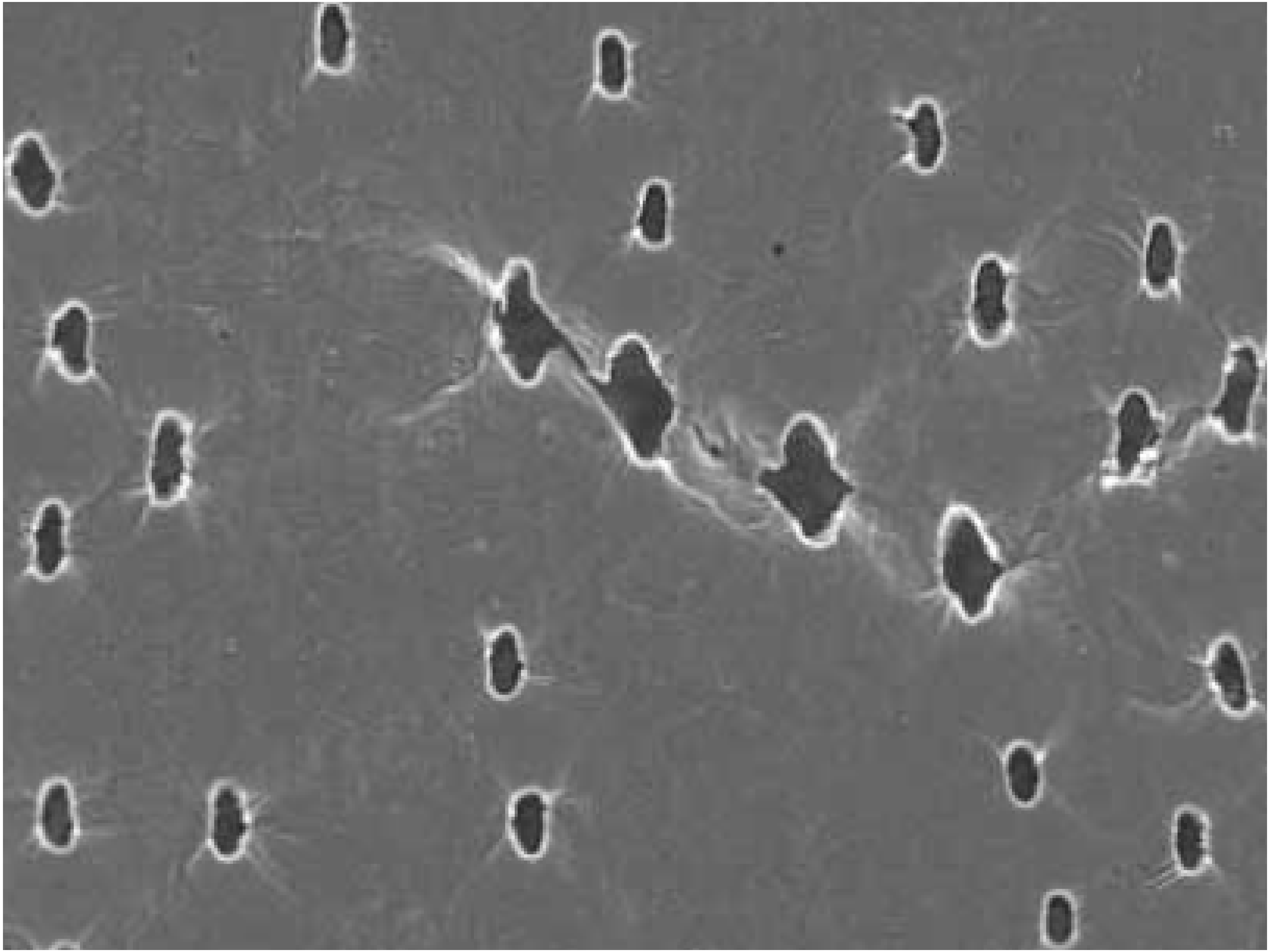
- A 100 μm thick metallic (aluminum alloy 5052) sheet
- The diameter of the laser-drilled holes is at least 10 μm.
- The pictures are taken in-situ in a scanning electron microscope (SEM)

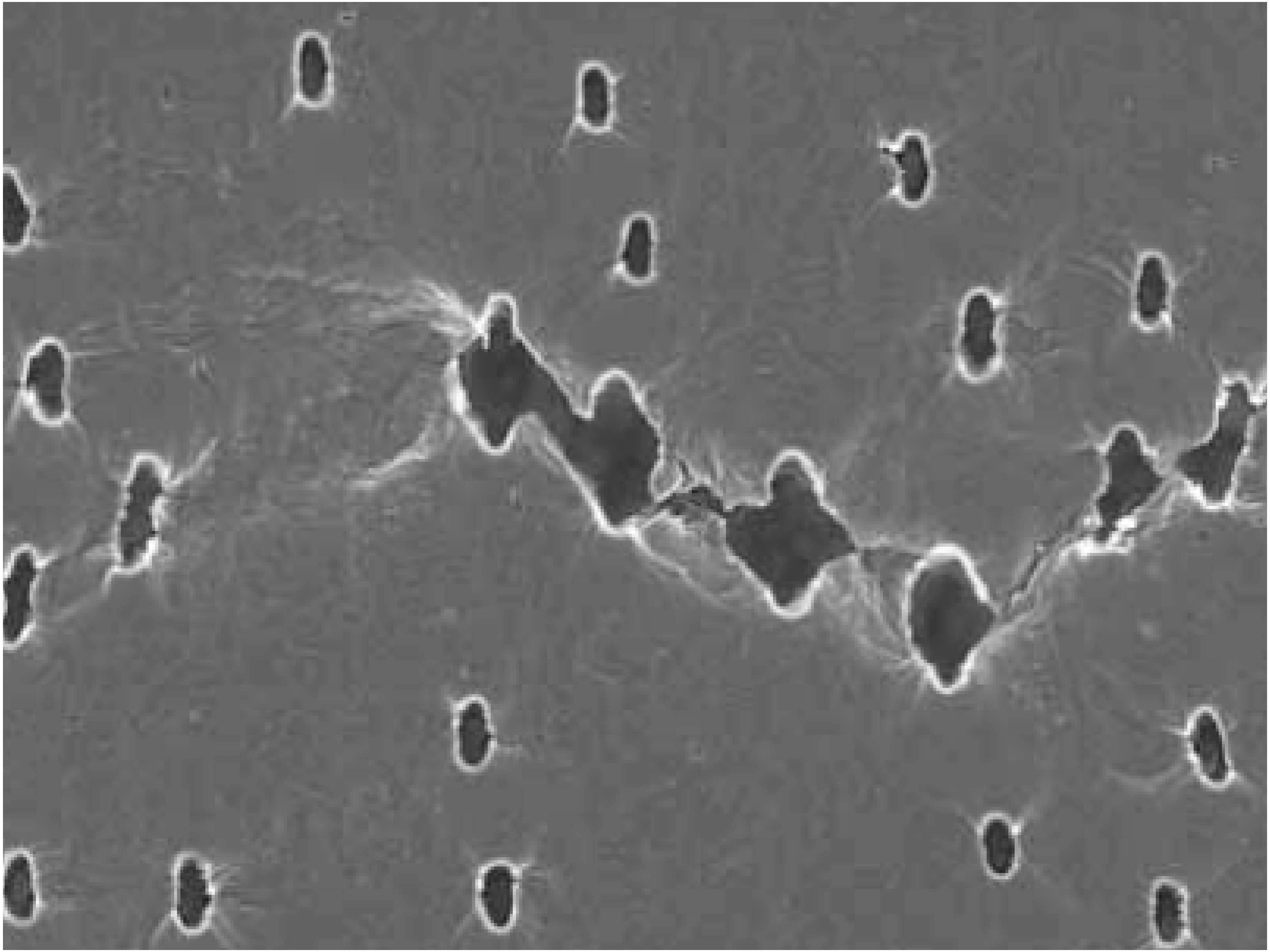
Arnaud Weck, PhD Thesis, McMaster University, 2007


35 µm

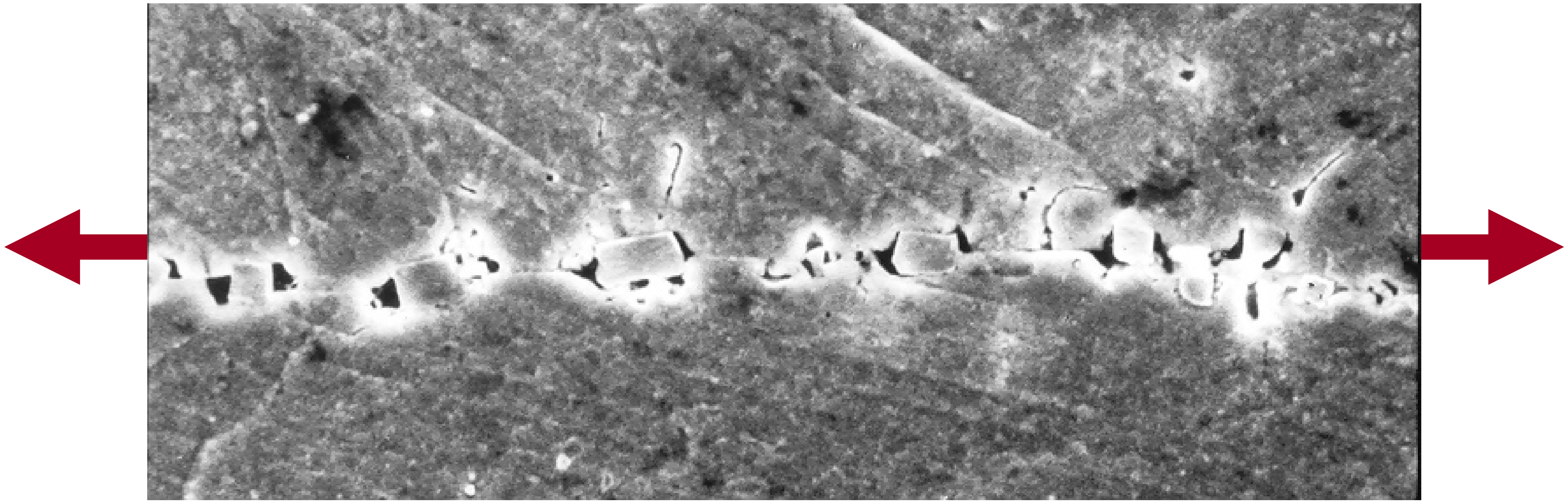






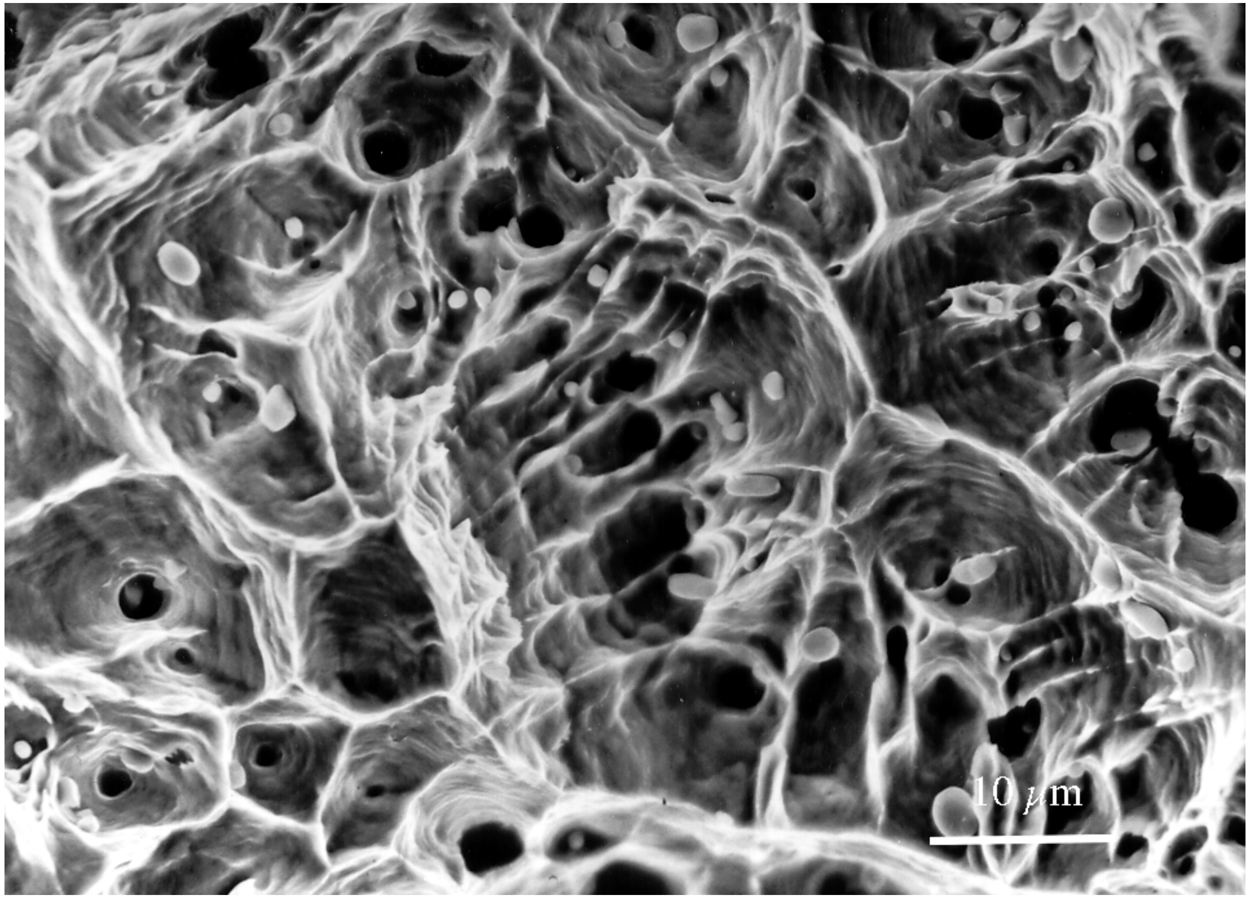


More anecdotic coalescence mechanism : coalescence in column

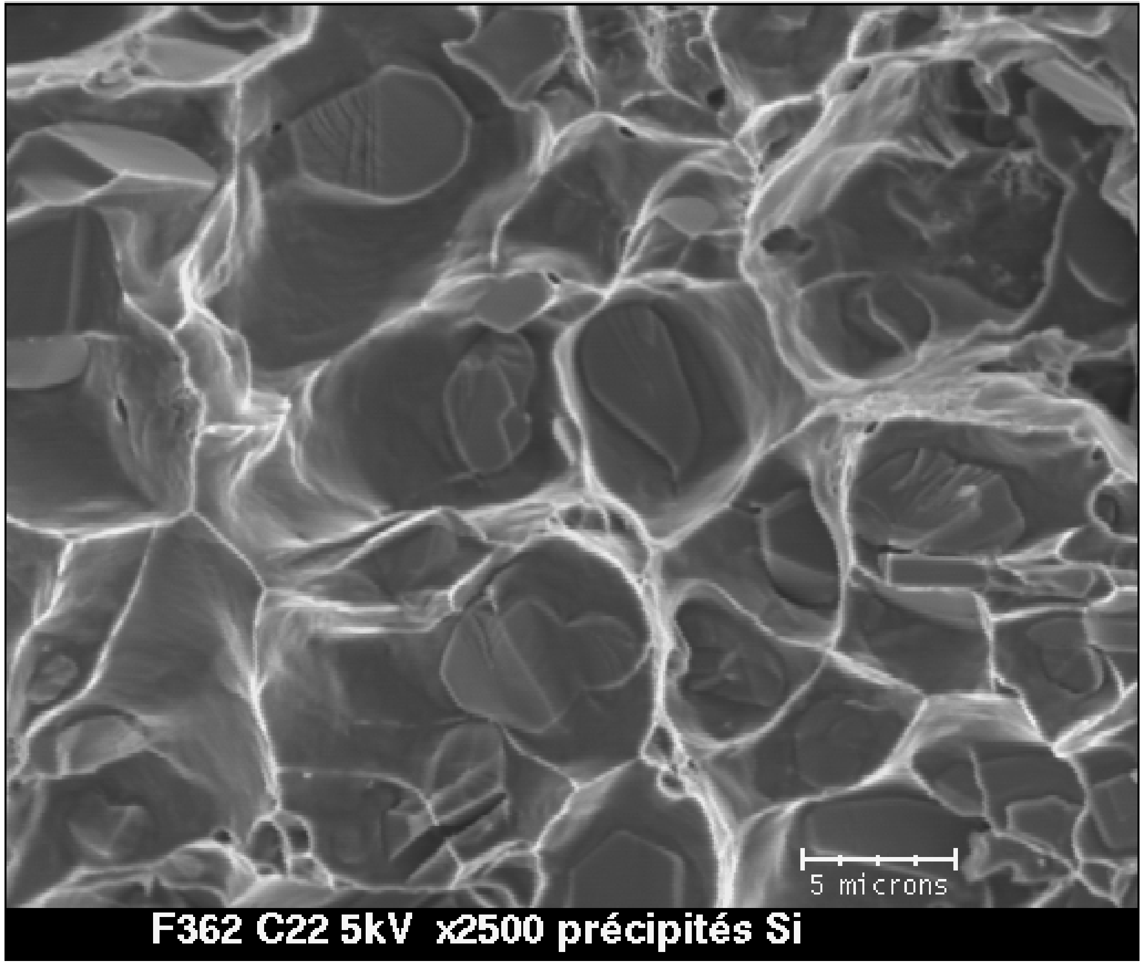


Micrograph : *metallographically prepared specimen made of extruded Cu bars, deformed in tension up to large strains (Pardoen, 1998)*

Final configuration : the fracture surface



Typical of ductile fracture - here voids have nucleated by decohesion of CuO inclusions inside Cu, see inclusions inside the dimples.

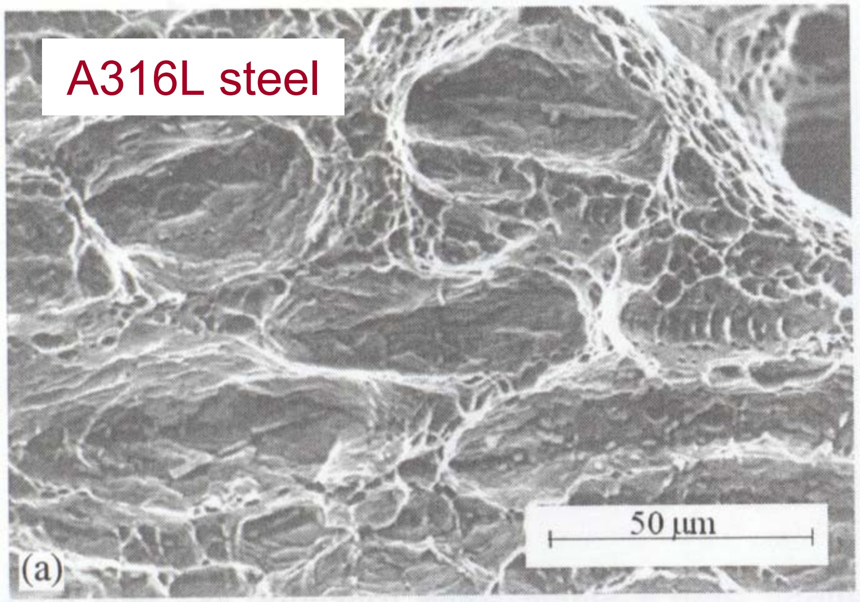


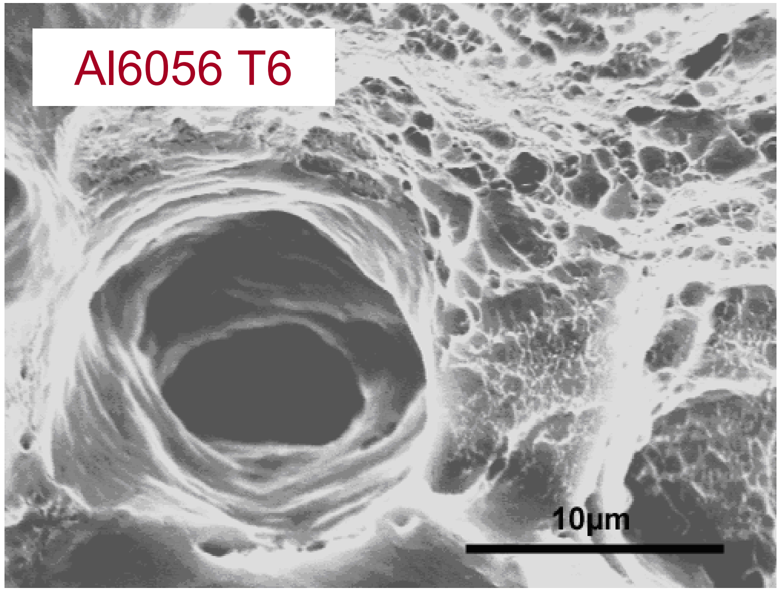
Fracture surface with dimples, typical of ductile fracture (here voids have nucleated by fracture of Si inclusions inside an Al matrix, see broken inclusions inside the dimples)

Secondary voids are often observed on fracture surfaces of metallic alloys

Second populations of voids result from nucleation on submicron sized particles at large strains, e.g.

