Identifier des contraintes sans modèle pour identifier un modèle ensuite?

Adrien Leygue & Friends

GTs – Apprentissage – Mesures de champs et identification

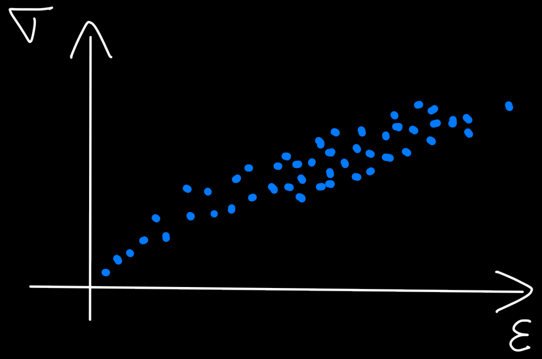
cnrs

GeM Institut de Recherche en Génie Civil et Mécanique

CENTRALE NANTES

Nantes Université

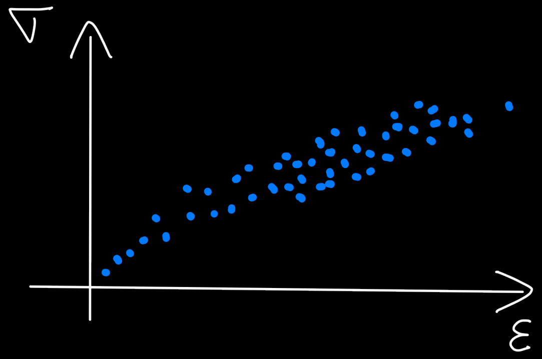
Model-free Mechanics of Materials



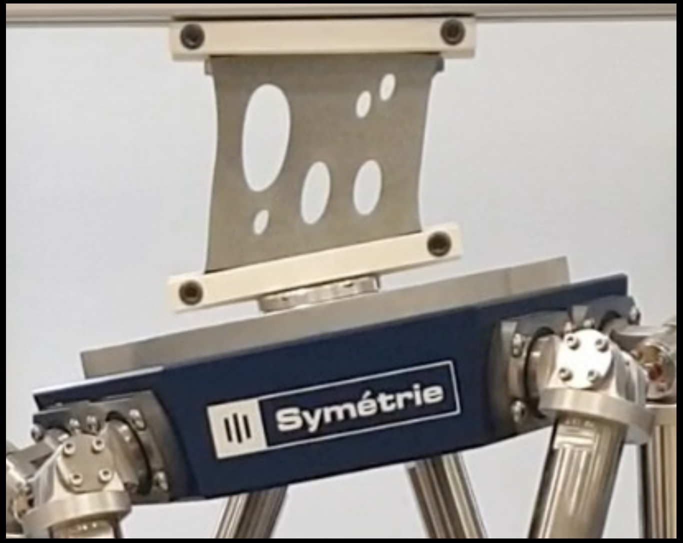
Mechanics of materials

Simple experiments

Model-free Mechanics of Materials

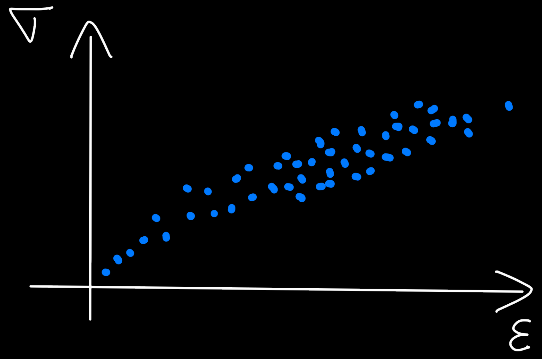


Mechanics of materials

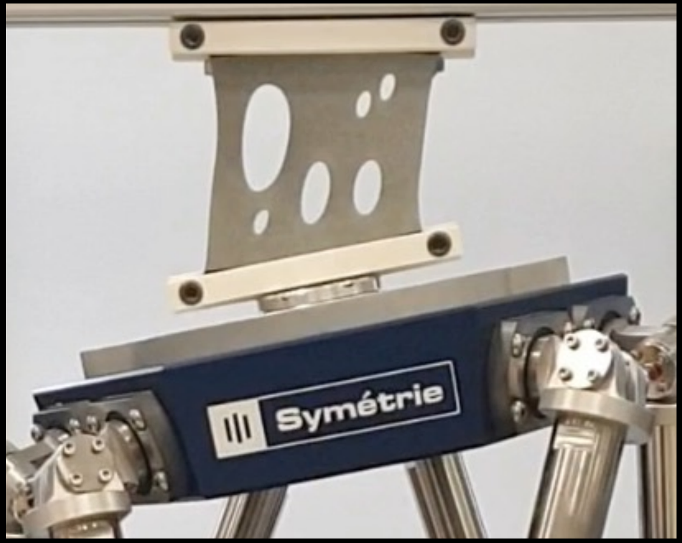