- carbides in steel
- dispersoïds in Al alloys
- small α particles in α/β Ti alloys





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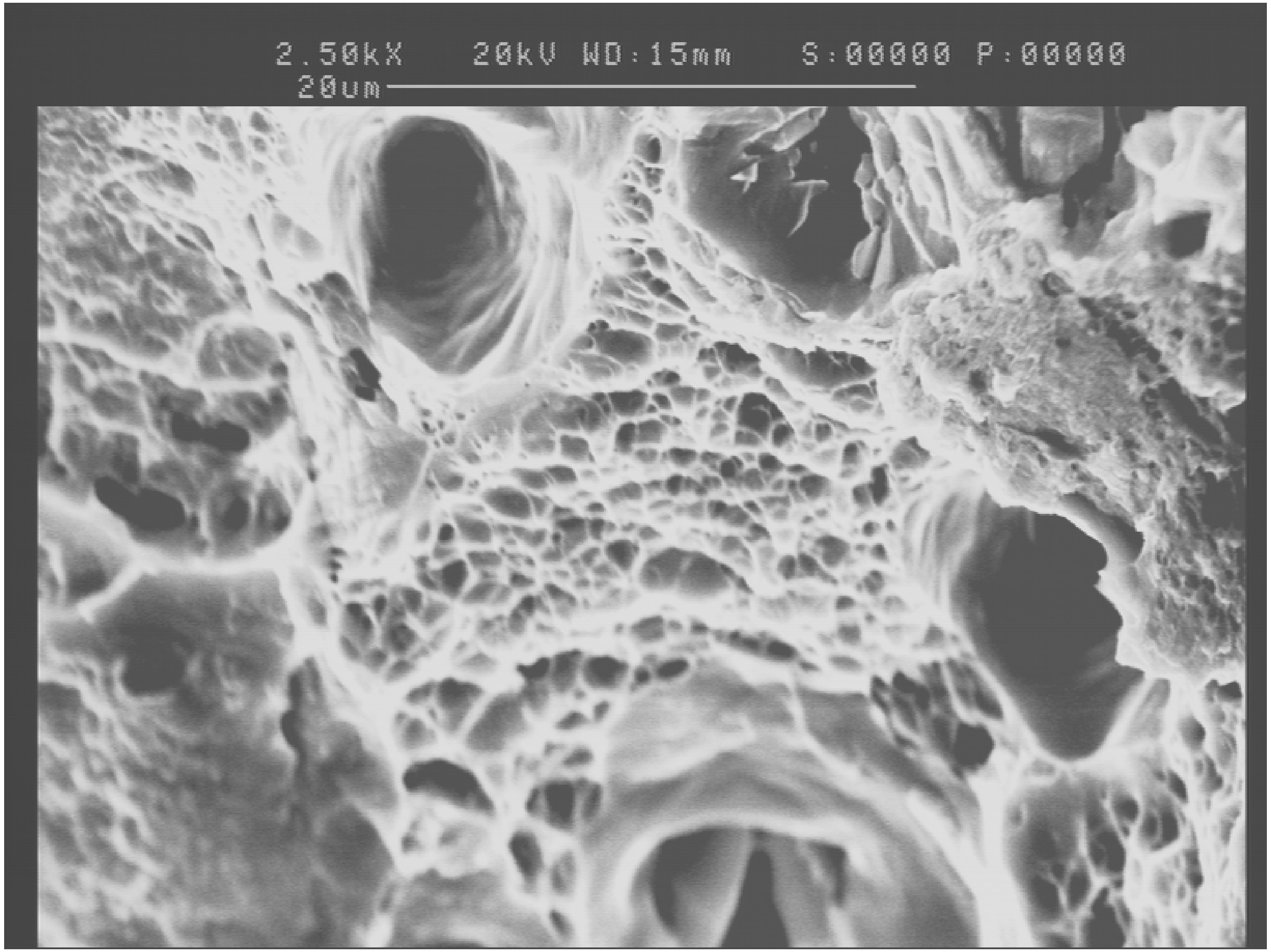
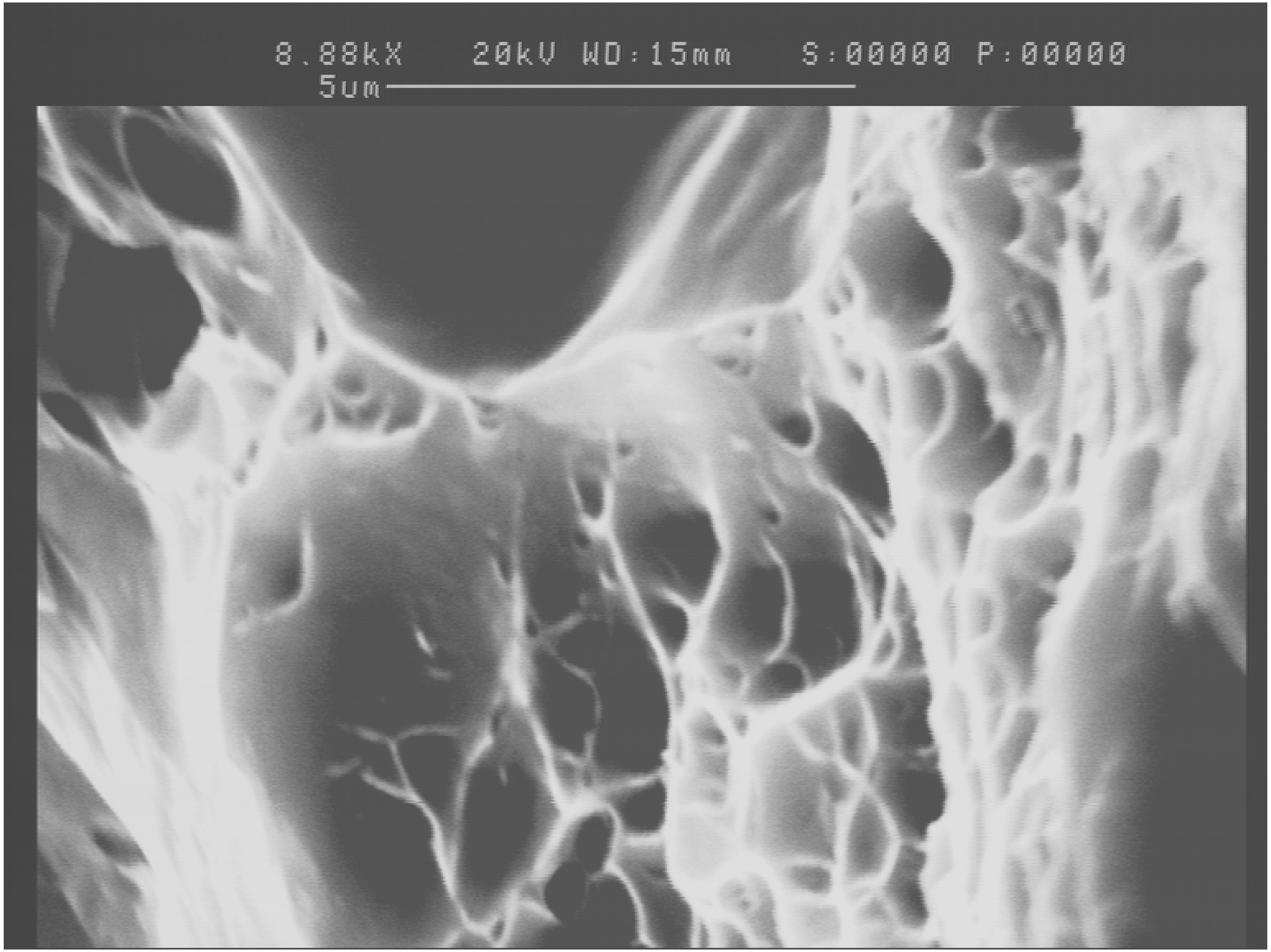
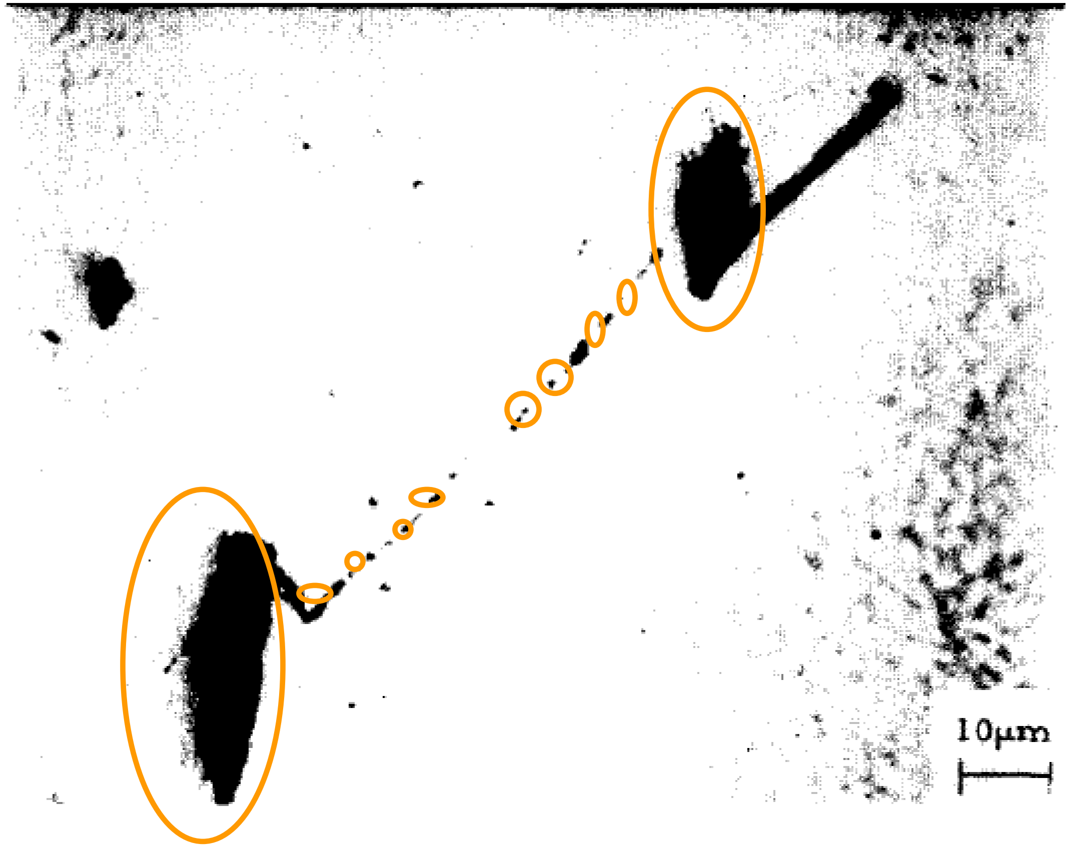



8.88kX 20kV WD:15mm S:00000 P:00000
5um



Cox, T. B. and Low, J. R., Jr. (1974) An investigation of the plastic fracture of AISI 4340 and 18 Nickel–200 grade maraging steels. *Metallurgical Transactions* **5**, 1457–1470.

Summary

• Void coalescence can be seen as a second stage of void growth but with the plastic flow localized in the intervoid ligament

• Void coalescence can take place
(1) ± normal to the main loading direction « internal necking »
(2) ± in the direction of maximum shear « coalescence in shear»
(3) in columns (… anecdotic)

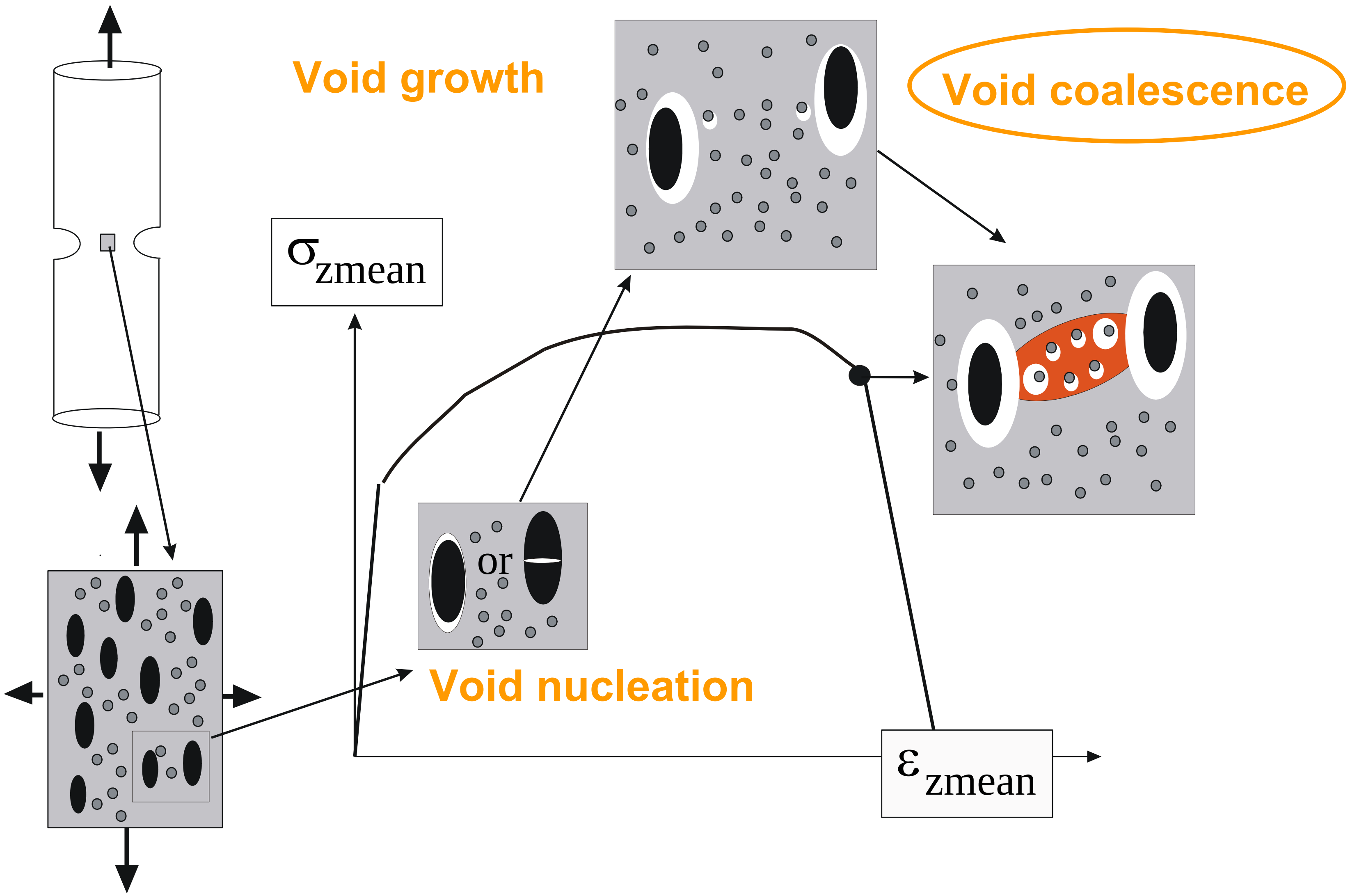
depending on the void arrangement, loading mode and strain hardening capacity

• Void coalescence can be accelerated by the presence of a second population of voids (in shear : « void sheeting »)

• Distinguish « onset of coalescence » and « coalescence »

• Void coalescence should not be confused with (meso) plastic localization mechanisms resulting from the damage induced softening, involving effects of the structure and boundary conditions

Summary



Void growth
Void coalescence
σzmean
or
Void nucleation
εzmean

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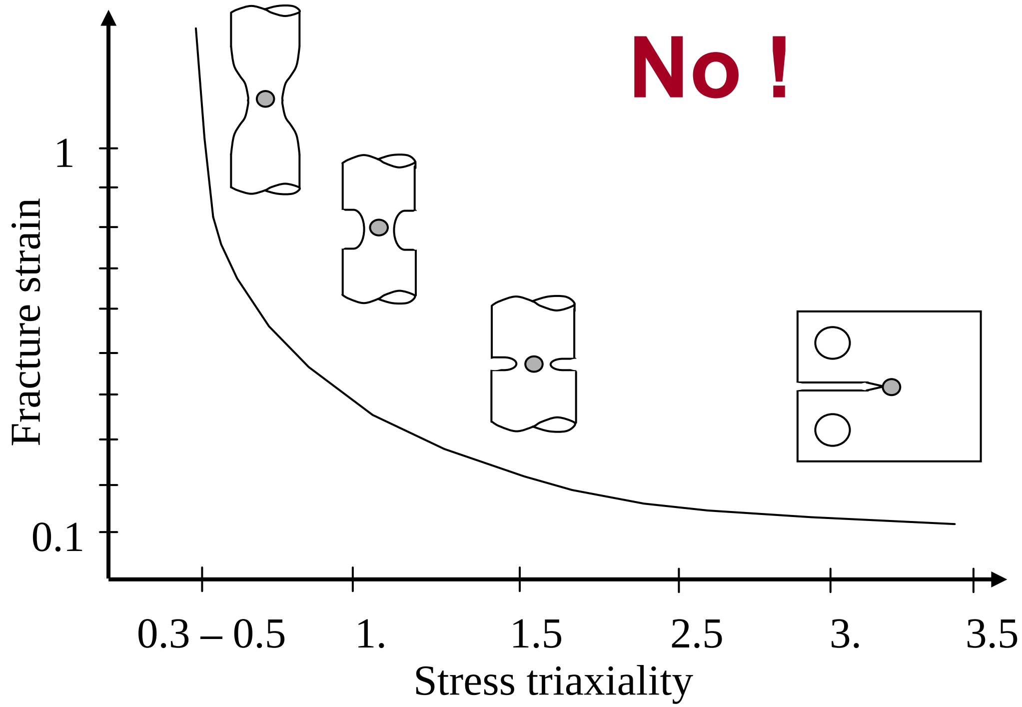
Empirical models for the onset of void coalescence based on macroscopic quantities

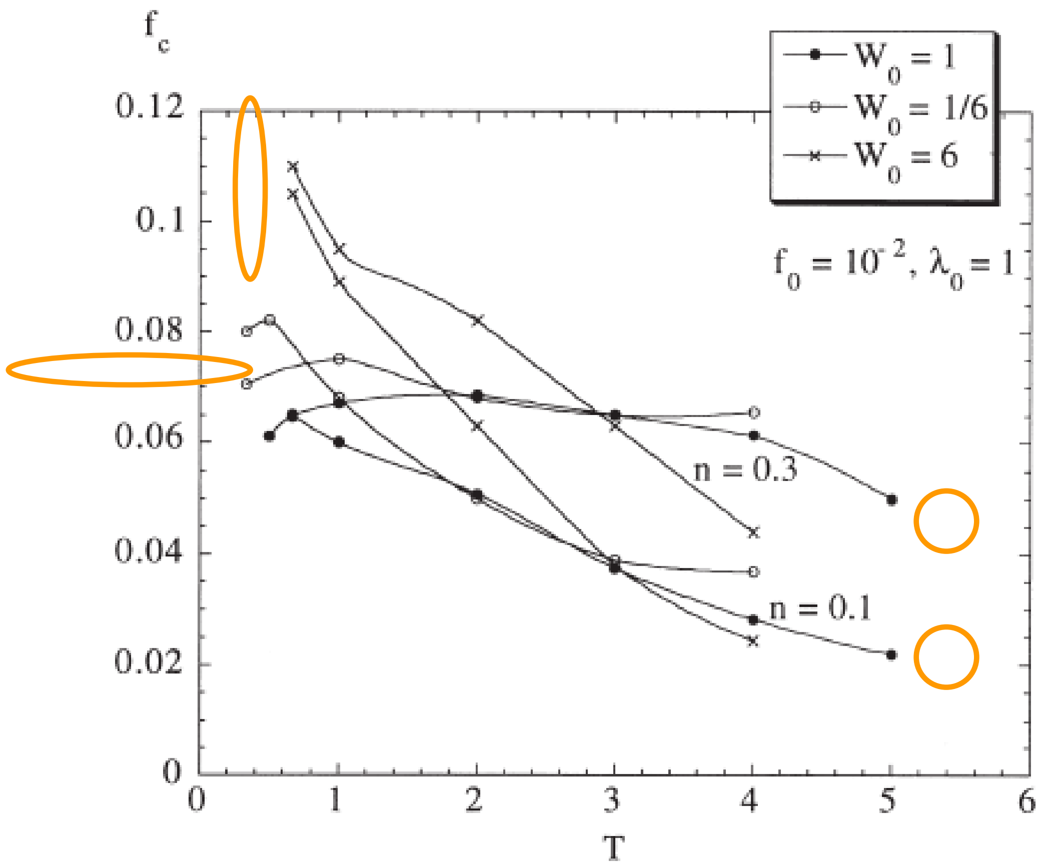
Critical strain ?