Rich experiments

Model-free Mechanics of Materials



Mechanics of materials



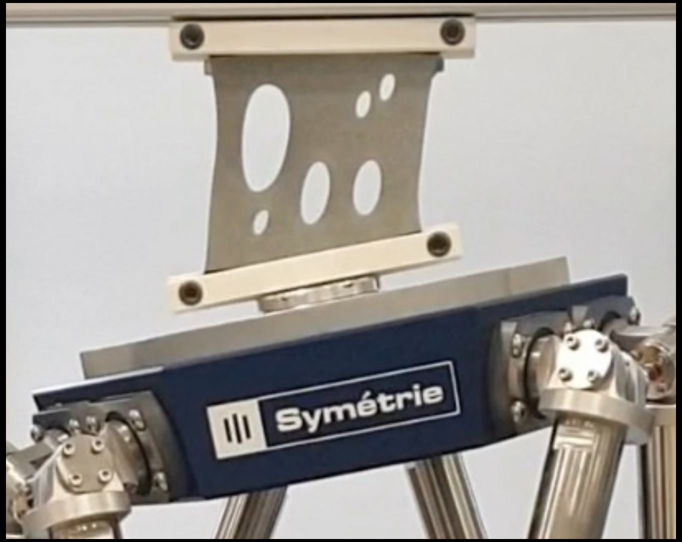
Rich experiments

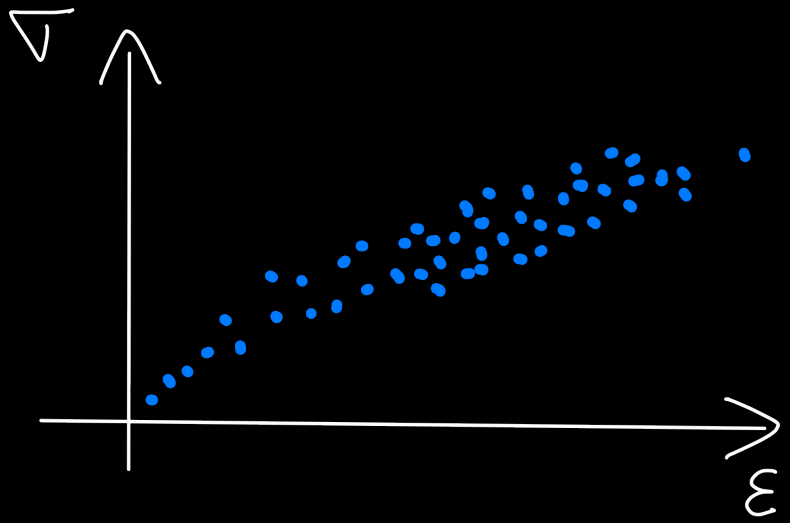
$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_p \frac{\mu_p}{\alpha_p} \left(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3 \right)$$

No constitutive models

Outline

- Getting the data





- Learning *from* the data

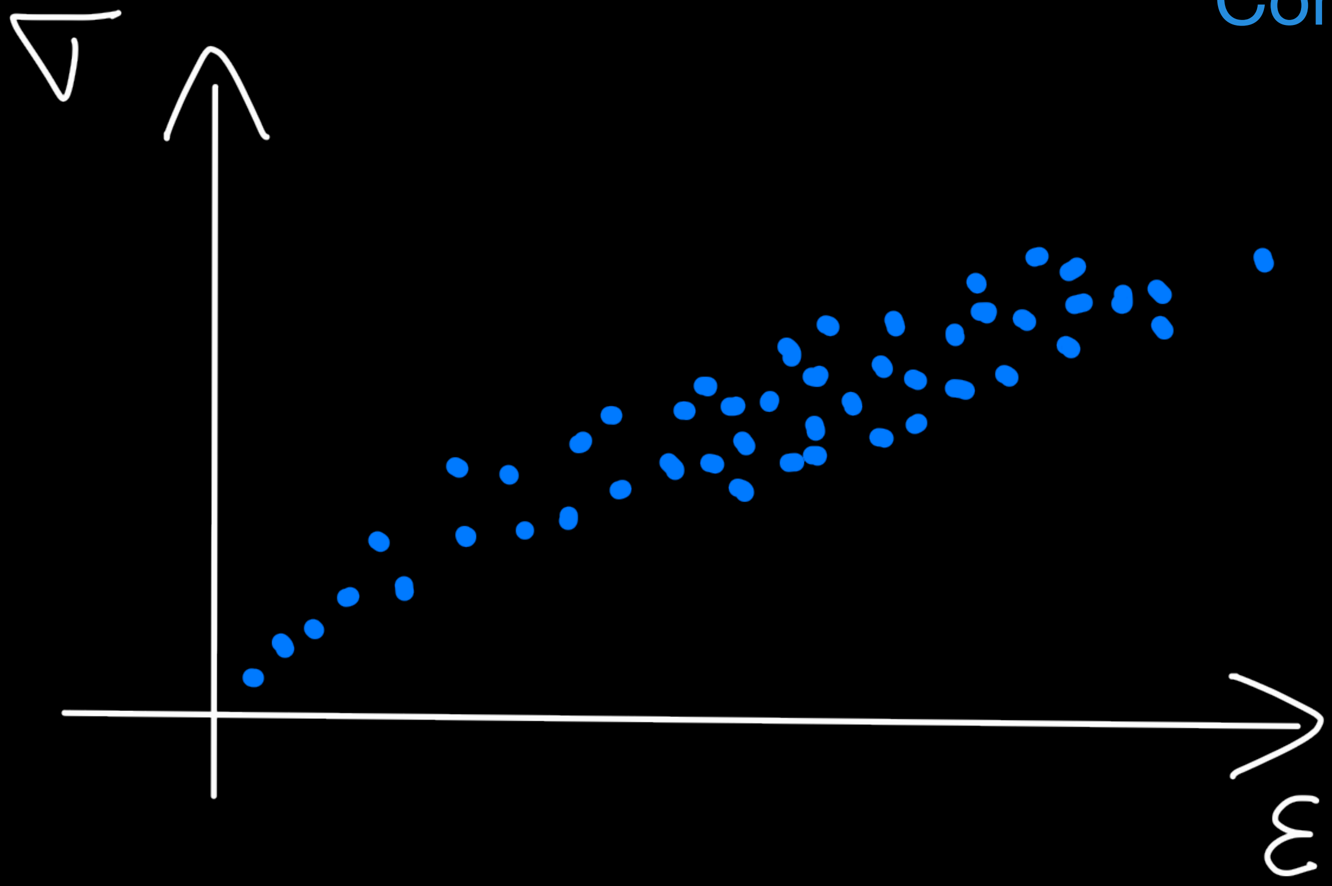
$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_p \frac{\mu_p}{\alpha_p} \left(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3\right)$$

Getting the data

Acquiring strain-stress data

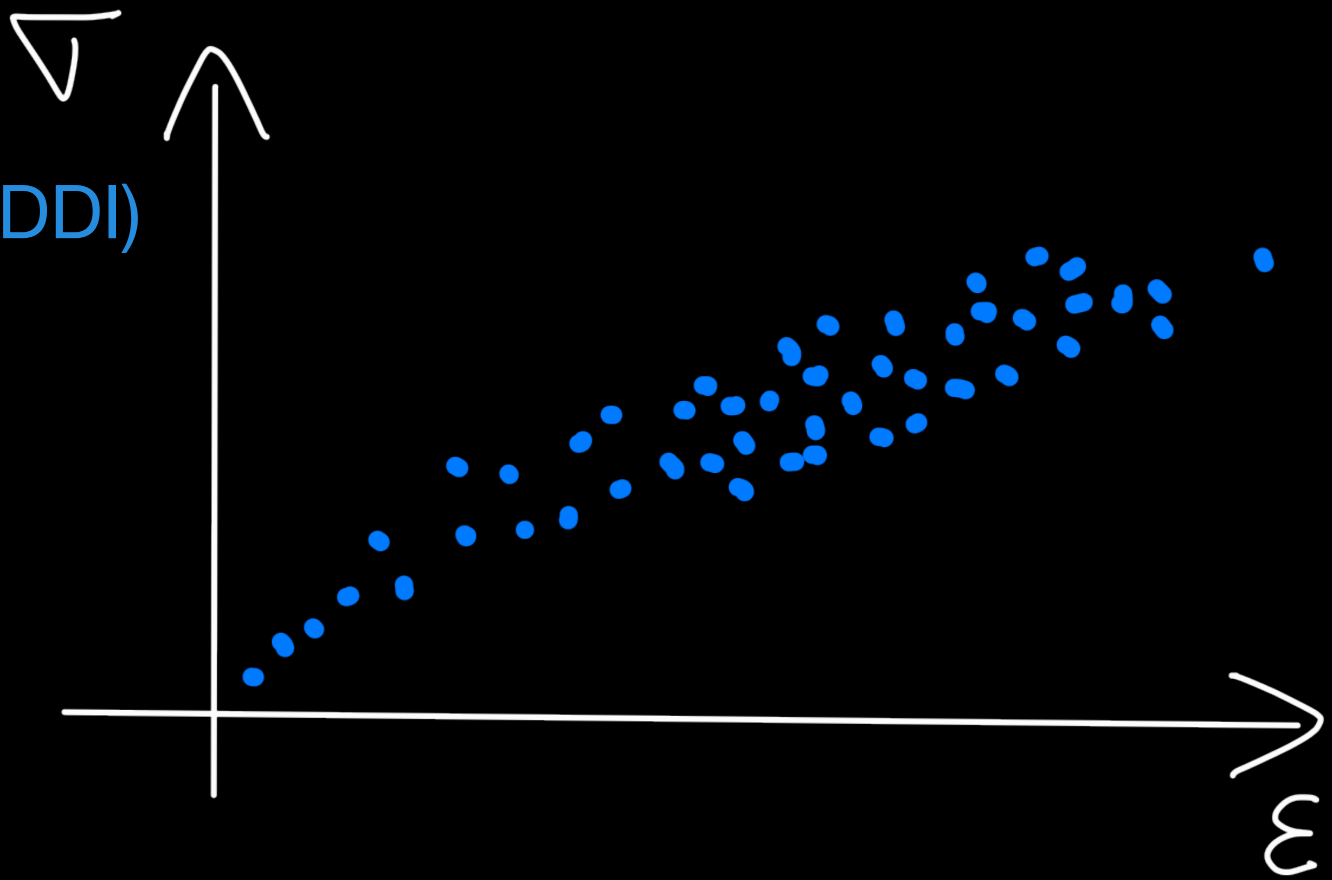
- Complex geometry
- Complex loading path
- Optical camera (not shown)
- 6-axis load cell measurements
- Multiple loading steps: snapshots