Other « critical » combinations of stress and strain (e.g. critical energy density, …) ? always miss the most important information : **void spacing**

Critical porosity model

Better, but it does moderately vary with stress triaxiality

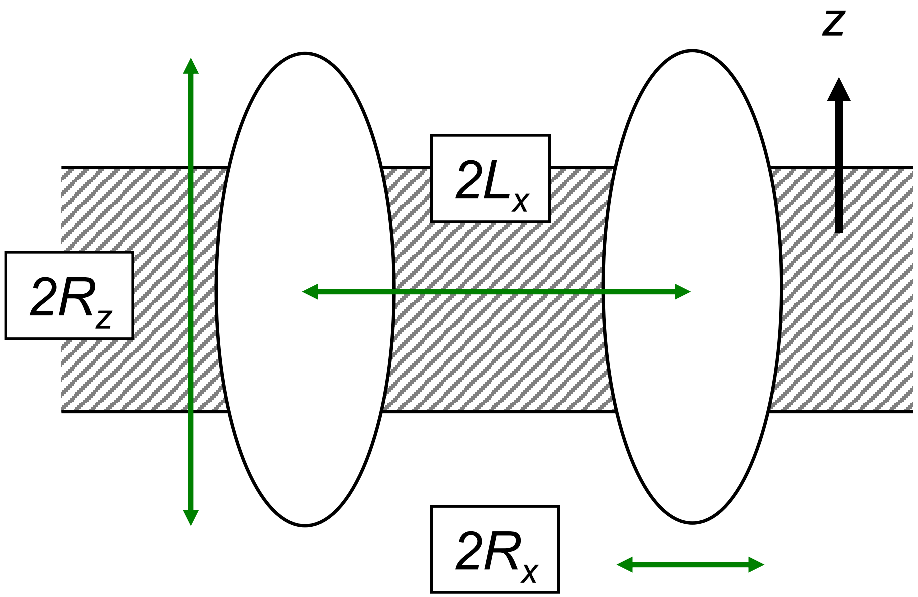


Pardoen & Hutchinson, *JMPS* 2000

also misses the most important information : **void spacing**

Prefer a micromechanical approach for the onset of void coalescence

Geometry

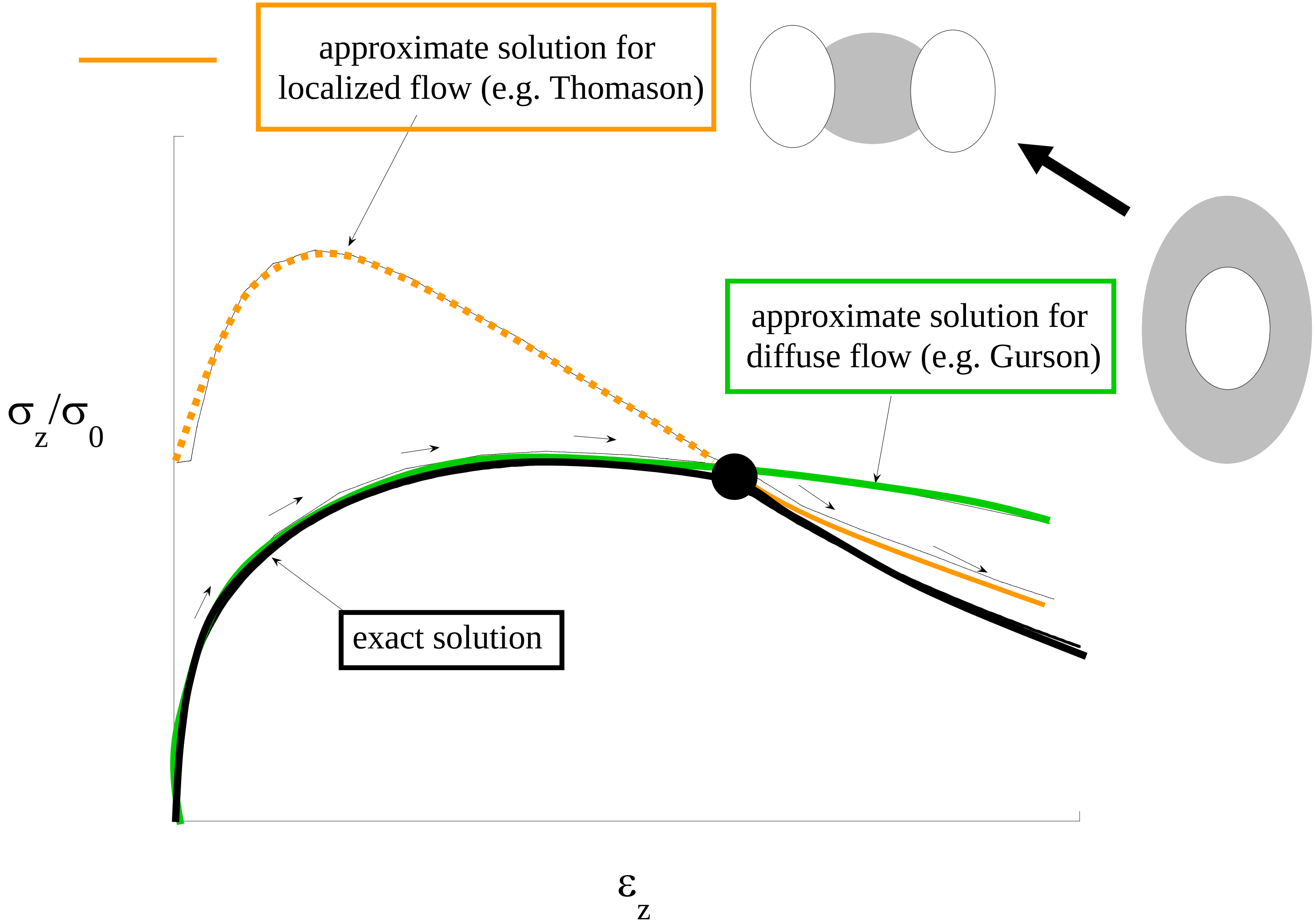


$\chi = R_x/L_x$ (relative void spacing)

$W = R_z/R_x$ (void aspect ratio)

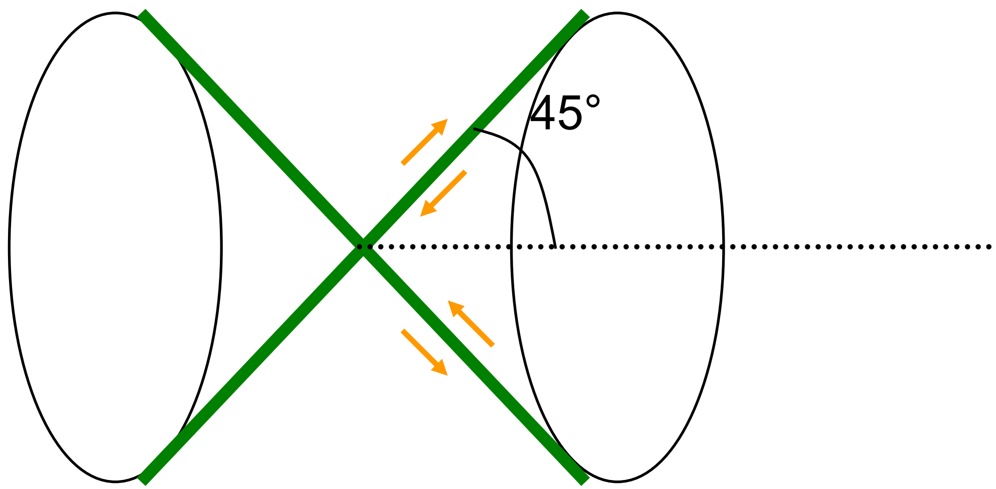
Construction of a solution for a localized mode of deformation

Void coalescence : transition to a localized mode of plastic deformation confined in the ligament between neighbouring voids



Brown and Embury (1973)

Coalescence (internal necking) sets in when two 45° shear bands can connect the two neighbouring voids

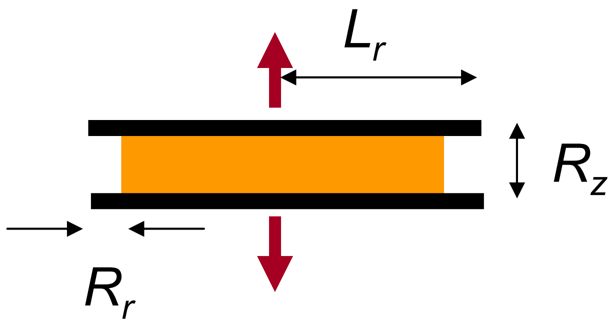


$$\chi \sqrt{1+W^2} = 1$$

Hill (1950)

An approximate analysis for the limit load of this configuration, with associated average true stress, can be carried out along the lines of Hill's (1950) plane strain analysis of a thin plastic layer welded to and squeezed by two rigid platens. The analysis assumes the material in the disk moves outward flowing in shear and otherwise supporting only hydrostatic tension such that the three normal stresses are approximately equal. Radial equilibrium (with the approximation that the normal stresses at the void are zero) provides the applied stress as a function of the current geometry

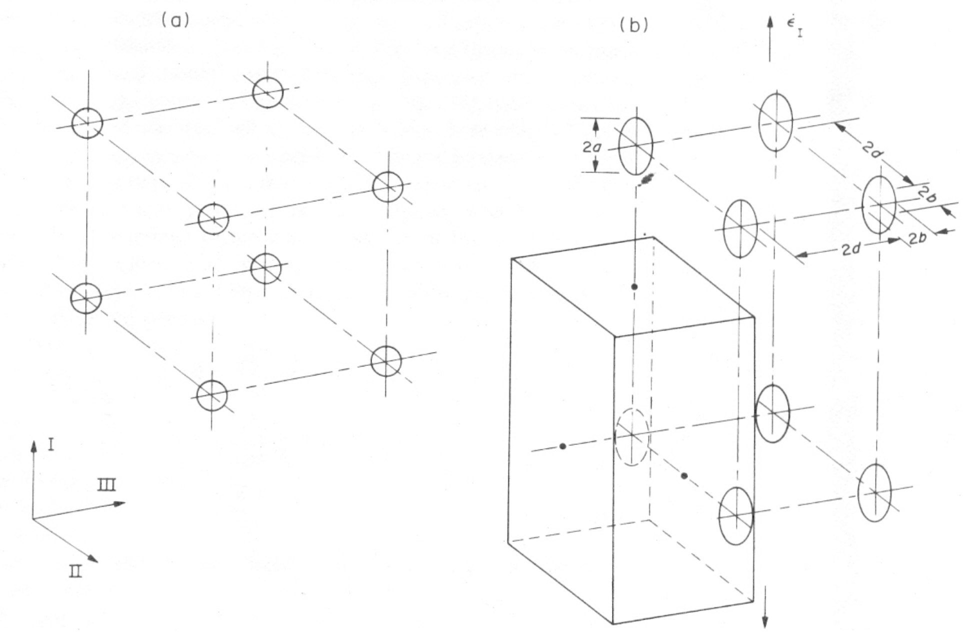
$$\frac{\Sigma_z}{\sigma_0} = \frac{2}{3\sqrt{3}} \frac{L_r}{R_z} \left(1 - \frac{R_r}{L_r}\right)^2 \left(2 + \frac{R_r}{L_r}\right)$$



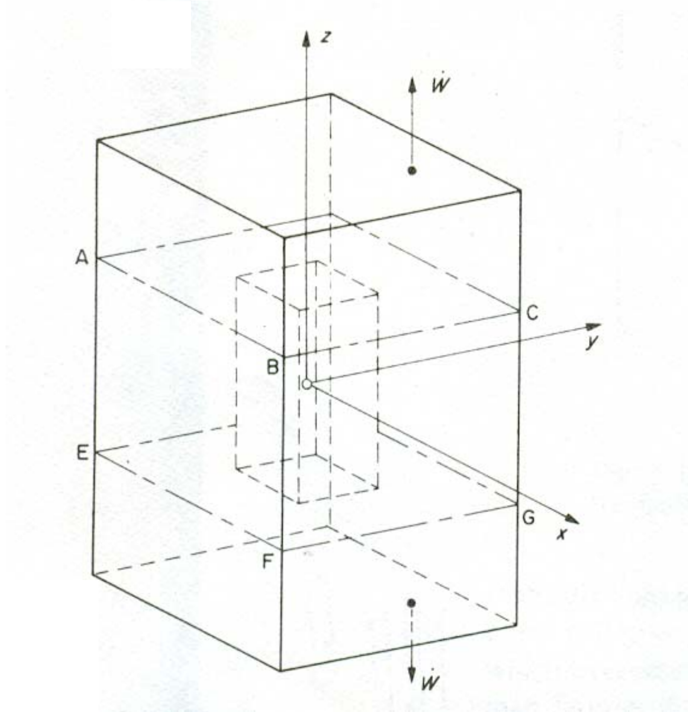
Volume conservation
$$\frac{R_z}{R_{z0}} = \frac{1 - (R_{r0} / L_r)^2}{1 - (R_r / L_r)^2}$$

$$E_z = \ln\left[(R_z - R_{z0}) / L_{z0}\right]$$

Thomason (1968,1985)



Initial geometry

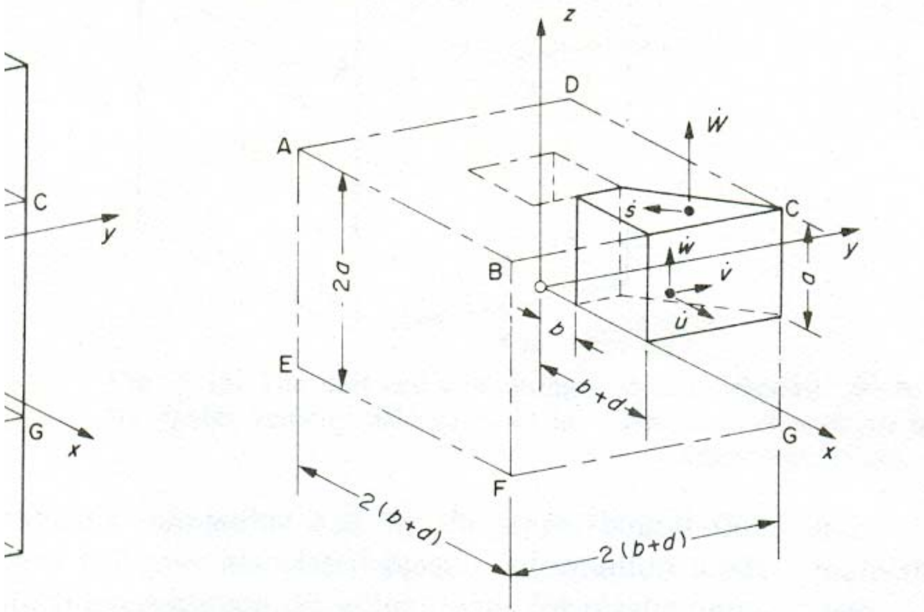


Ellipsoidal voids simplified as square prismatic voids

Thomason (1968,1985)

Upper bound analysis – propose kinematically admissible velocity fields

« parallel » velocity field
(solution 1)

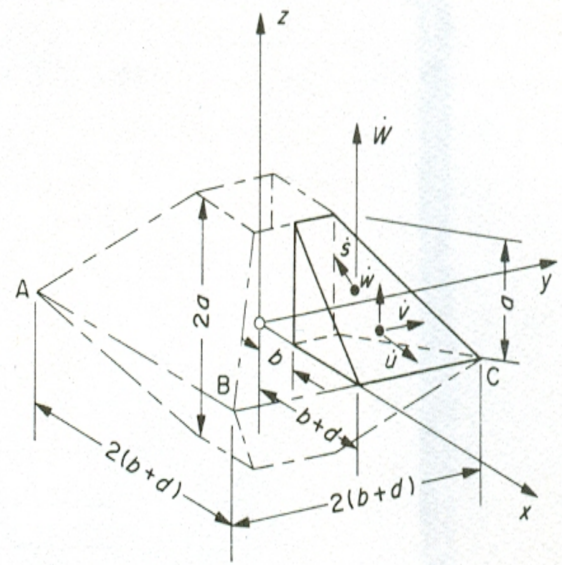


$a = R_z$

$b = R_x$

$d = L_x - R_x$

« triangular » velocity field
(solution 2)



$$\dot{u}(x,y,z) = \frac{\dot{W}}{2R_z x}\left(L_x^2 - x^2\right)$$

$$\dot{v}(x,y,z) = \frac{\dot{W}}{2R_z x^2}\left(L_x^2 - x^2\right)$$

$$\dot{w}(x,y,z) = \frac{\dot{W}z}{R_z}$$

+ associated tangent velocity discontinuity $\dot{s}$ at rigid/plastic boundary (see Thomason 1985)

$$\dot{u}(x,y,z) = \frac{\dot{W}(L_x - R_x)}{2R_z x}(L_x + x)$$

$$\dot{v}(x,y,z) = \frac{\dot{W}(L_x - R_x)y}{2R_z x^2}(L_x + x)$$

$$\dot{w}(x,y,z) = -\frac{\dot{W}z(L_x - R_x)}{2R_z x}$$

Thomason (1968,1985)

Rate of internal work

$$\dot{I} = \sqrt{\frac{2}{3}}\sigma_0 \int_V \sqrt{\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{zz}^2 + \frac{1}{2}\left(\dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{yz}^2 + \dot{\varepsilon}_{xz}^2\right)} dV + \frac{\sigma_0}{\sqrt{3}} \int_S \dot{s} dS$$

=

Rate of external work

$$\dot{E} = \sigma_n A_n \dot{W}$$

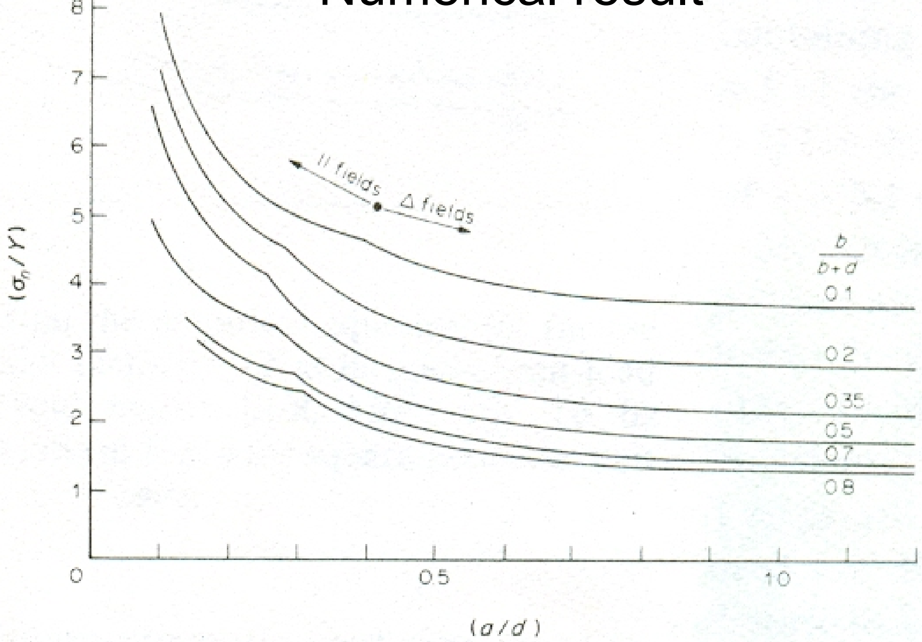
with σ_n applied on the non porous (effective) surface

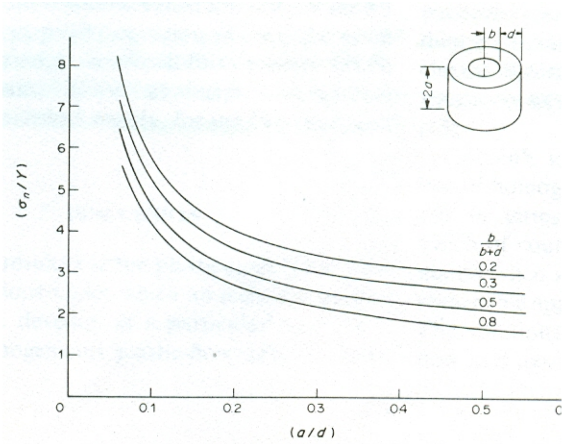
using $$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i}\right)$$

and $u_1 = u$, $u_2 = v$, $u_3 = w$

Thomason (1968,1985)

Numerical result





Note : ressemble cylindrical solution

with η a function of void distribution

$$\frac{\sigma_n}{\sigma_0} = \left[0.1 \left(\frac{1-\chi}{\chi W} \right)^2 + 1.2 \sqrt{\frac{1}{\chi}} \right]$$

$$\frac{\sigma_z}{\sigma_0} = \left(1 - \eta \chi^2\right) \left[0.1 \left(\frac{1-\chi}{\chi W} \right)^2 + 1.2 \sqrt{\frac{1}{\chi}} \right]$$

χ is the key parameter

Note the extension by Benzerga, Besson, Pineau (2000)

$$\frac{\Sigma_{22}}{\sigma_0} = (1-\eta\chi^2)\left[\alpha\left(\frac{\chi^{-1}-1}{W^2+0.1\chi^{-1}+0.02\chi^{-2}}\right)^2+\beta\frac{1}{\sqrt{\chi}}\right]$$

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Journal of the Mechanics and Physics of Solids 60 (2012) 1363–1381

Contents lists available at SciVerse ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

A criterion for the onset of void coalescence under combined tension and shear

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A R T I C L E I N F O

Article history:
Received 24 June 2011
Received in revised form
23 January 2012
Accepted 19 February 2012
Available online 24 February 2012

Keywords:
Ductility
Fracture mechanisms
Voids and inclusions
Finite elements
Limit-load analysis

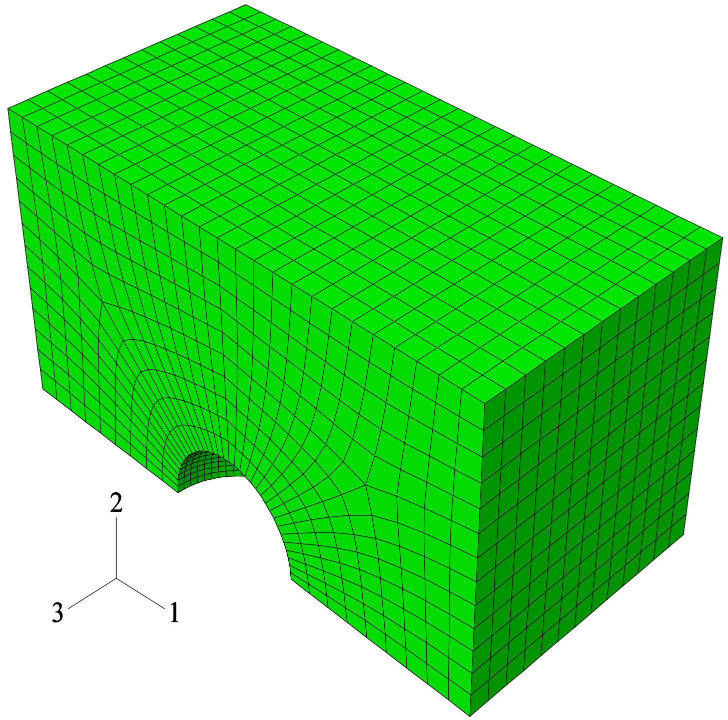
A B S T R A C T

Depending on the relative positions of voids and on the loading conditions, shear loading components can play an important role in the void coalescence process leading to ductile fracture. Yet, most void coalescence criteria including the original criterion of Thomason, and its various extensions/improvements, take only normal loads into account and neglect the contribution from shear loads to coalescence. Shear can affect both the stress/strain at the onset of coalescence and the direction of deformation localization. In this paper, first, the predictive capabilities of different coalescence criteria without shear effect are critically assessed and the expressions involved in the original Thomason criterion are fine-tuned by comparing with 3D finite element calculations performed on a unit cell containing a spheroidal void. Then, the improved Thomason criterion is theoretically extended—by using limit load analysis—to incorporate the effect of shear. The predictions of this new coalescence criterion are in good agreement with the results produced by 3D finite element calculations, for both loadings involving or not a shear component.

Limit Load Analysis with FE Calculations

Elastic perfectly plastic material

$$\frac{E}{\sigma_0} = 444:5;\ \nu = 0:49$$



Predominant axial stress state

$$\Sigma_{22} > \Sigma_{11}\ (= \Sigma_{33})$$

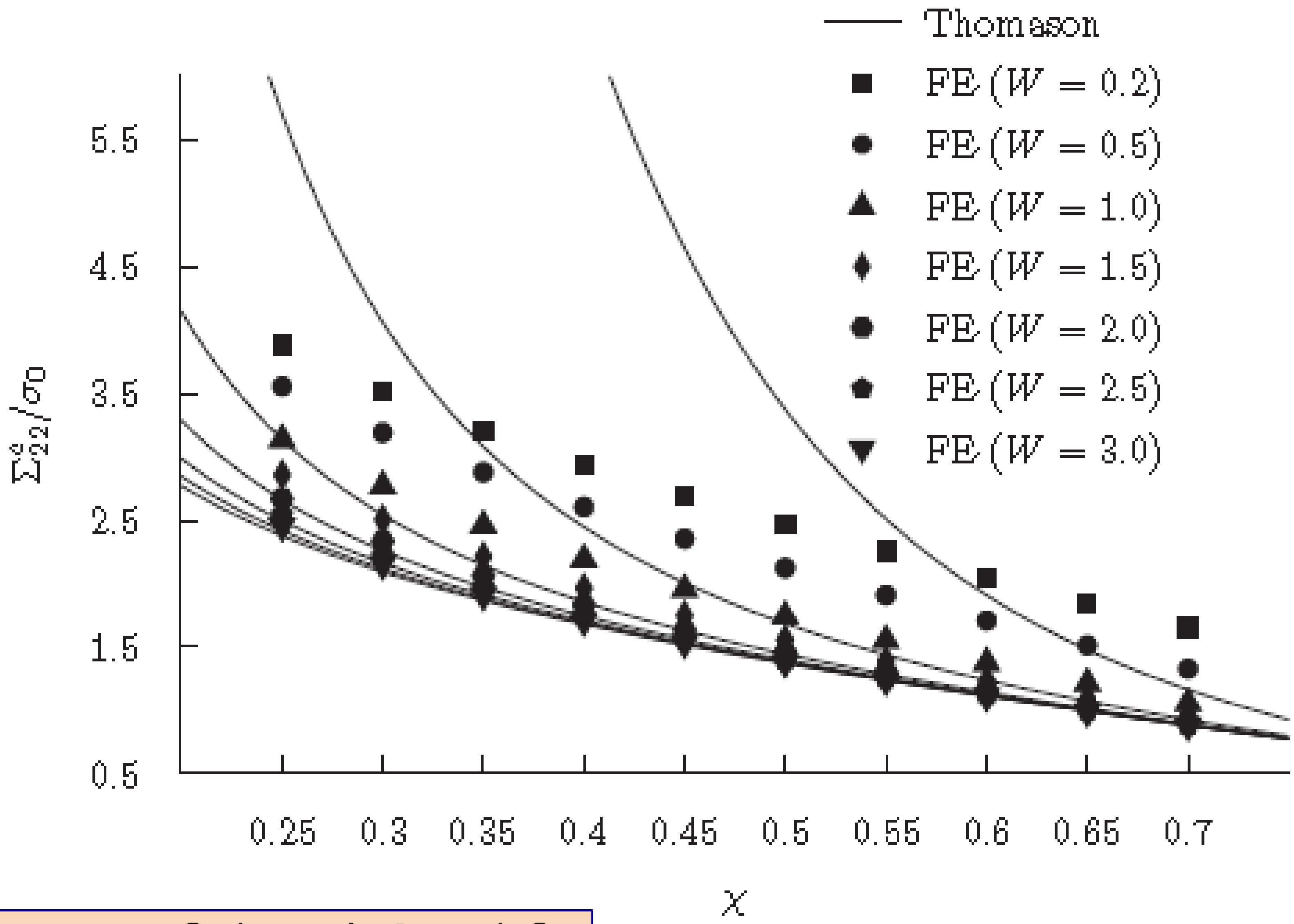
Displacement BC's are applied

$$E_{22} > E_{11} (= E_{33})$$

Equilibrium eqs. are solved on the initial configuration

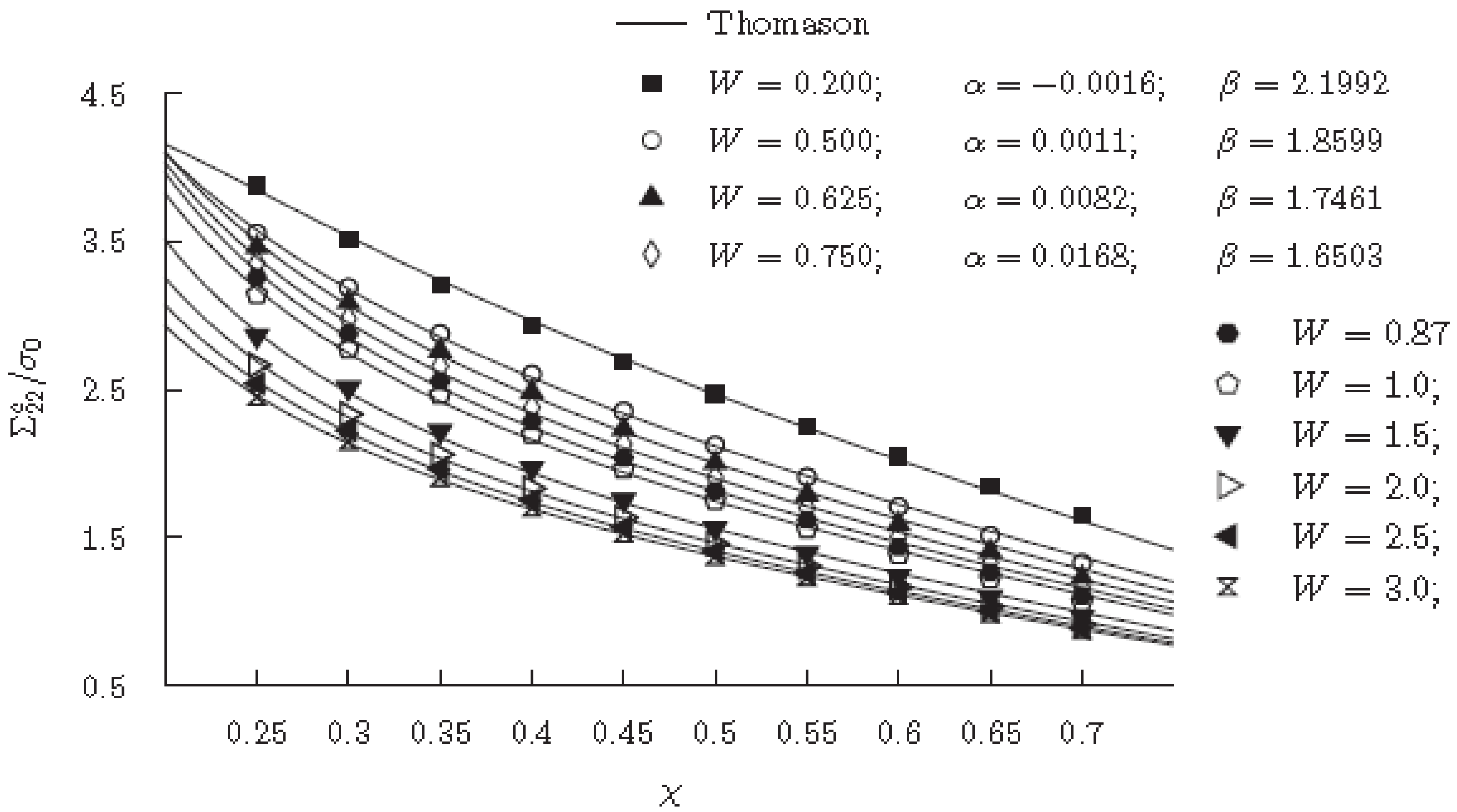
$$W = W_0 \text{ and } A = A_0$$

Onset of coalescence (Thomason versus FE)



with α = 0.1 and β = 1.2

Modified Thomason versus FE



$$\frac{\Sigma_{22}}{\sigma_0} = (1-\eta\chi^2)\left[\alpha^{Th}(W)\left(\frac{1-\chi}{W\chi}\right)^2 + \beta^{Th}(W)\frac{1}{\sqrt{\chi}}\right],$$

$$\alpha^{Th}(W) = 0.0819W - 0.0373,\ \ \beta^{Th}(W) = 0.0036W^5 - 0.0030W^4 - 0.1694W^3 + 0.8499W^2 - 1.6743W + 2.5022$$

Modified Thomason versus FE

This is a very recent contribution which was thus, unfortunately, not used in the other enhancements described hereafter.

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4. **Extension to strain hardening**

5. Extension to a second population

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Incorporate strain hardening

$$\frac{\sigma_z}{\sigma_y} = \left(1-\eta\chi^2\right)\left[0.1\left(\frac{1-\chi}{\chi W}\right)^2 + 1.2\sqrt{\frac{1}{\chi}}\right]$$

?

First approach (2000)

σ_y is defined as the average yield stress of the matrix material and computed using the energy balance (Gurson, 1977)

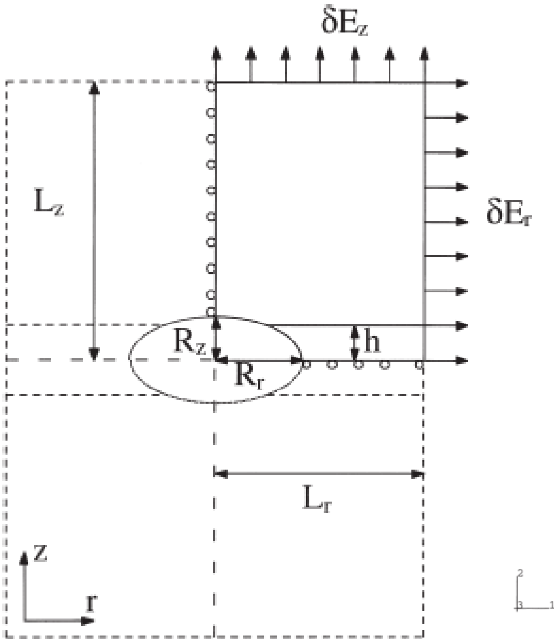
$$\sigma_y\,\dot{\varepsilon}_y^p\left(1-f\right) = \sigma_{ij}\,\dot{\varepsilon}_{ij}^p$$

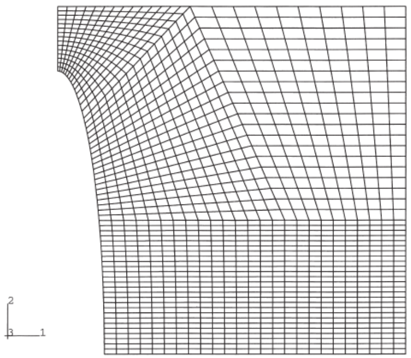
Pardoen & Hutchinson, *JMPS* 2000

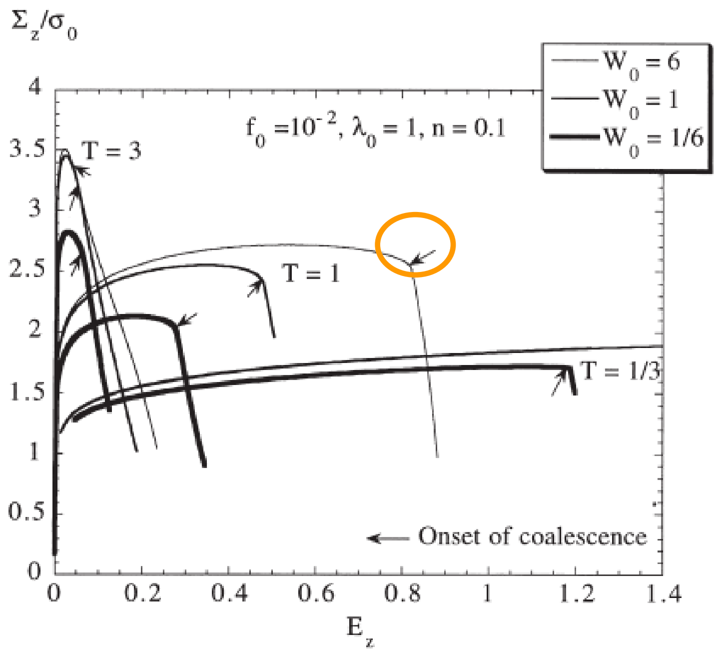
Validation of first approach

$$\frac{\sigma_z}{\sigma_y} = \left(1-\eta\chi^2\right)\left[0.1\left(\frac{1-\chi}{\chi W}\right)^2 + 1.2\sqrt{\frac{1}{\chi}}\right]$$

a. run FE void cell calculations





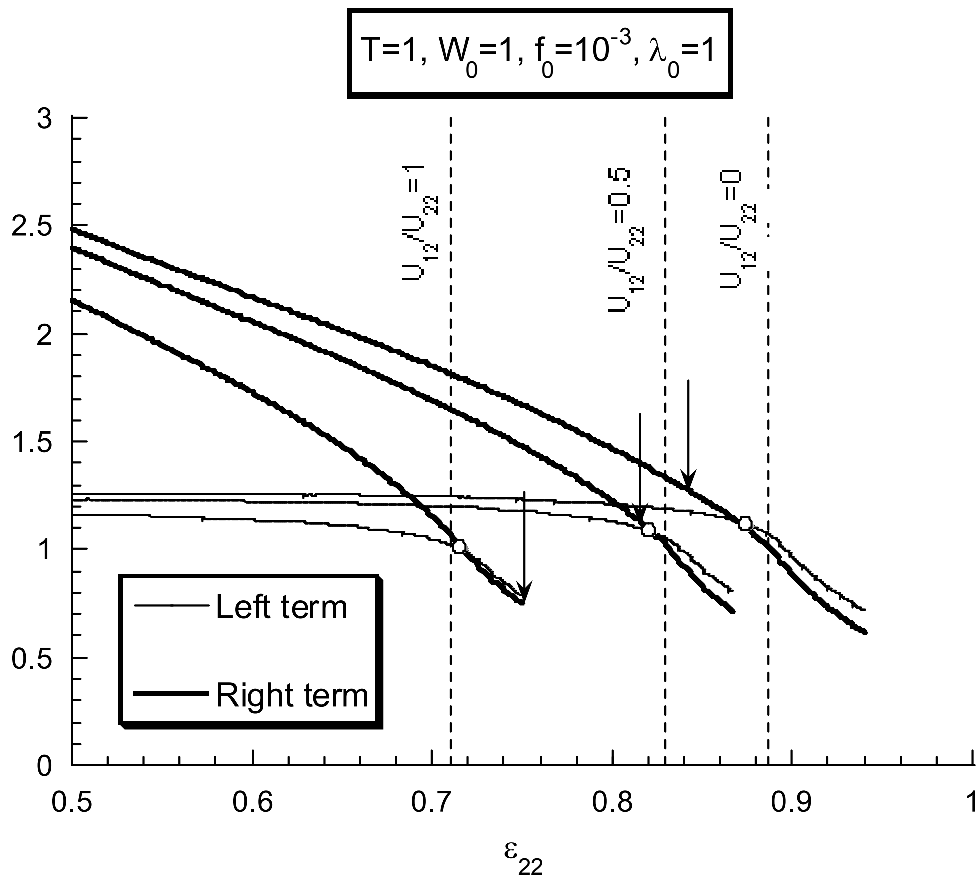


b. extract from the cell calculation the evolution of *W*, χ, σ_z and σ_y

Pardoen & Hutchinson, *JMPS* 2000

Plug the values into the Thomason model and detect when the criterion is fulfilled

$$\frac{\sigma_z}{\sigma_y} = \left(1 - \eta\chi^2\right)\left[0.1\left(\frac{1-\chi}{\chi W}\right)^2 + 1.2\sqrt{\frac{1}{\chi}}\right]$$