Acquiring strain-stress data



Acquiring strain-stress data

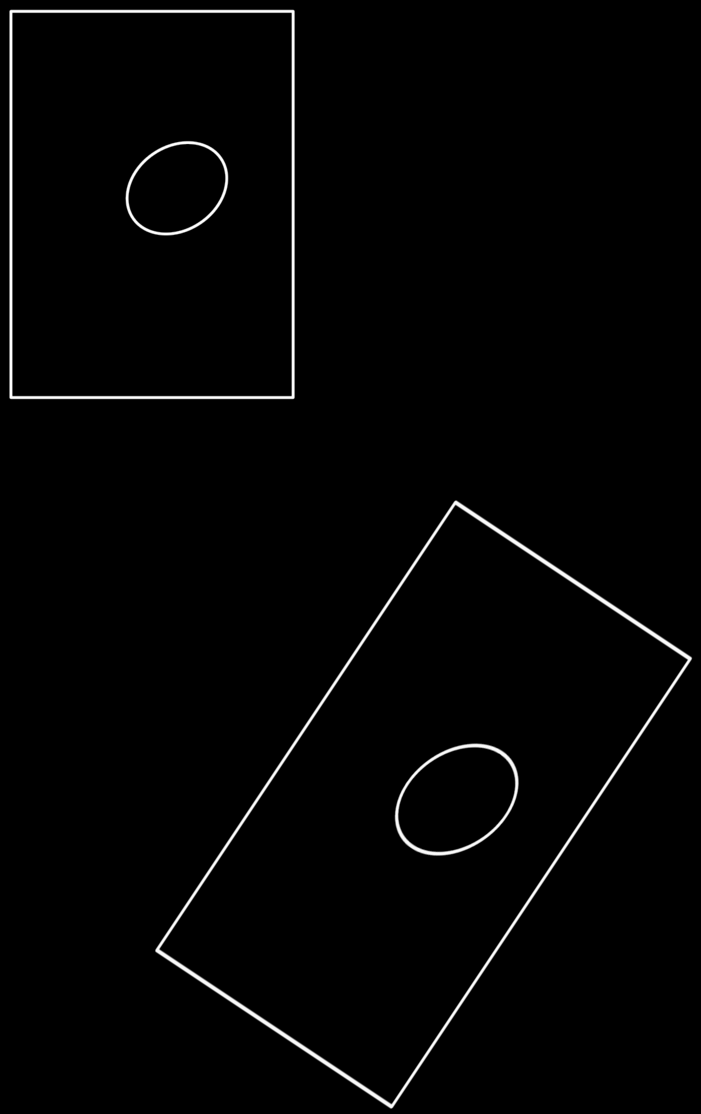
Data-Driven Identification (DDI)



Digital Image Correlation (DIC)

Sutton et al (2009)
Leygue et al (2018)

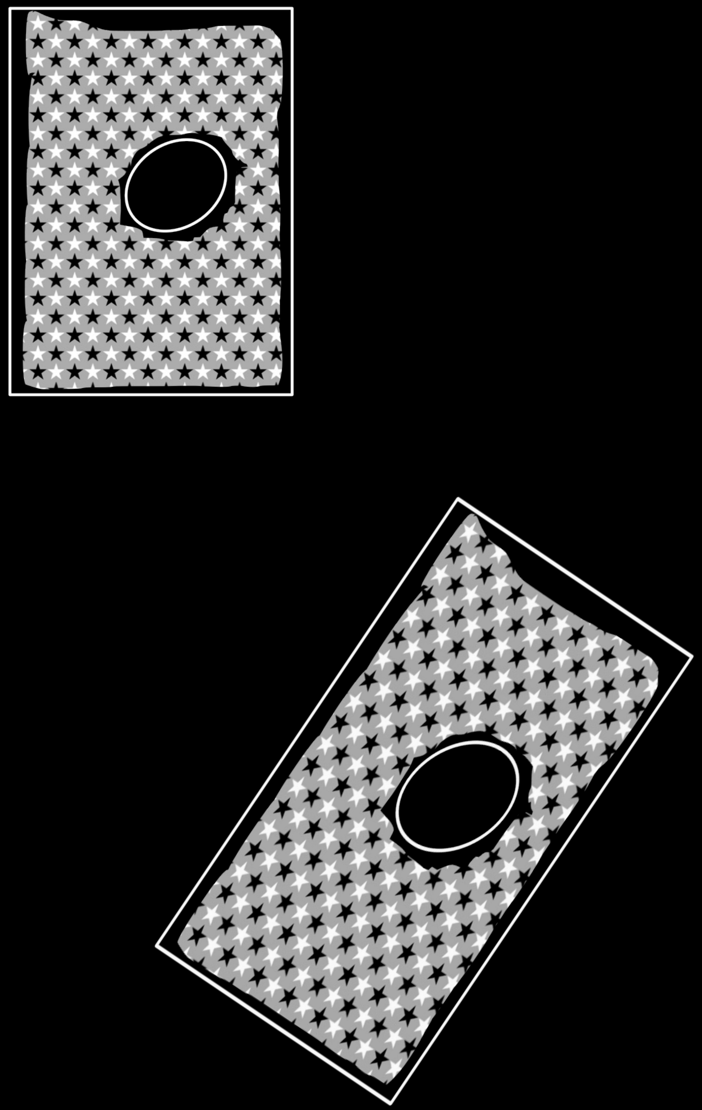
Digital Image Correlation (DIC)



- Quantity of interest

$$\vec{u}$$

Digital Image Correlation (DIC)