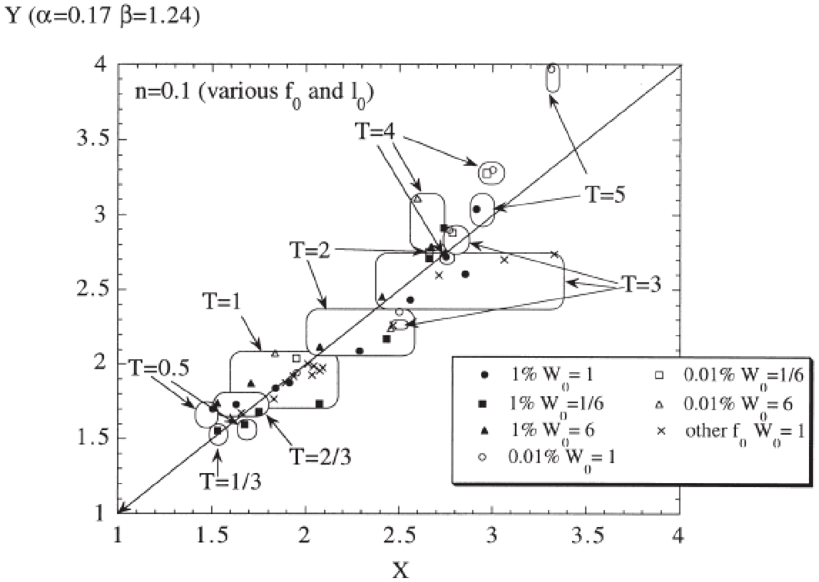


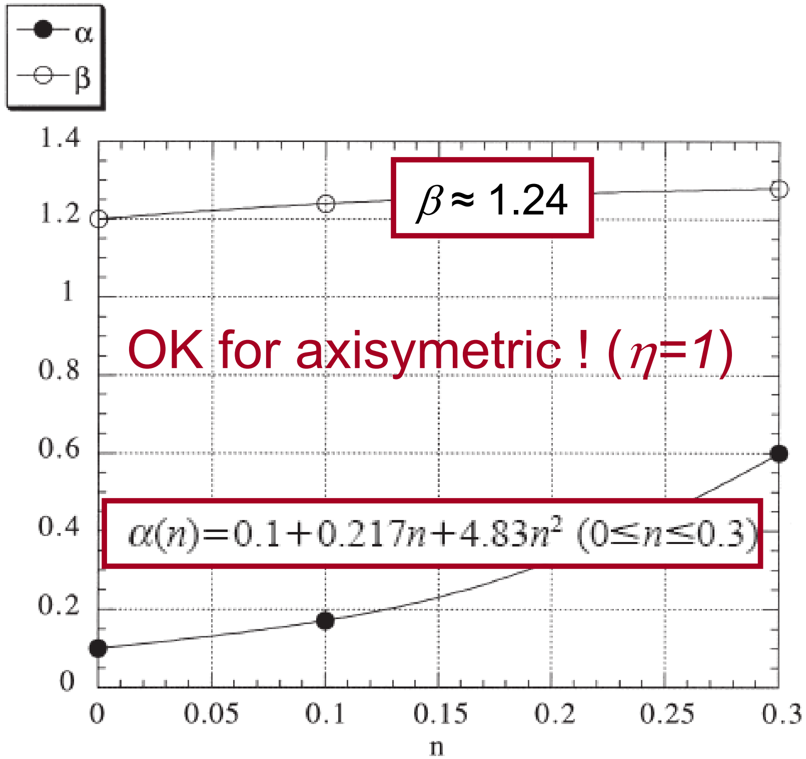
Pardoen & Hutchinson,
JMPS 2000

Result : a strain hardening dependent parameter is required to allow accurate predictions

$$\frac{\sigma_z}{\sigma_y} = \left(1-\eta\chi^2\right)\left[\alpha(n)\left(\frac{1-\chi}{\chi W}\right)^2 + \beta(n)\sqrt{\frac{1}{\chi}}\right]$$

$$\sigma_y = \sigma_0\left(1+\frac{\varepsilon_y^p}{\varepsilon_0}\right)^n$$

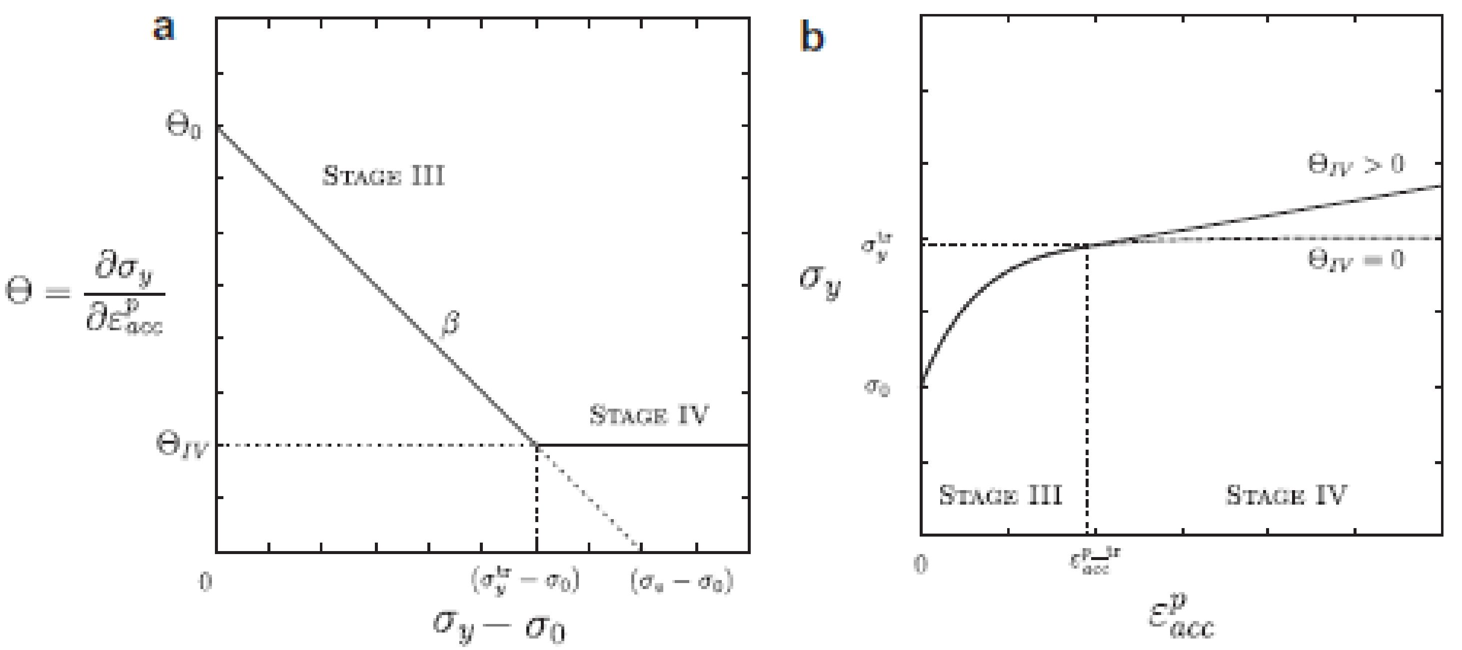




But, not elegant + what do we do when power law hardening does not apply?

Pardoen & Hutchinson, *JMPS* 2000

What if a more physical hardening Voce law with stage IV ?

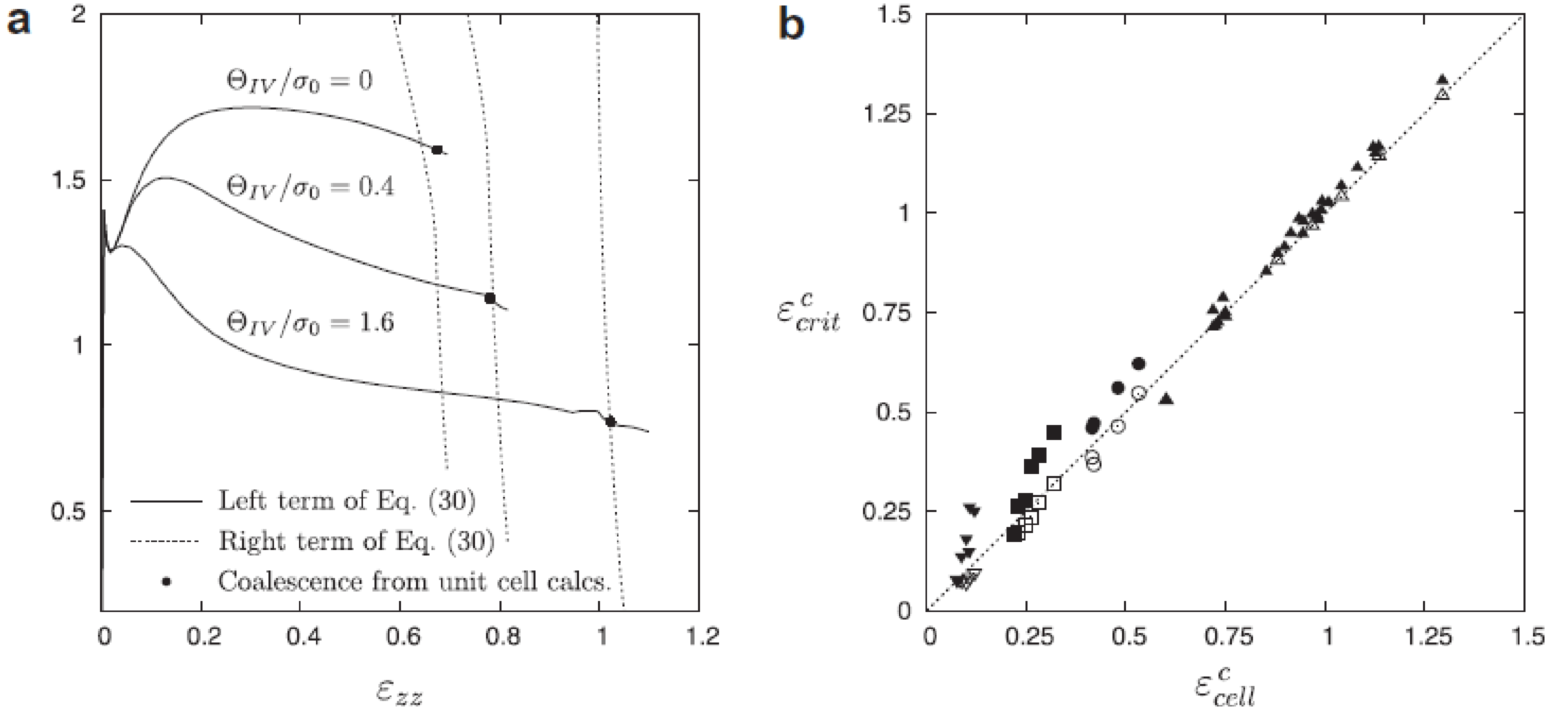


$$\sigma_y = \begin{cases} \sigma_0 + \frac{\Theta_0}{\beta}[1 - \exp(-\beta \varepsilon^p_{acc})] & \text{for } \sigma_y \leqslant \sigma_y^{tr} \quad \text{(stage III)} \\ \sigma_y^{tr} + \Theta_{IV}\left(\varepsilon^p_{acc} - \varepsilon^{p\_tr}_{acc}\right) & \text{for } \sigma_y > \sigma_y^{tr} \quad \text{(stage IV)} \end{cases}$$

L. Lecarme, C. Tekoglu and T. Pardoen, IJP 2011

Works well when using the current incremental n_ε strain hardening value in $\alpha(n)=0.1+0.217n+4.83n^2 \ (0\leq n\leq 0.3)$

$$\frac{\sigma_{zz}}{\sigma_y} = (1-\chi^2)\left[\alpha(n_\varepsilon)\left(\frac{1-\chi}{\chi W}\right)^2 + 1.24\frac{1}{\sqrt{\chi}}\right].$$



L. Lecarme, C. Tekoglu and T. Pardoen, IJP 2011

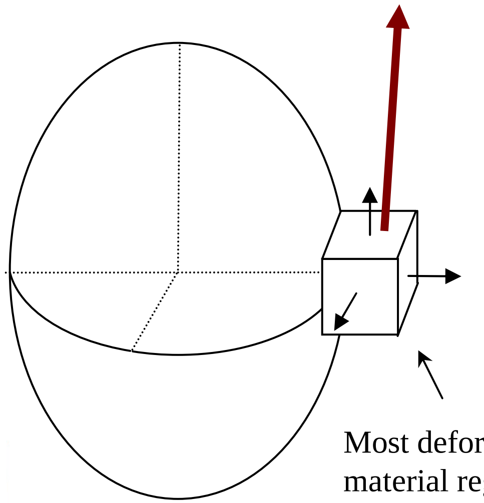
Second approach (2008-2009)

$$\frac{\sigma_z}{\sigma_y^{loc}} = \left(1-\eta\chi^2\right)\left[0.1\left(\frac{1-\chi}{\chi W}\right)^2 + 1.24\sqrt{\frac{1}{\chi}}\right]$$

Localization is controlled by the local value of the yield stress

constant again !

$$\varepsilon_y^{ploc} = \int \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{ploc}\dot{\varepsilon}_{ij}^{ploc}}\,dt \xrightarrow{\text{hardening law}} \sigma_y^{loc}$$



$$\dot{\varepsilon}_{33}^{ploc} = \frac{\dot{R}_3}{R_3} = \frac{1}{3}\left(\frac{\dot{f}}{f} - \frac{\dot{W}}{W} + \dot{\varepsilon}_{22}^{\infty} + \dot{\varepsilon}_{11}^{\infty}\right)$$

$$\dot{\varepsilon}_{22}^{ploc} = \frac{\dot{R}_2}{R_2}\frac{P_e}{4R_2} \approx \frac{1}{3}\left(\dot{\varepsilon}_{22}^{\infty} + \dot{\varepsilon}_{11}^{\infty} + \frac{\dot{f}}{f} + 2\frac{\dot{W}}{W}\right)\left(\frac{\pi}{2}\sqrt{\frac{1}{2}\left(\frac{1}{W^2}+1\right)}\right)$$

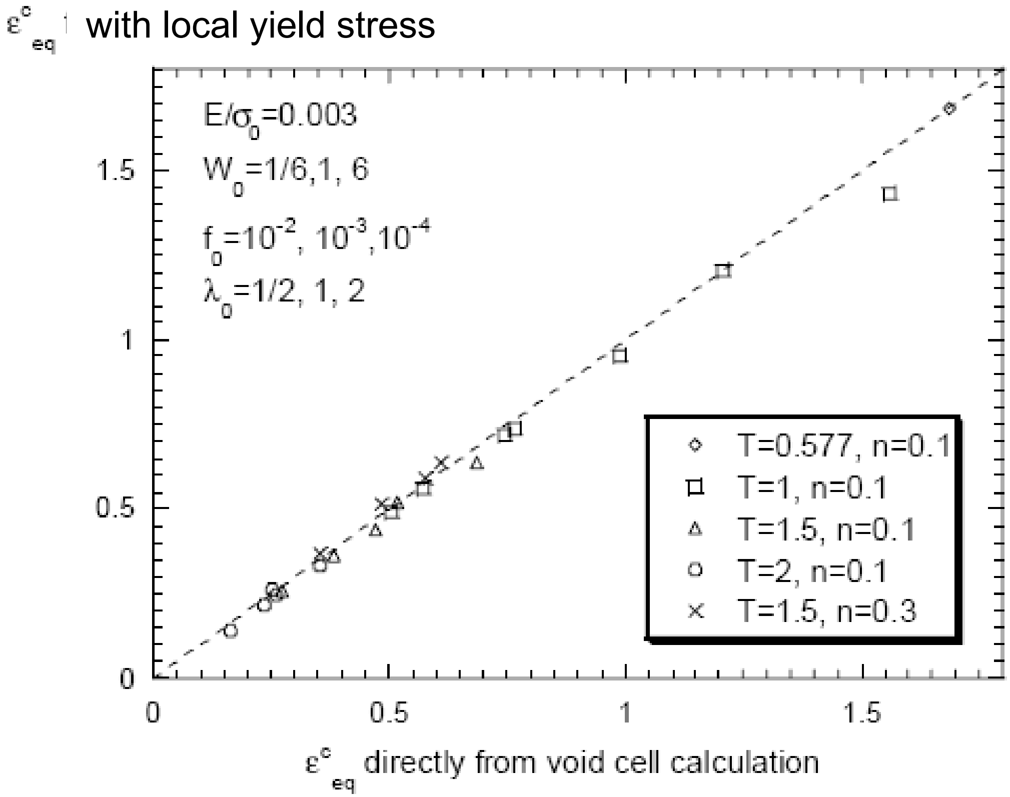
$$\dot{\varepsilon}_{11}^{ploc} = -\dot{\varepsilon}_{22}^{ploc} - \dot{\varepsilon}_{33}^{ploc}$$

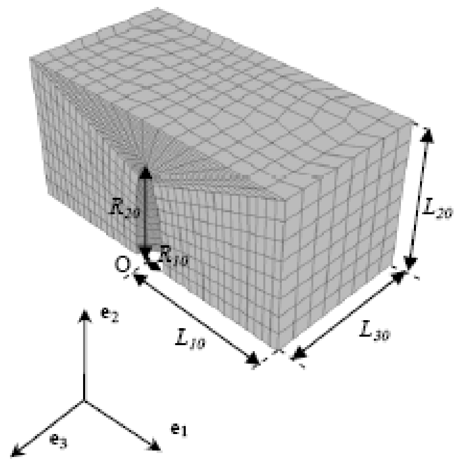
Kinematic model very accurate !

F. Scheyvaerts, P.R. Onck, and T. Pardoen, *JMPS 2011*

Validation 1 : for power law hardening

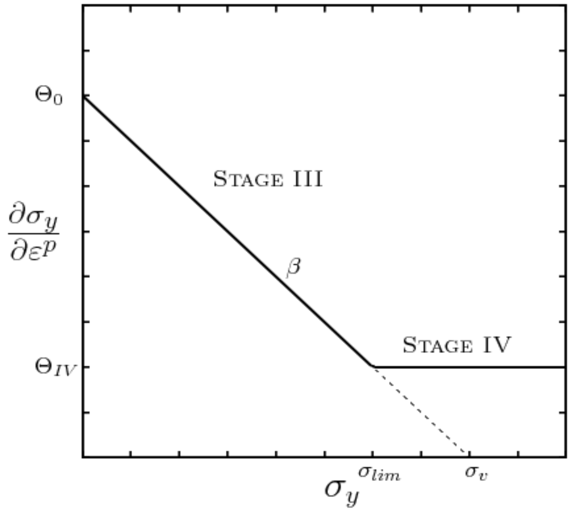
$$\sigma_y = \sigma_0\left(1+\frac{\varepsilon_y^p}{\varepsilon_0}\right)^n$$





F. Scheyvaerts, P.R. Onck, and T. Pardoen, *JMPS 2011*

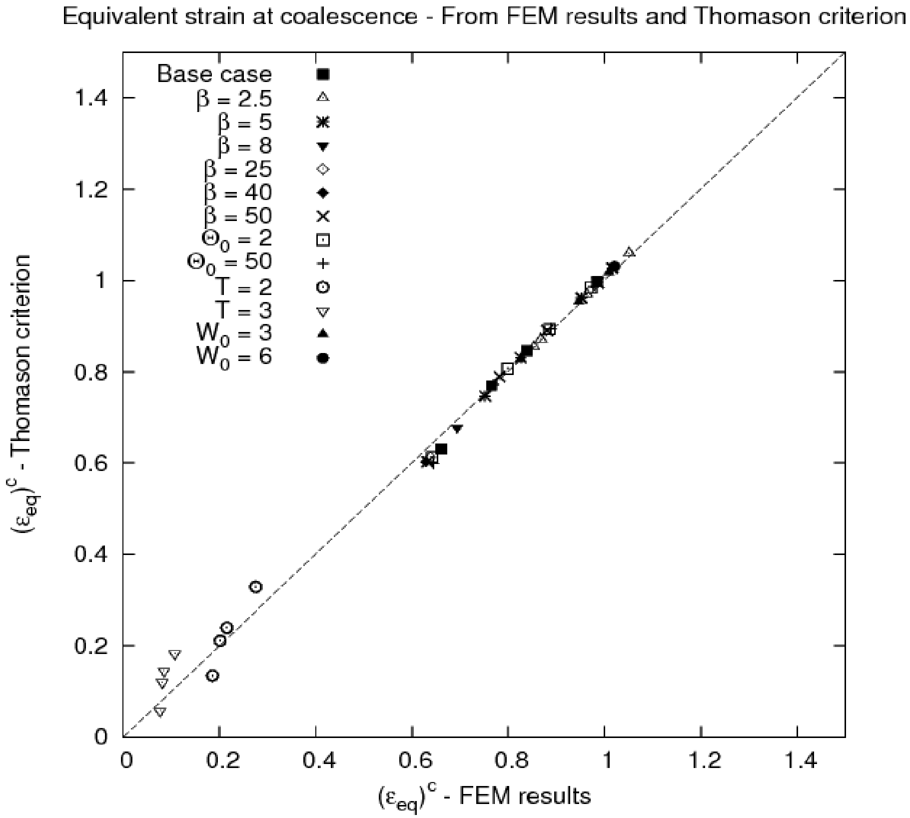
Validation 2 : Voce law with stage IV



$$\sigma_y = \sigma_0 + \frac{\Theta_0}{\beta}\left[1 - \exp(-\beta \cdot \varepsilon^p)\right] \quad \textit{[stage III]}$$

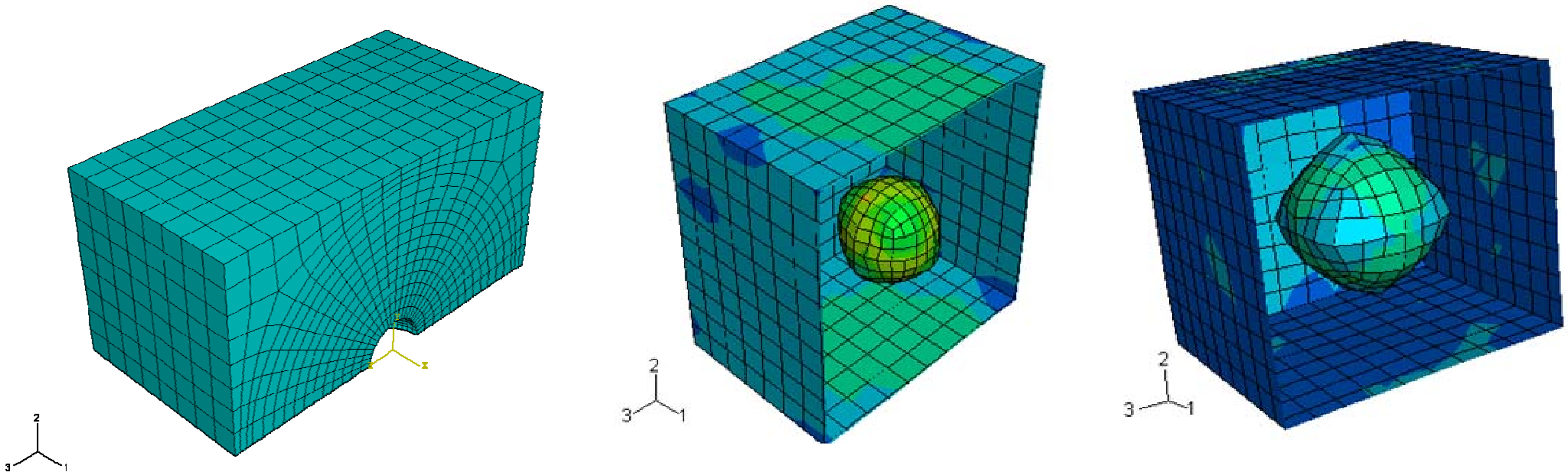
$$\sigma_y = \sigma_{lim} + \Theta_{IV} \cdot (\varepsilon^p - \varepsilon^p_{lim}) \quad \textit{[stage IV]}$$

Excellent agreement with FE void cell simulations except at high triaxiality



L. Lecarme, C. Tekoglu and T. Pardoen, IJP 2011

Validation 3 : single crystal plasticity with power law hardening

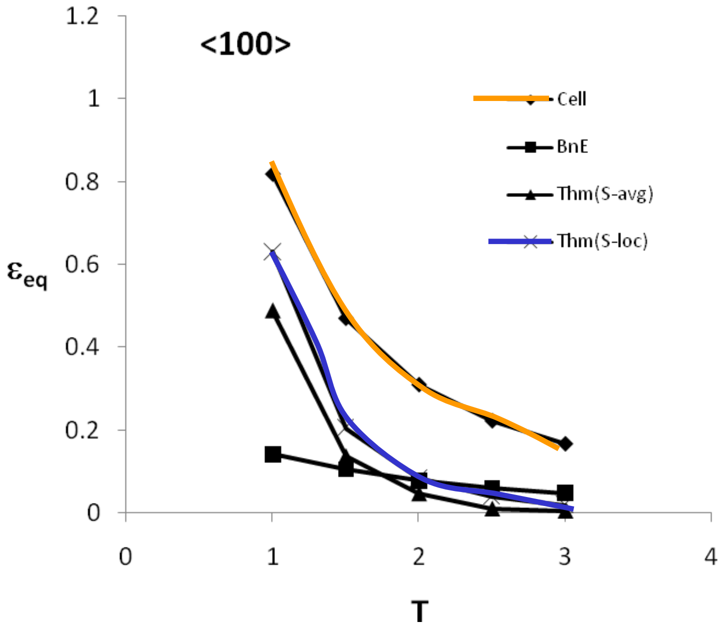


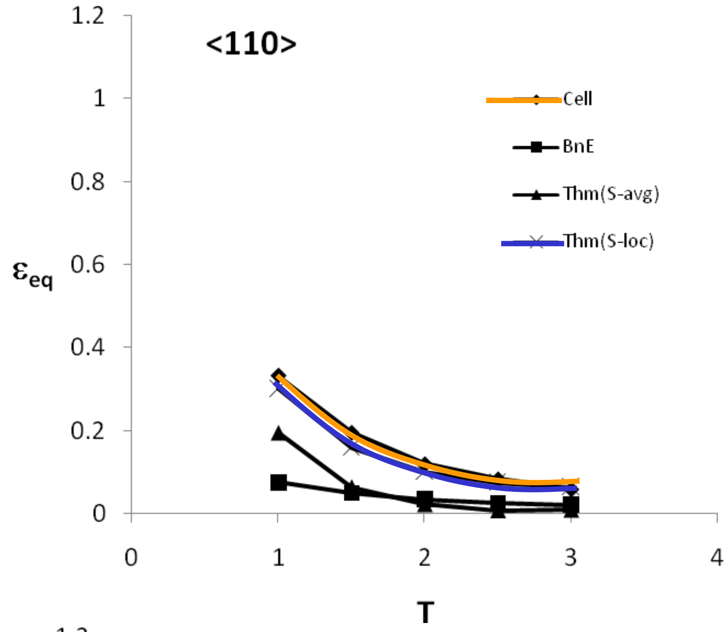
3D void cell simulations (Abaqus)
Fully periodic BC
Constant stress triaxiality
BCC single crystal (48 slip systems)
Different crystal orientation
Power law hardening for CRSS

$$\tau_c = \tau_{co}\left(1+\frac{\Gamma}{\Gamma_o}\right)^n$$

S. Kumar Yerra, C. Tekoglu, F. Scheyvaerts, L. Delannay, A. Van Bael, P. Van Houtte, T. Pardoen, IJSS, 2010

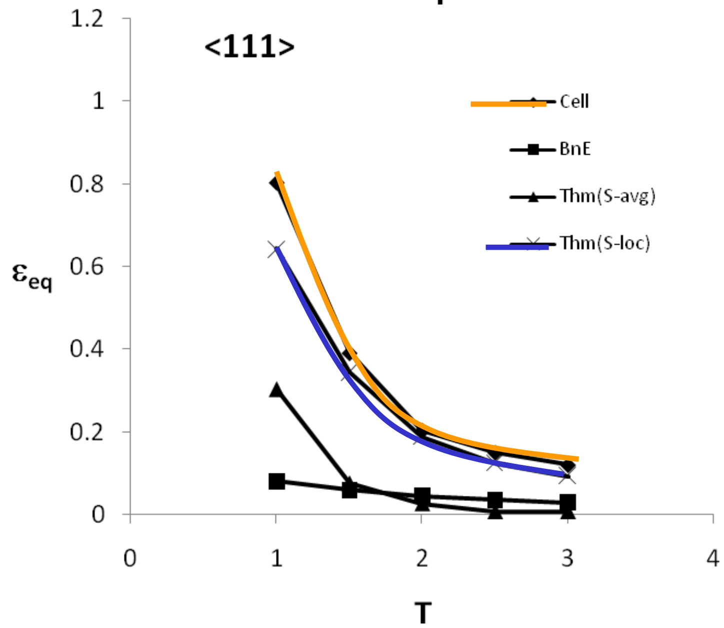
Assessment of new Thomason criterion with local yield stress





Works very well

… except for <100> at high triax (local shear strain not taken into account)

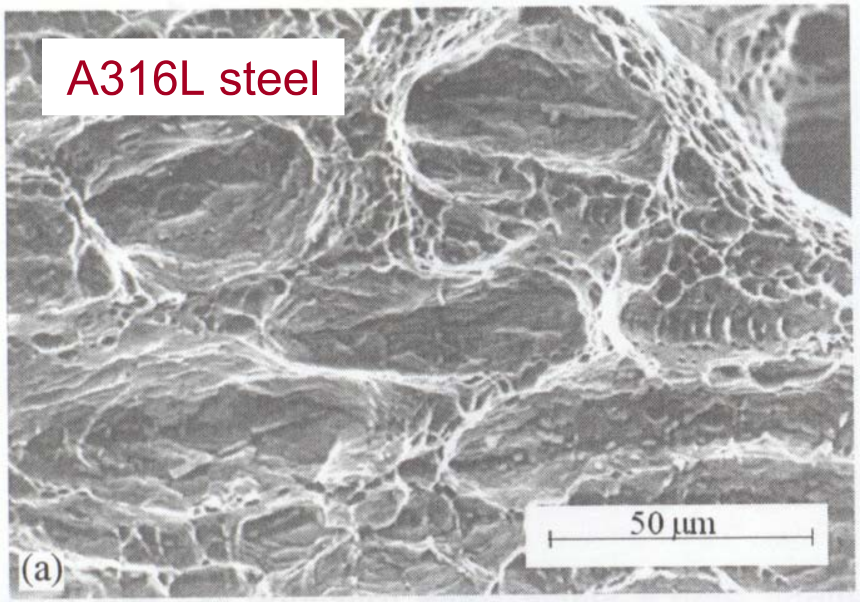


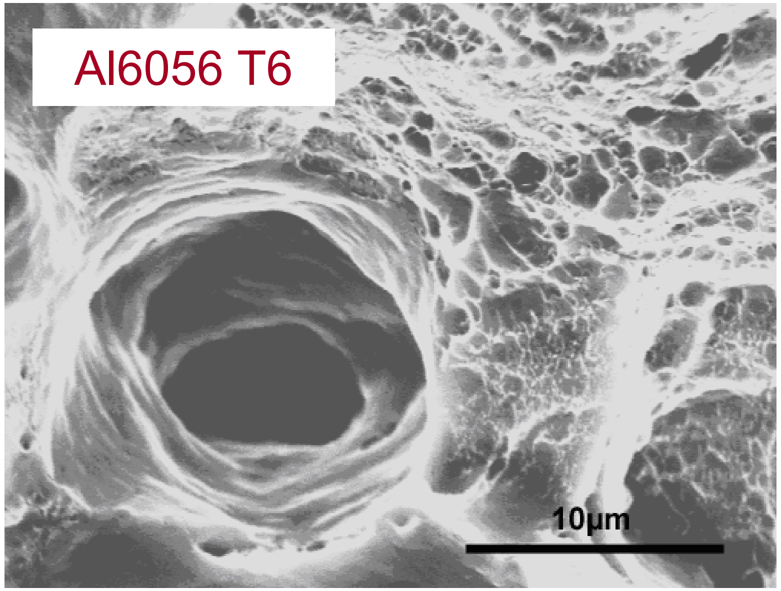
OUTLINE

Secondary voids are often observed on fracture surfaces of metallic alloys

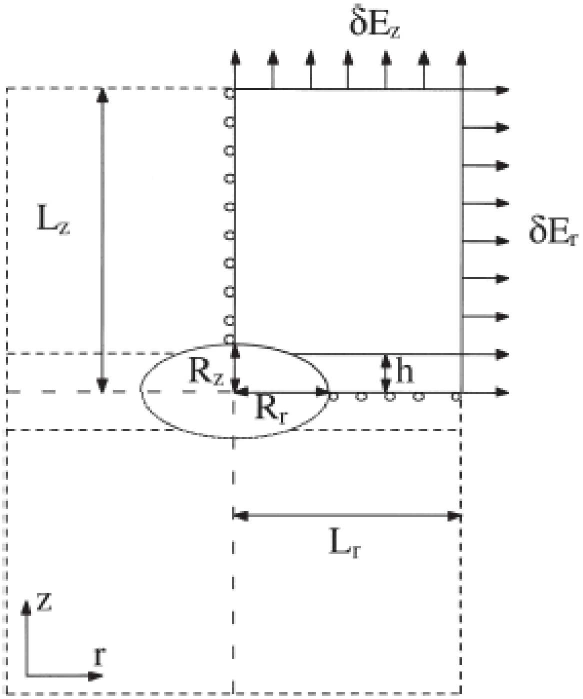
Second populations of voids result from nucleation on submicron sized particles at large strains, e.g.

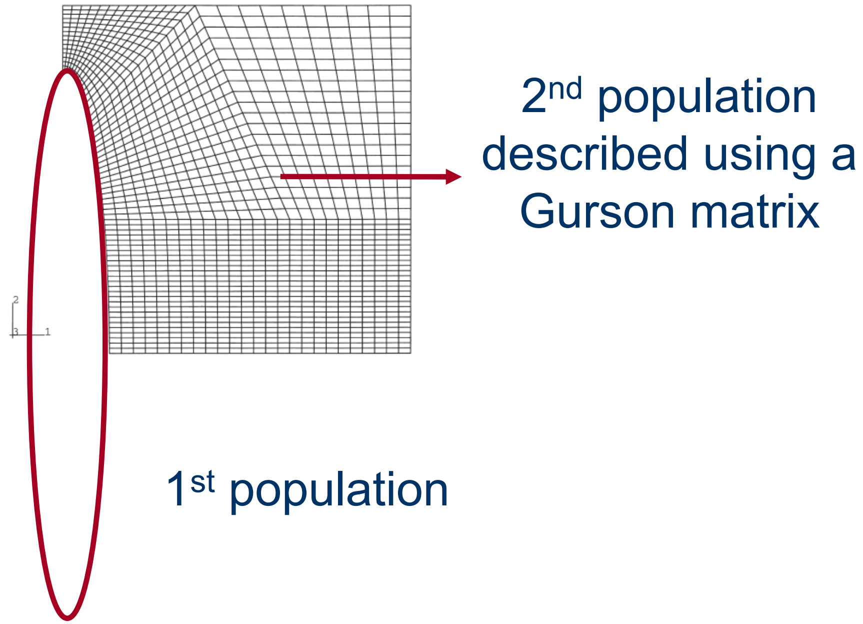
- carbides in steel
- dispersoïds in Al alloys
- small α particles in α/β Ti alloys





FE void cell calculations with two populations of voids

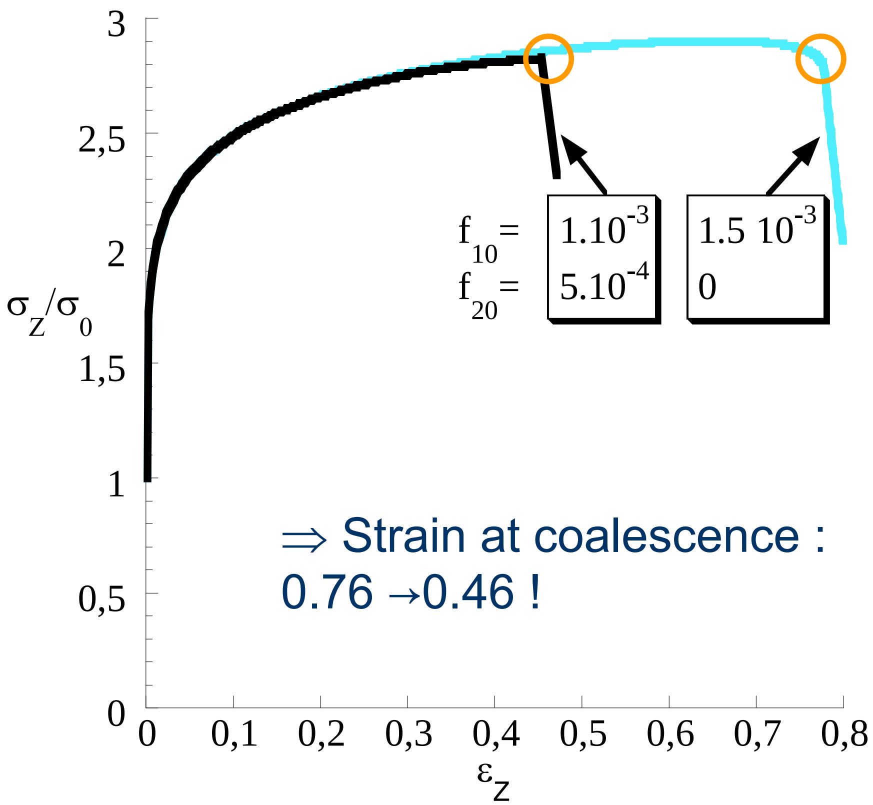




2D axisym void cell simulations (Abaqus)
Periodic BC & constant stress triaxiality
Gurson matrix
Different primary and secondary void vol. fr.
Primary voids already present
Strain controlled nucleation of secondary voids

Fabrègue & Pardoen, *JMPS* 2008

Major influence of the presence of a second population on the onset of void coalescence, but no effect on the growth of the primary voids



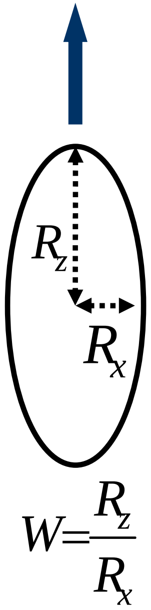
⇒ Strain at coalescence : 0.76 →0.46 !

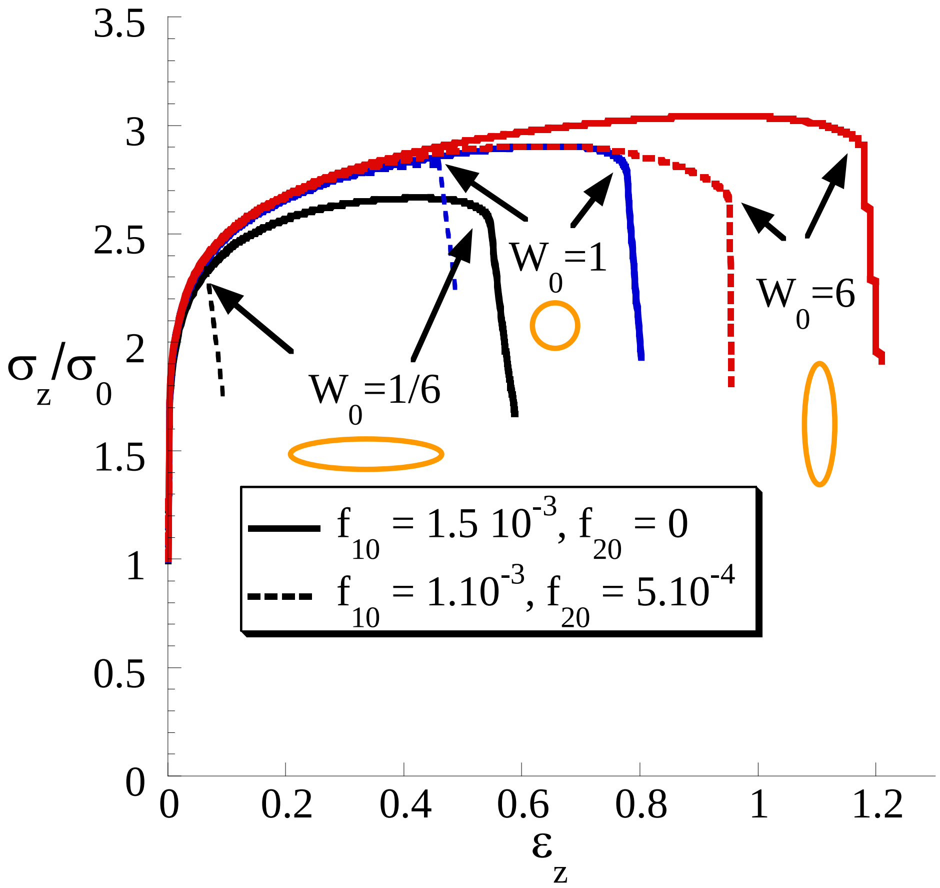
Triaxiality = 1
2nd population present from the beginning
1st population spherical

$$E/\sigma_0 = 500$$

$$n = 0.1$$

The effect of the presence of secondary voids on triggering coalescence is much larger with flat voids