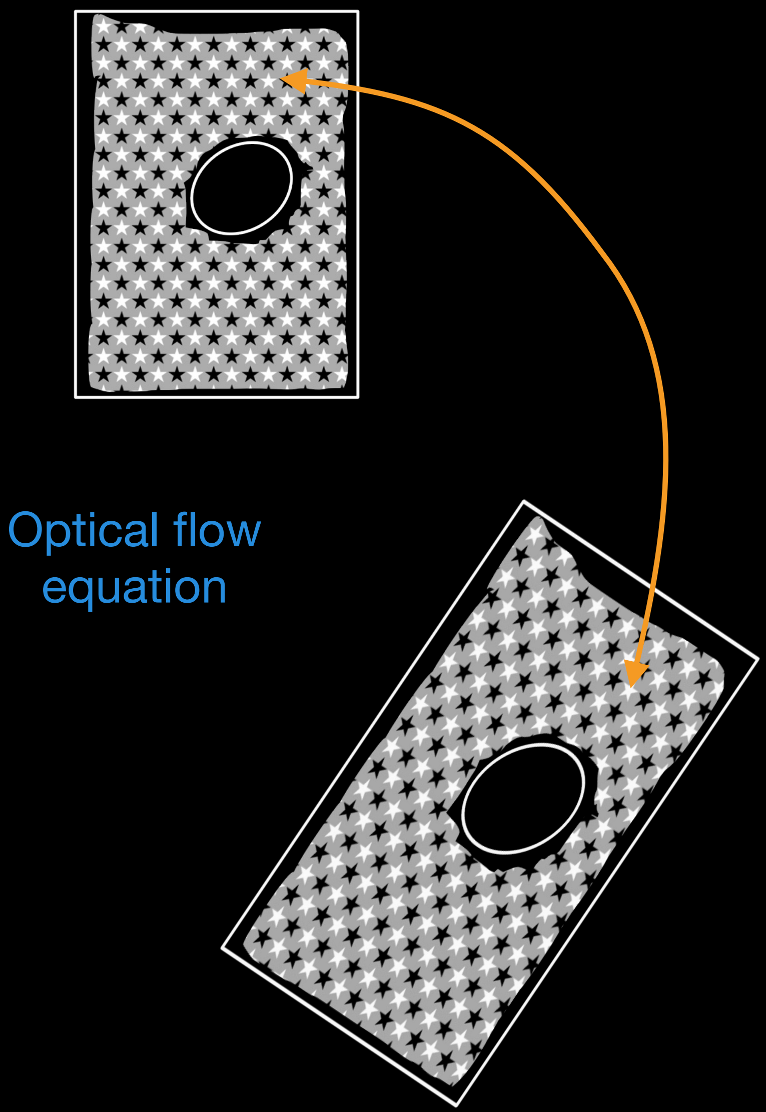
- Quantity of interest

$$\vec{u}$$

- Auxiliary field

$$I(x)$$

Digital Image Correlation (DIC)



- Quantity of interest

$$\vec{u}$$

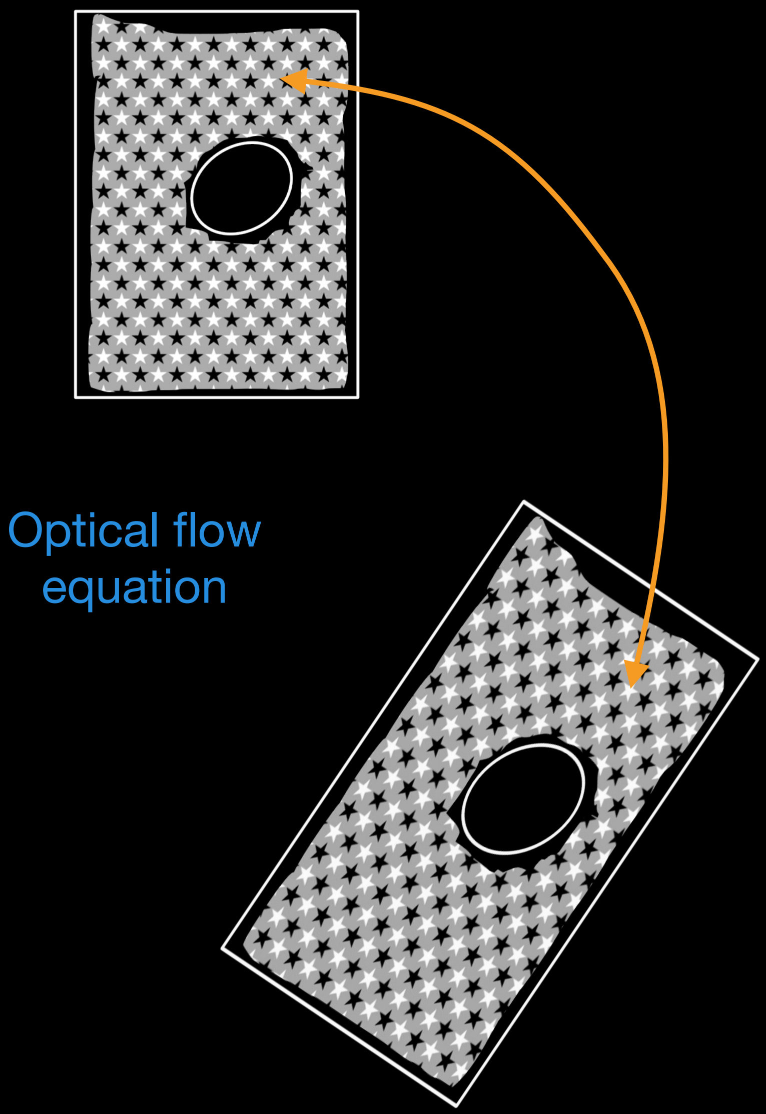
- Auxiliary field

$$I(x)$$

- Balance principle

$$I(\vec{x}, t) = I(\vec{x} + \vec{u}, t + \Delta t)$$

Digital Image Correlation (DIC)



- Quantity of interest

$$\vec{u}$$

- Auxiliary field

$$I(x)$$

- Balance principle

$$\vec{\mathrm{grad}}(I) \cdot \vec{u} + \Delta I = 0$$

Digital Image Correlation (DIC)

- Quantity of interest

$$\vec{u}$$

- Auxiliary field

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$$\vec{\mathrm{grad}}(I) \cdot \vec{u} + \Delta I = 0$$

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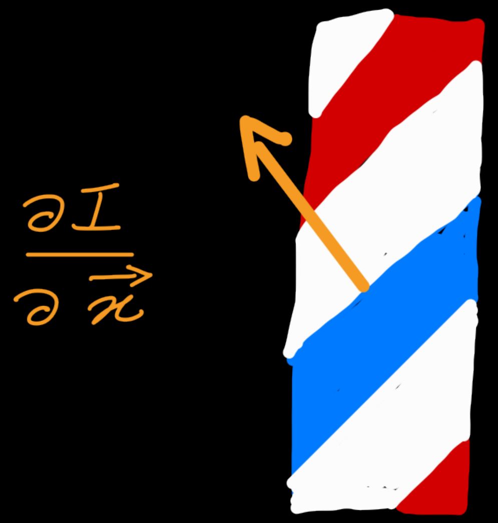