Triaxiality = 1

2nd population present from the beginning

$E/\sigma_0 = 500$

$n = 0.1$

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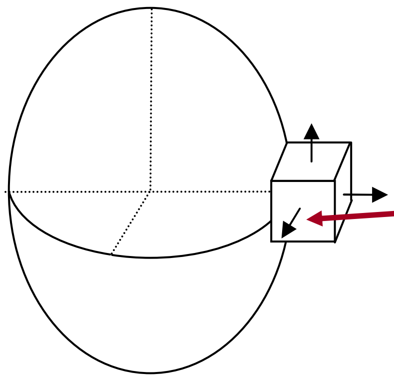
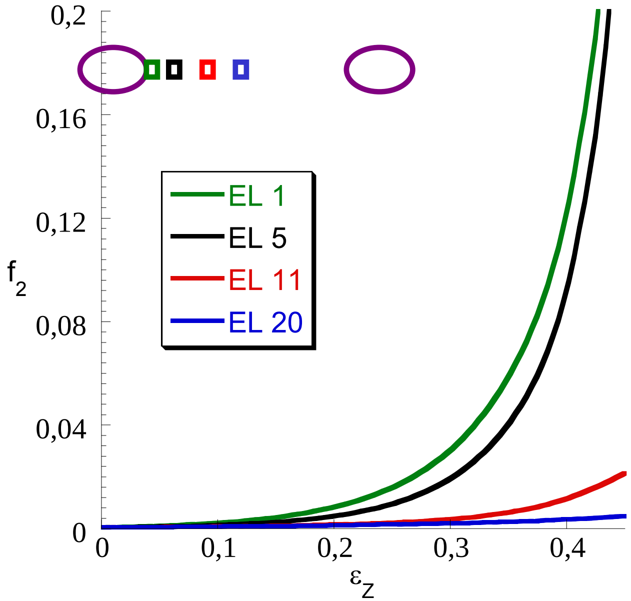
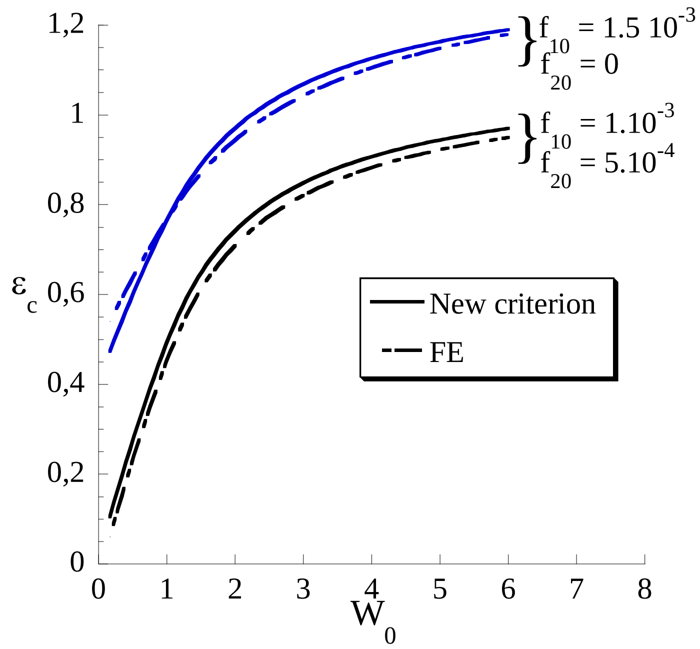
Introduce second population in the void coalescence criterion



$$\frac{\sigma_z}{\sigma_y(1-f_2)} = (1-\eta\chi^2)\left[\alpha\left(\frac{1-\chi}{\chi W}\right)^2 + 1.24\sqrt{\frac{1}{\chi}}\right]$$



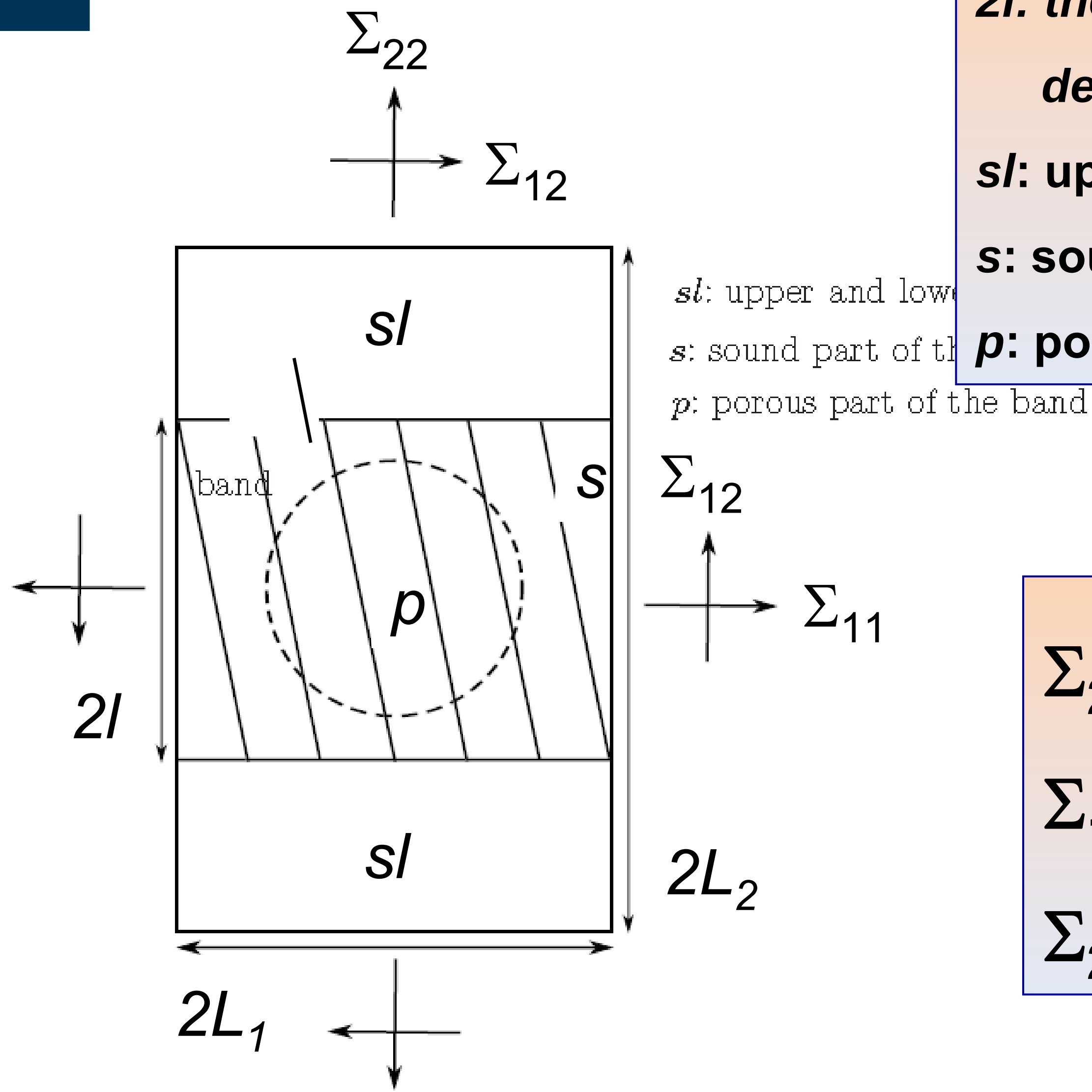


Fabrègue & Pardoen, *JMPS* 2008

OUTLINE

1. Experimental observations
2. « Elementary » models for the onset of void coalescence
3. Fine tuning of Thomason model
4. Extension to strain hardening
5. Extension to a second population
6. **Extension to combined shear/tension coalescence**
7. Models for the coalescence stage

Coalescence Criterion for General Loadings



2l*: *the thickness of the localized deformation band

***sl*: upper and lower sound layers**

***s*: sound part of the band**

***p*: porous part of the band**

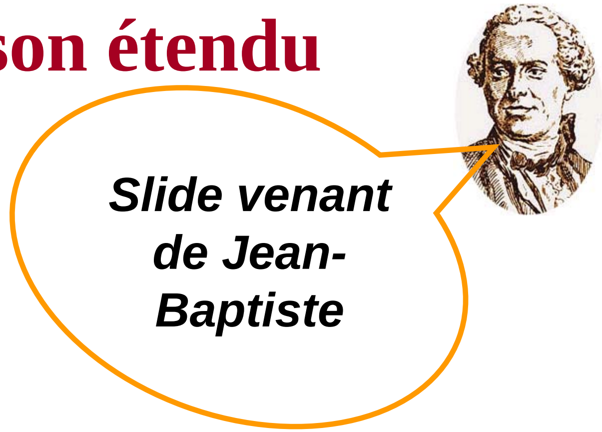
$$\Sigma_{22} > \Sigma_{11};\ \Sigma_{22} > \Sigma_{33}$$

$$\Sigma_{12} = \Sigma_{21} \neq 0$$

$$\Sigma_{23} = \Sigma_{32} \neq 0$$

Tekoglu, Leblond, Pardoen, JMPS 2012

Critère de coalescence de Thomason étendu



$$\left(\frac{\Sigma_{22}}{\Sigma_{22}^{Th}}\right)^2 + \frac{3\left(\Sigma_{21}^2 + \Sigma_{23}^2\right)}{(1-f_b)^2\sigma_0^2} = 1$$

Σ_{22}^{Th} : « contrainte de coalescence » en traction pure donnée par Thomason

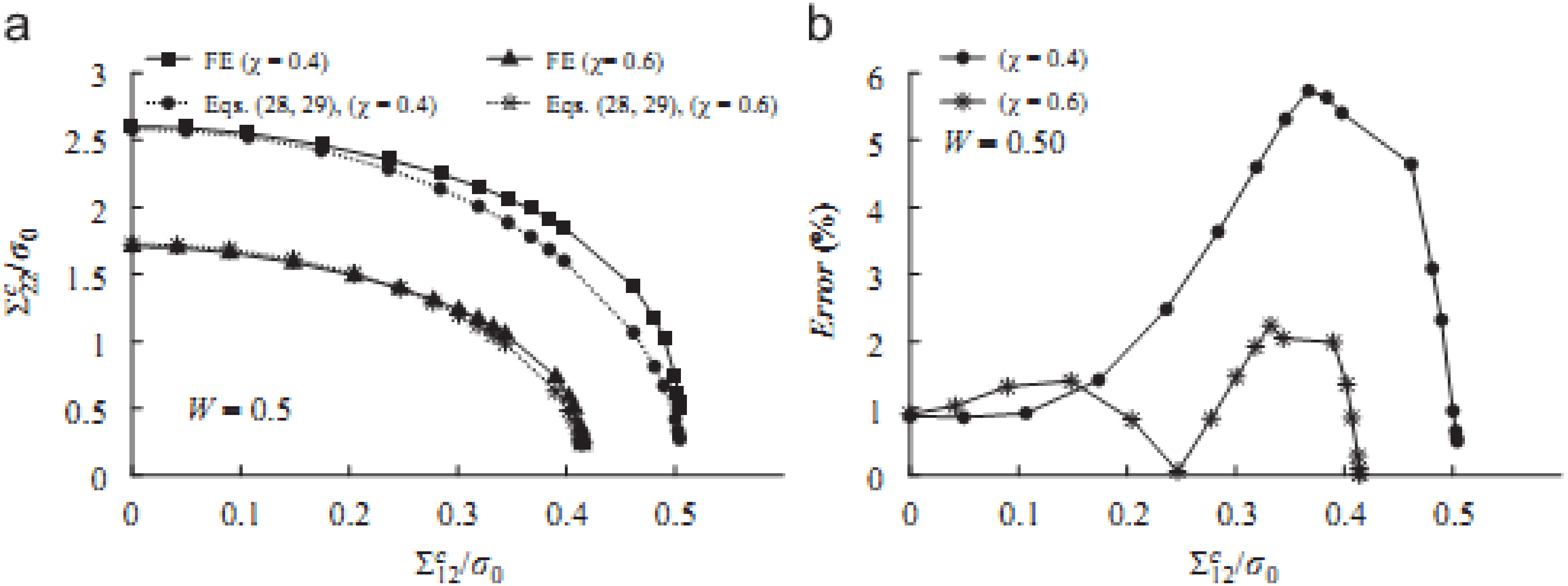
$f_b = f\,\dfrac{L_2}{\ell}$: porosité dans la bande

(ℓ est prise proportionnelle à la dimension verticale du vide)

Comparaison avec des simulations numériques micromécaniques

Meilleur accord quand $2l = 0.6R_2$, corresponding to an f_b value of

$$f_b = 1.455\frac{L_2}{R_2}f = 1.455\frac{L_2}{\chi W L_1}f.$$



a: critères de coalescence analytique et numérique

b: erreur relative

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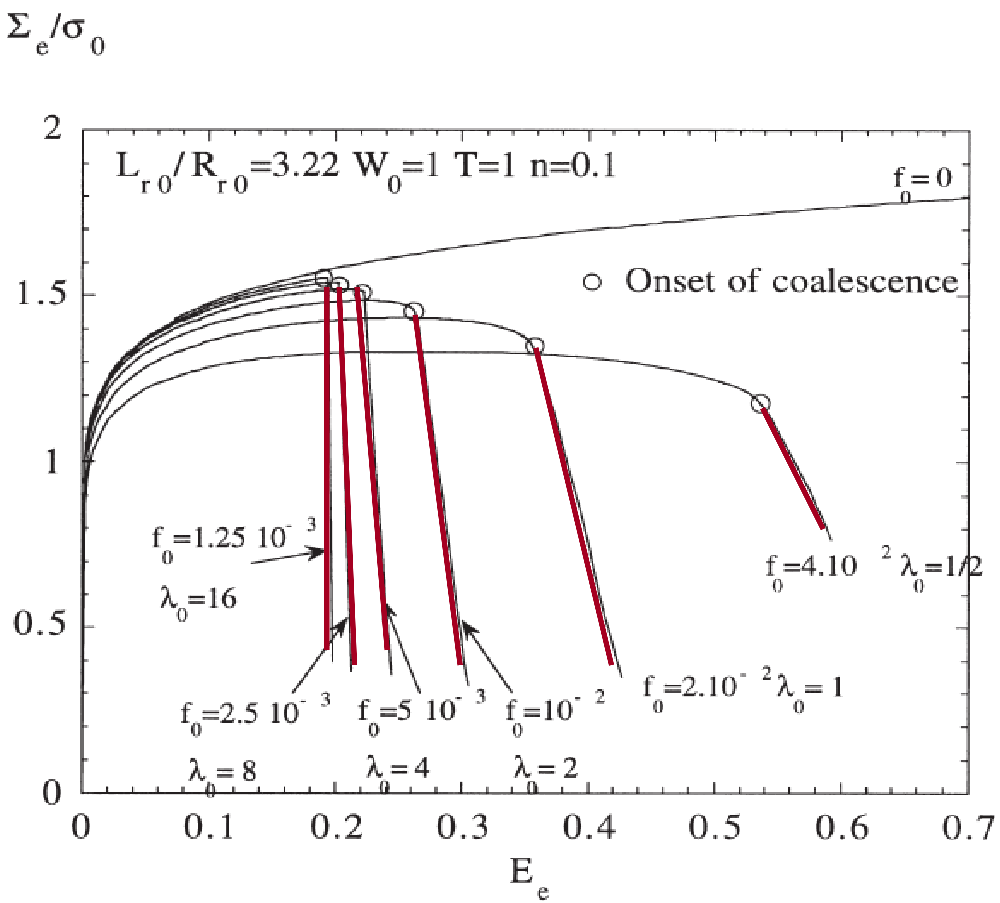
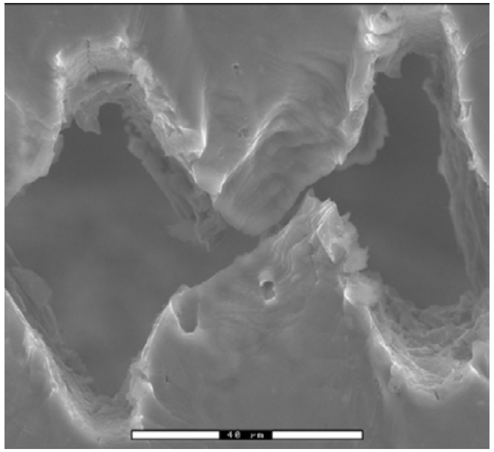
OUTLINE

Models for the coalescence stage



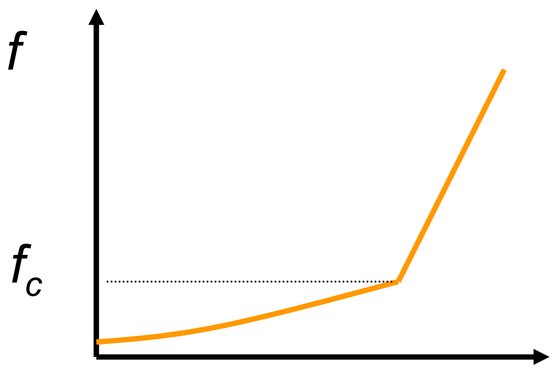


Weck et al.

Standard approaches

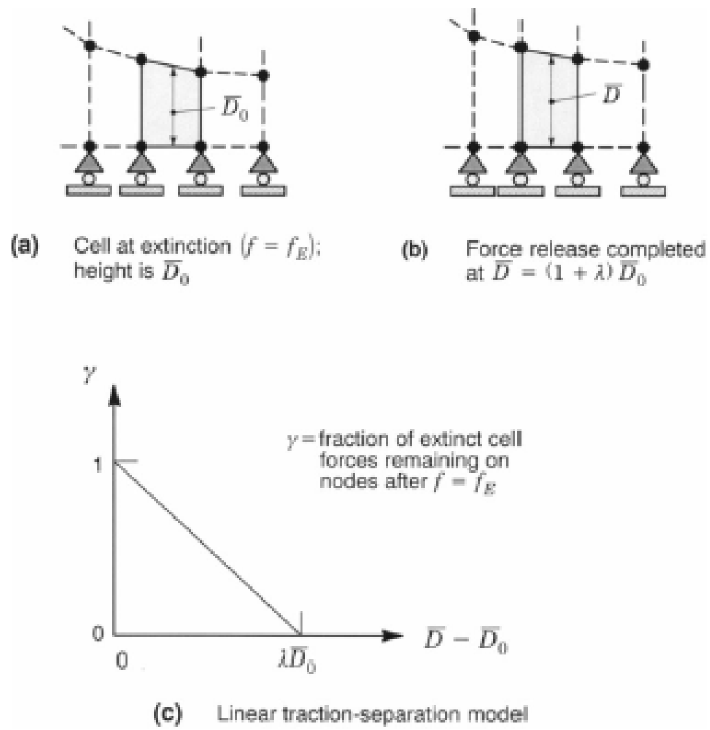
In the framework of Gurson model introduce an artifical acceleration factor for the porosity growth (Tvergaard and Needleman, 1984)

$$f^* = \begin{cases} f & \text{if } f < f_c \\ f_c + \delta (f - f_c) & \text{if } f \geq f_c \end{cases}$$

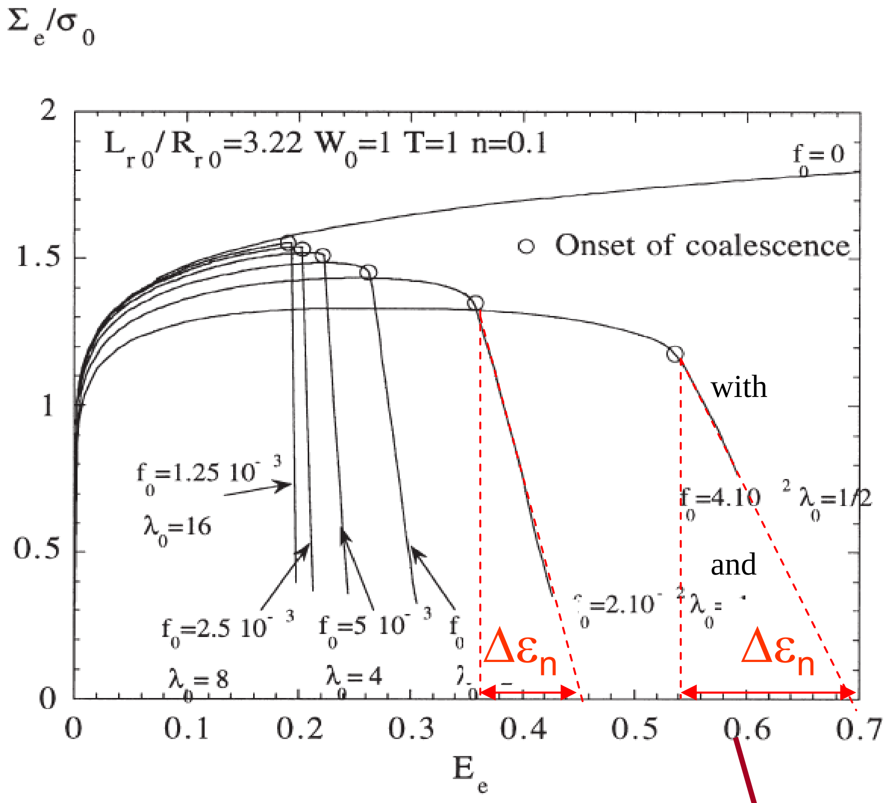


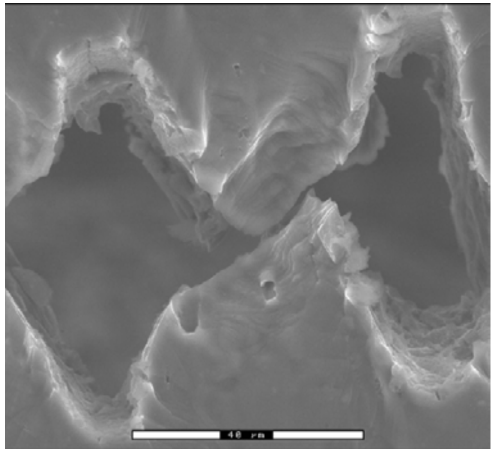
or

Prescribe unloading slope via a sort of cohesive zone model (Shih, Dodds, …)