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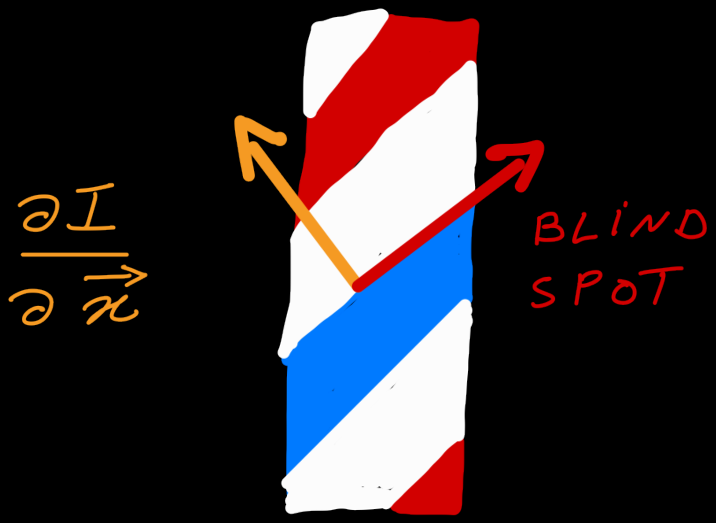

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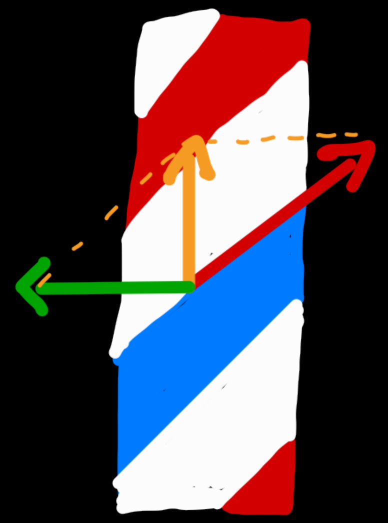

- Quantity of interest

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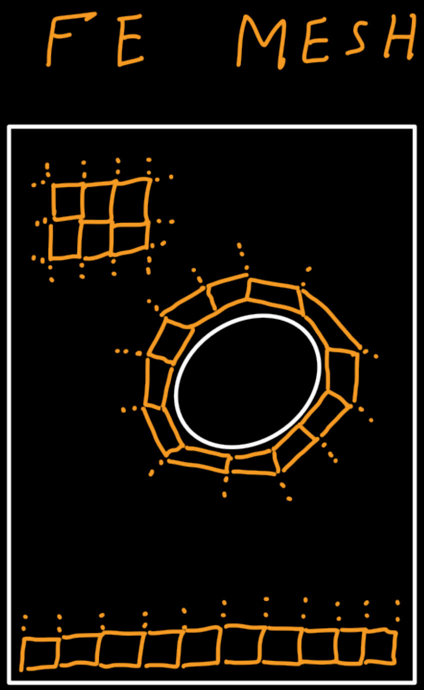


Close points in physical space
Have similar displacement

- Quantity of interest

$$\vec{u}$$

- Auxiliary field

$$I(x)$$

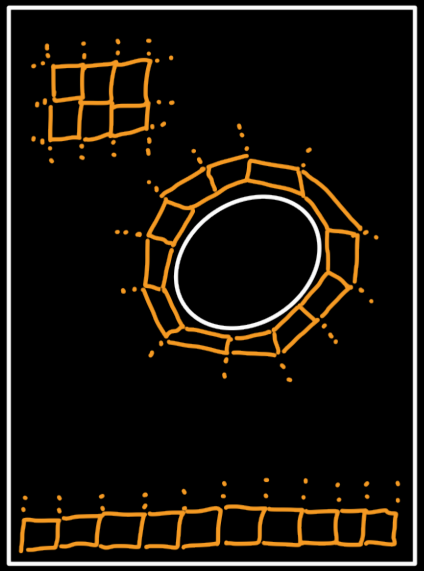
- Balance principle

$$\vec{\mathrm{grad}}(I) \cdot \vec{u} + \Delta I = 0$$

- Regularization

$$\vec{u}(\vec{x}) \simeq \Phi_i(\vec{x})\vec{U}_i$$

Digital Image Correlation (DIC)



Best regularizeddisplacement field?

- Quantity of interest

$$\vec{u}$$

- Auxiliary field

$$I(x)$$

- Balance principle

$$\vec{\text{grad}}(I) \cdot \vec{u} + \Delta I = 0$$

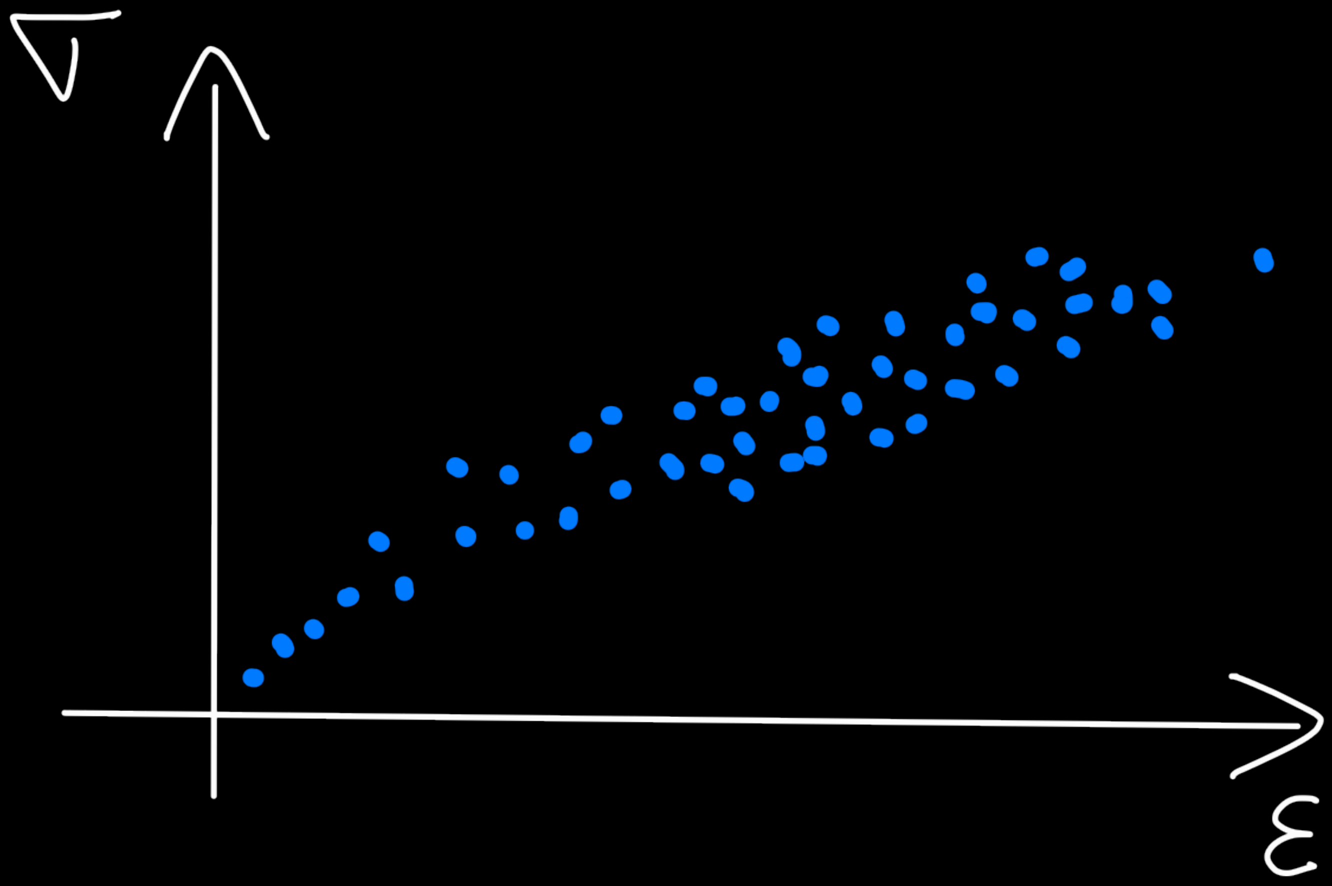
- Regularization

$$\vec{u}(\vec{x}) \simeq \Phi_i(\vec{x})\vec{U}_i$$

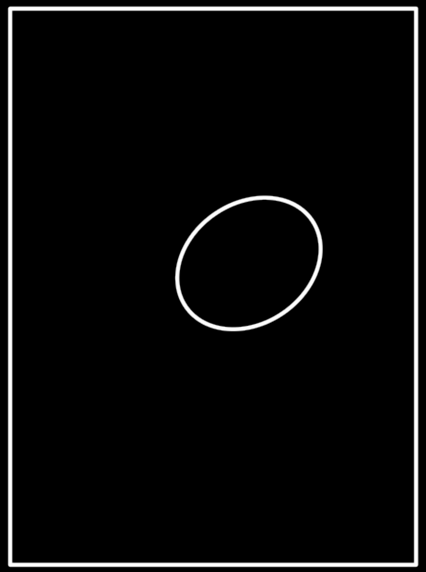
- Minimization

$$\min_{\vec{U}_i} \left\| \vec{\text{grad}}(I) \cdot \left(\Phi_k \vec{U}_k \right) + \Delta I \right\|_2^2$$

Acquiring strain-stress data



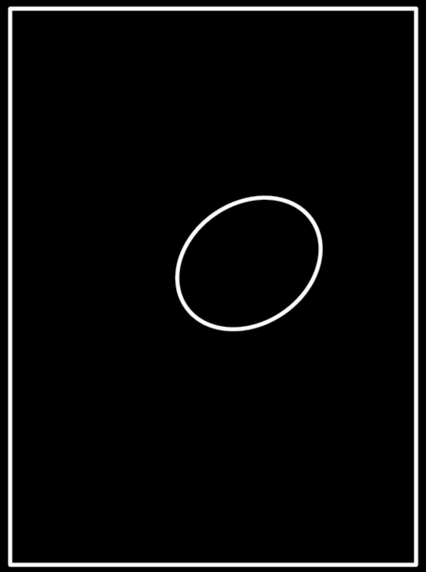
Data Driven Identification (DDI)



- Quantity of interest

$$\bar{\bar{\sigma}}$$

Data Driven Identification (DDI)