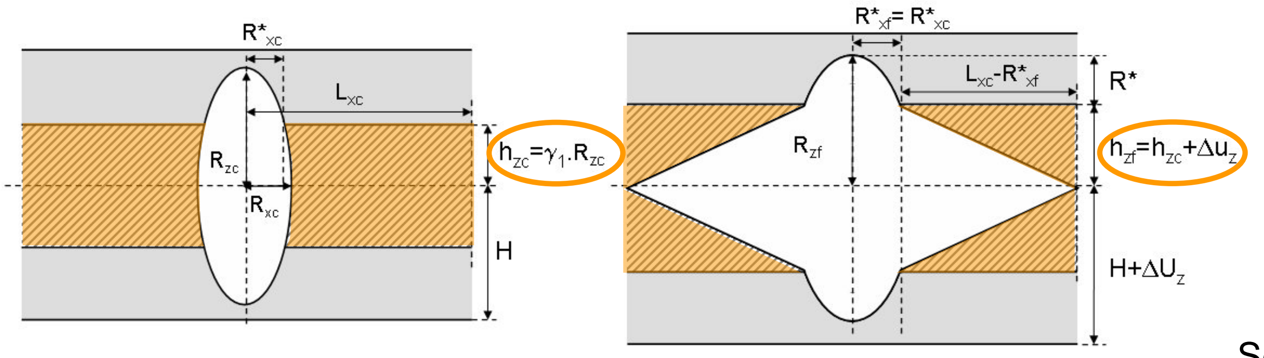
« geometric » approach





Weck et al.

$$\Delta\varepsilon_n = F\left(\chi_c, W_c, \text{strain state}\right)$$



Scheyvaerts et al., IJDM *2010*

Full constitutive model approach

Let's think in terms of two yield surfaces

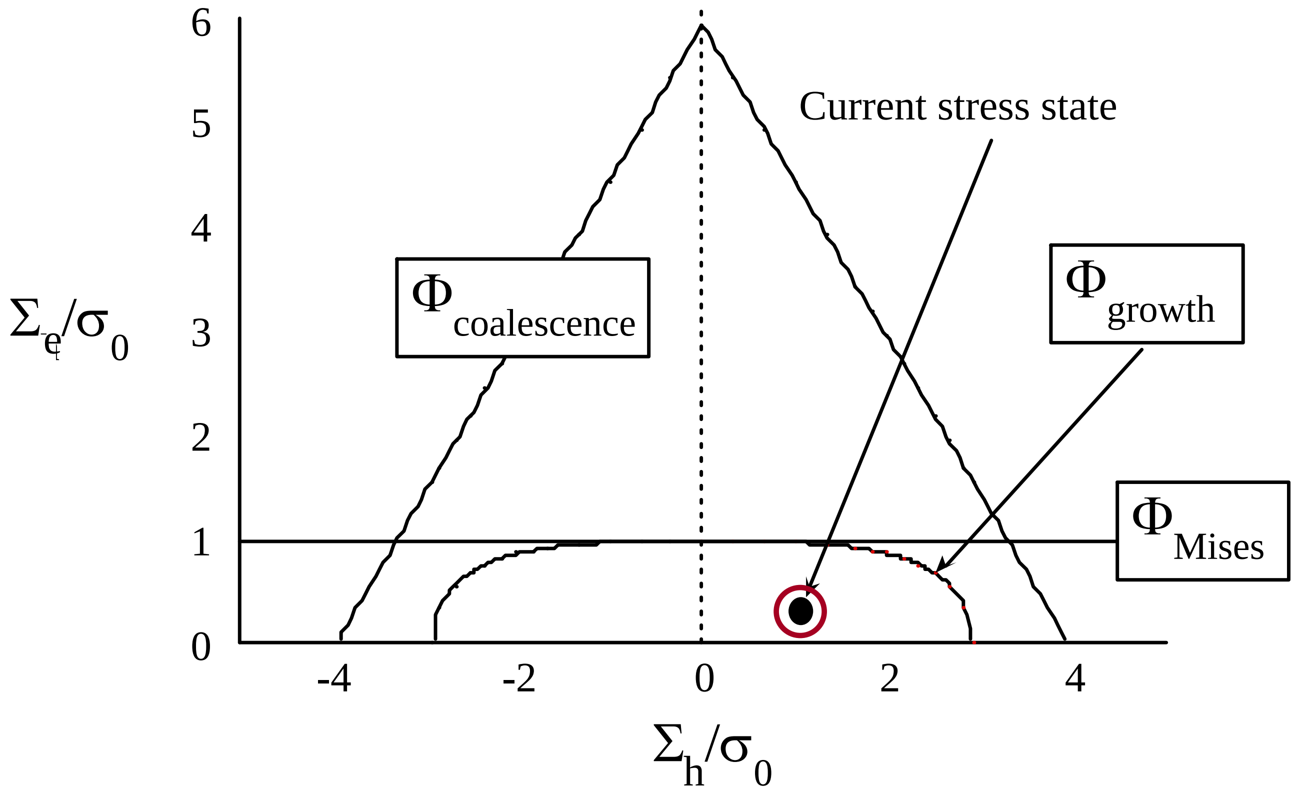
$$\Phi_{\text{coalescence}} \equiv \frac{\Sigma_e}{\sigma_f} + \frac{1}{2}\frac{\Sigma_{kk}}{\sigma_f} - F(S,\chi) = 0$$

See Benzerga or Pardoen and Hutchinson

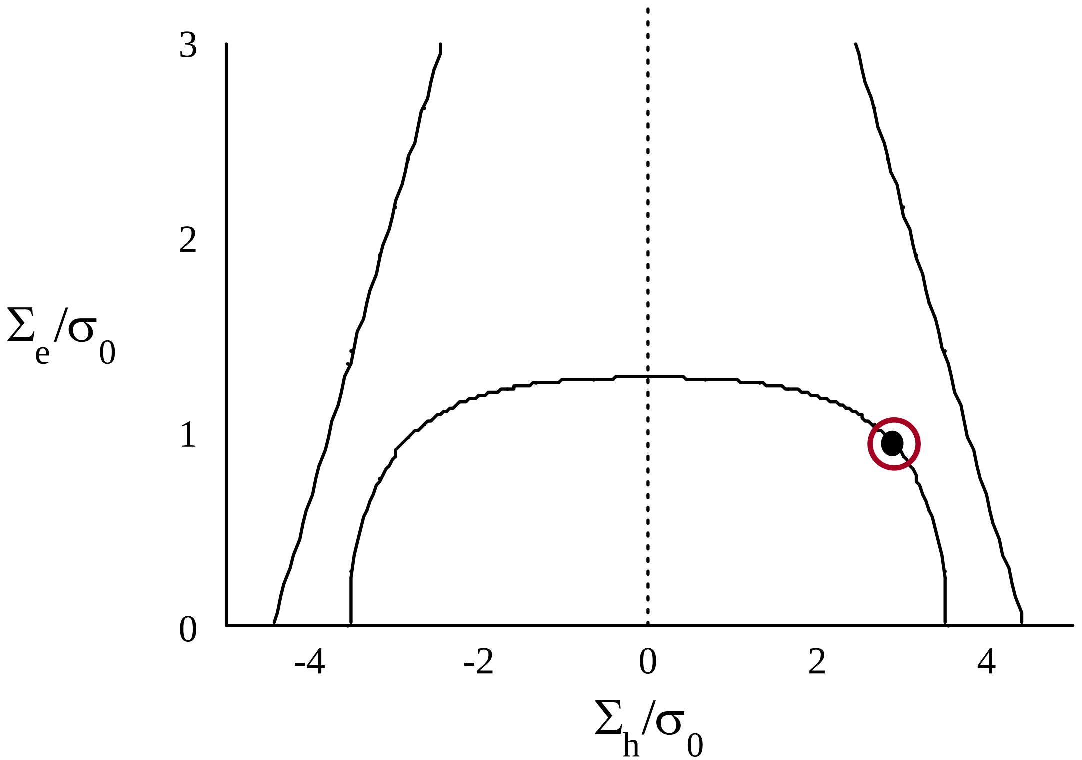
+ evolution laws for the internal variables

Elasticity

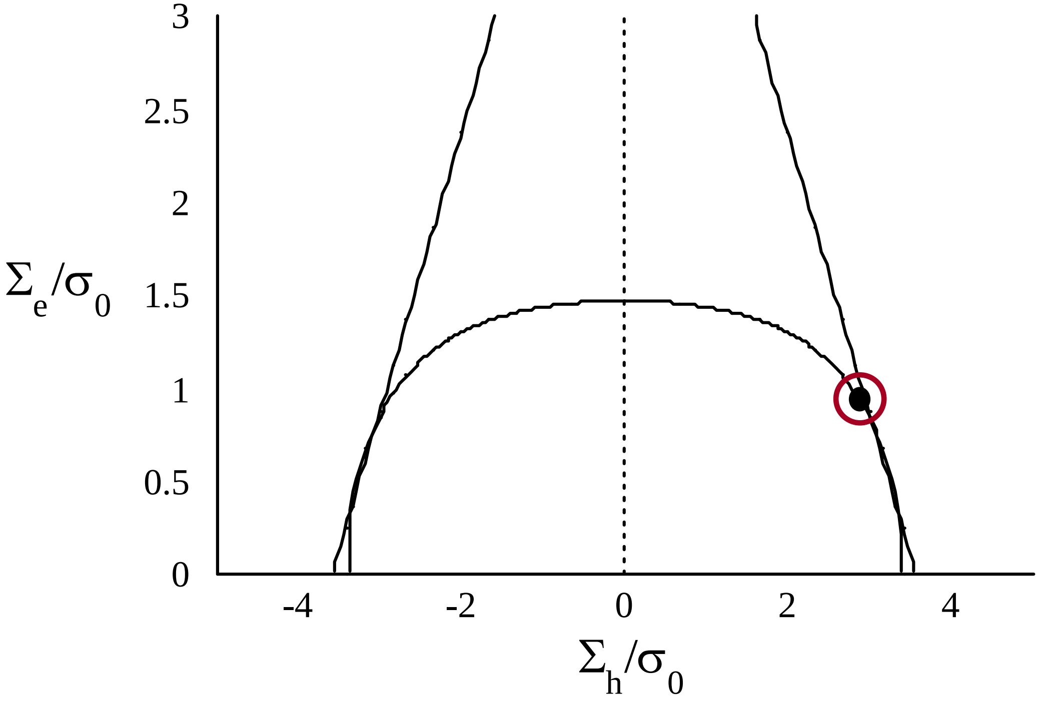
$$F(W,\chi) \equiv \frac{3}{2}\left(1-\eta\chi^2\right)\left[\alpha\left(\frac{1-\chi}{\chi W}\right)^2 + \beta\sqrt{\frac{1}{\chi}}\right]$$



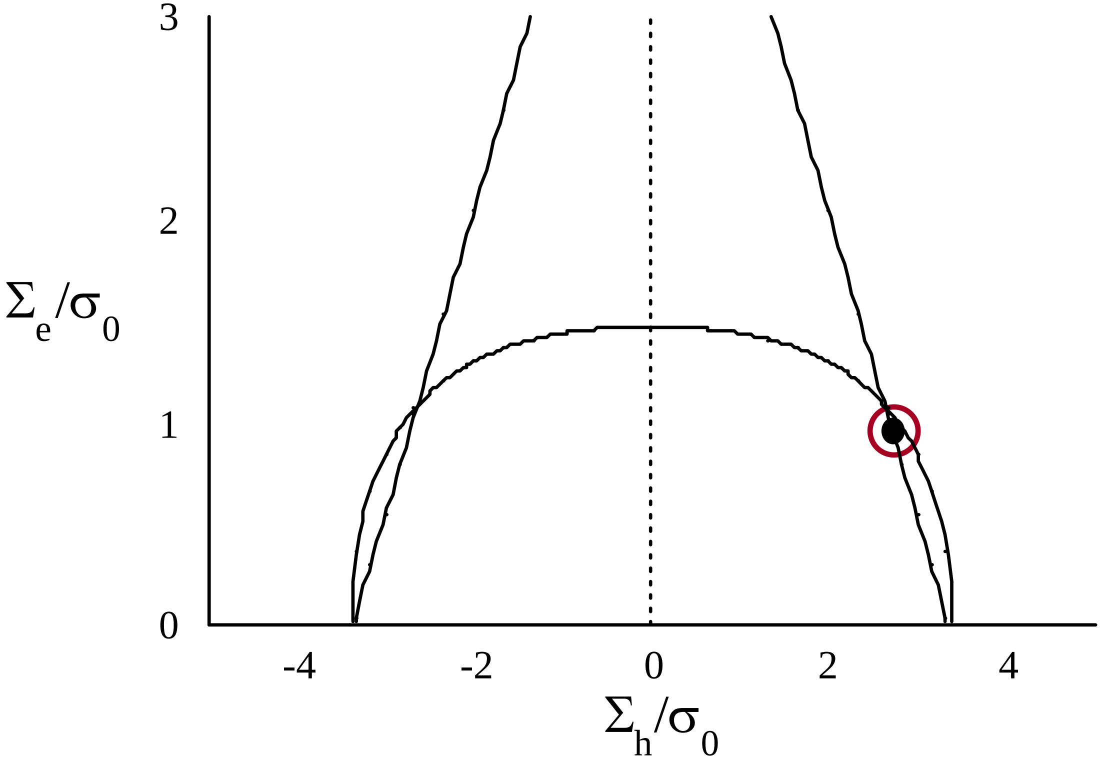
First, void growth, but strain hardening dominates damage softening



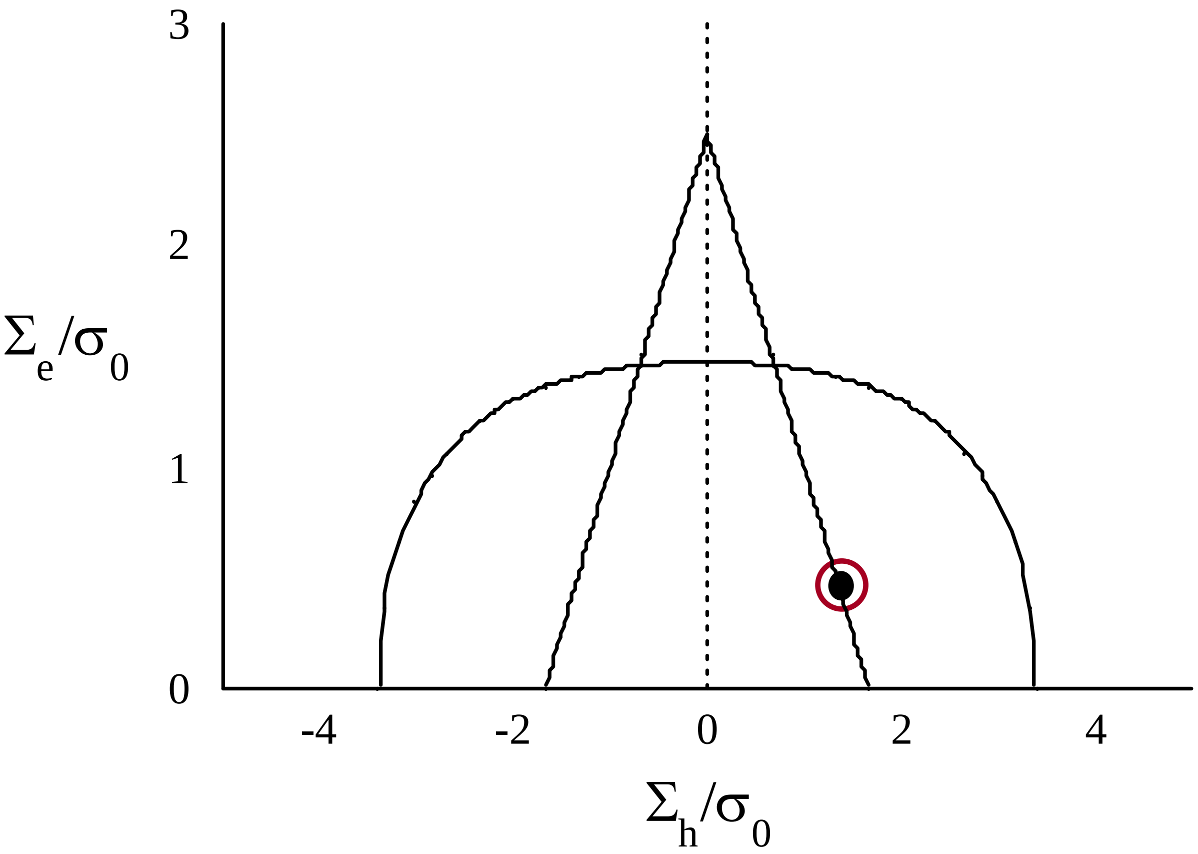
Just before void coalescence



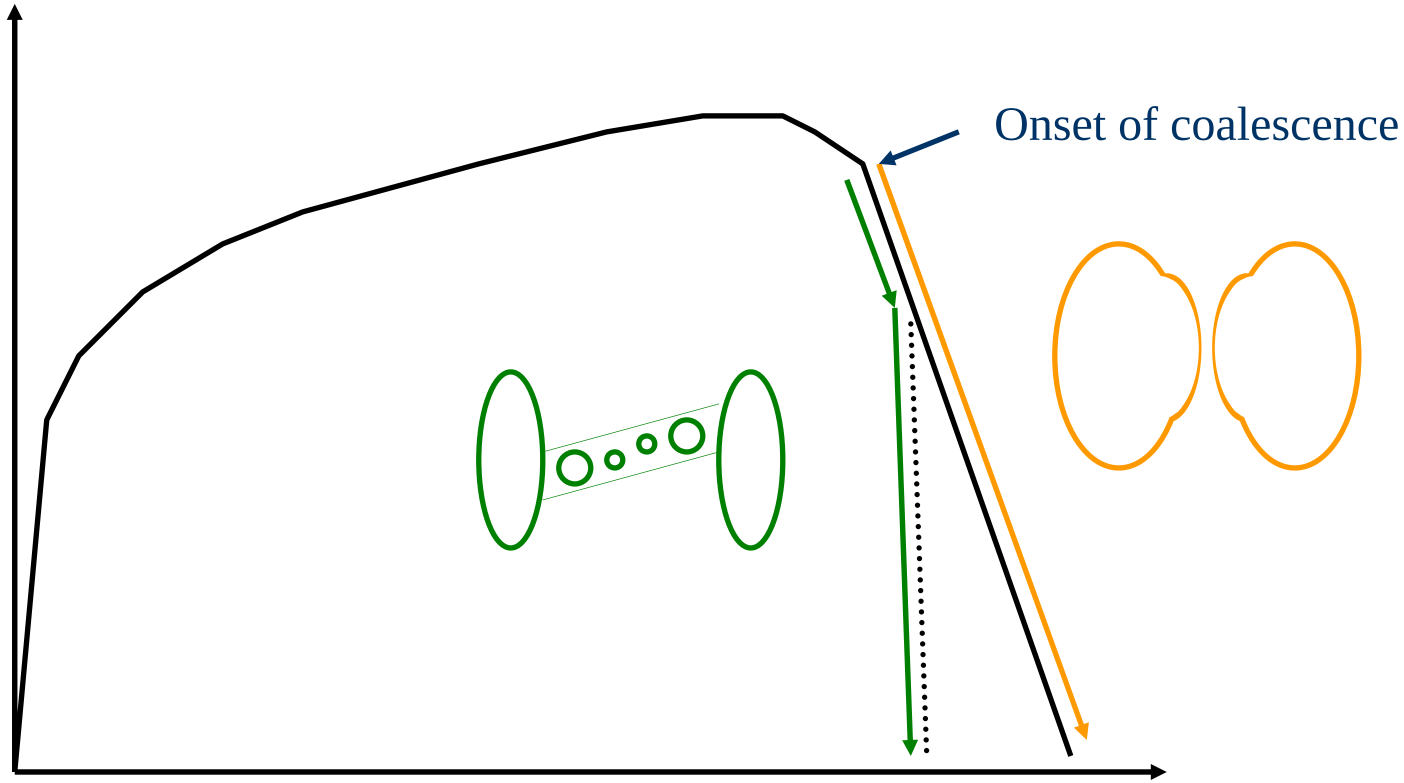
Just after the onset of void coalescence



During the coalescence phase



Additional effects taking place during coalescence



See e.g. Fabrègue and Pardoen, 2008

Conclusion

No time for conclusions !!!