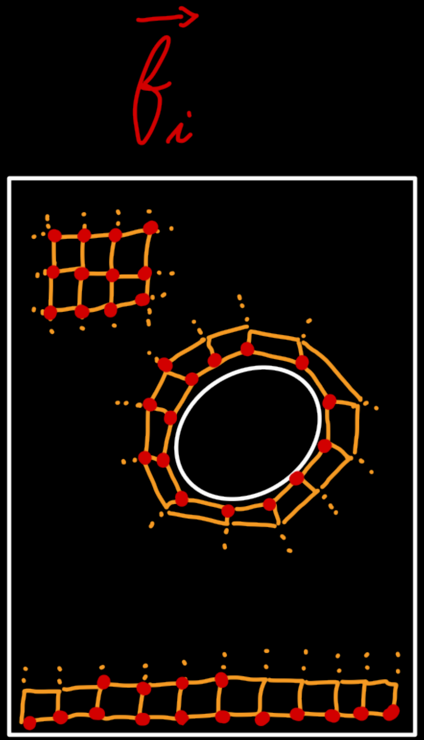


The stress field is identified simultaneously for all loading steps

- Quantity of interest

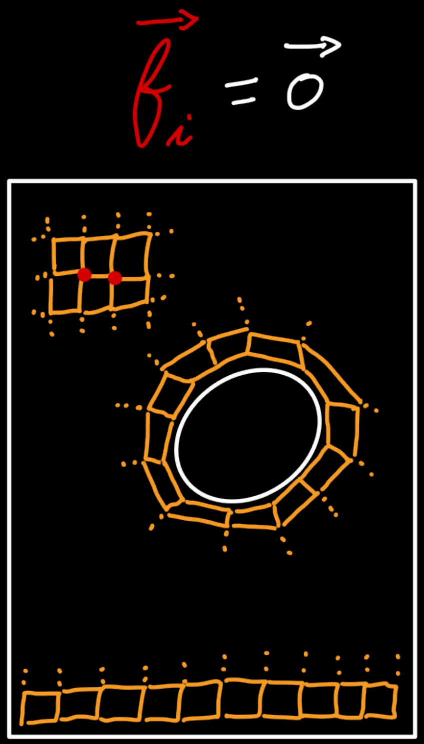
$$\bar{\bar{\sigma}}$$

Data Driven Identification (DDI)



- Quantity of interest $\bar{\bar{\sigma}}$
- Auxiliary field $\vec{f}(\vec{x})$

Data Driven Identification (DDI)



- Quantity of interest

$$\bar{\bar{\sigma}}$$

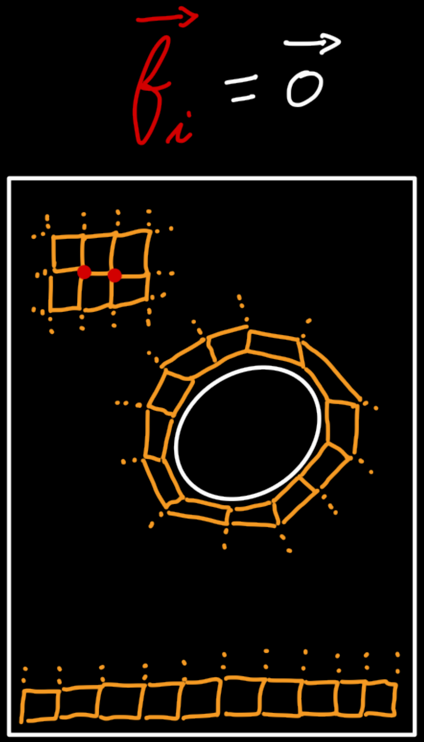
- Auxiliary field

$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{\text{div}}\,(\bar{\bar{\sigma}}) + \vec{f} = \vec{0}$$

Data Driven Identification (DDI)



- Quantity of interest

$$\bar{\bar{\sigma}}$$

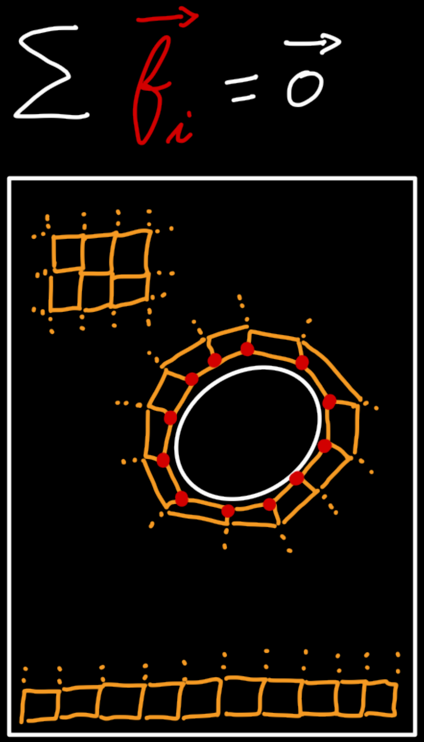
- Auxiliary field

$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

Data Driven Identification (DDI)



- Quantity of interest

$$\bar{\bar{\sigma}}$$

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$$\vec{f}(\vec{x})$$

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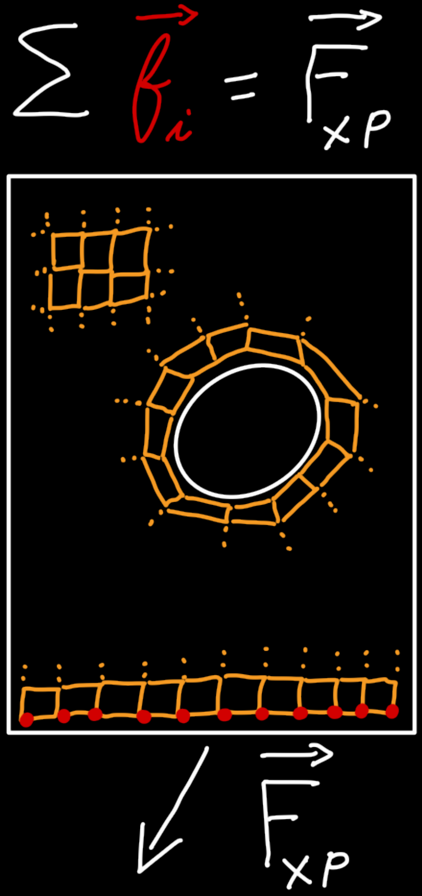
Dalémat et al. (2024)

Data Driven Identification (DDI)



- Quantity of interest

$$\bar{\bar{\sigma}}$$

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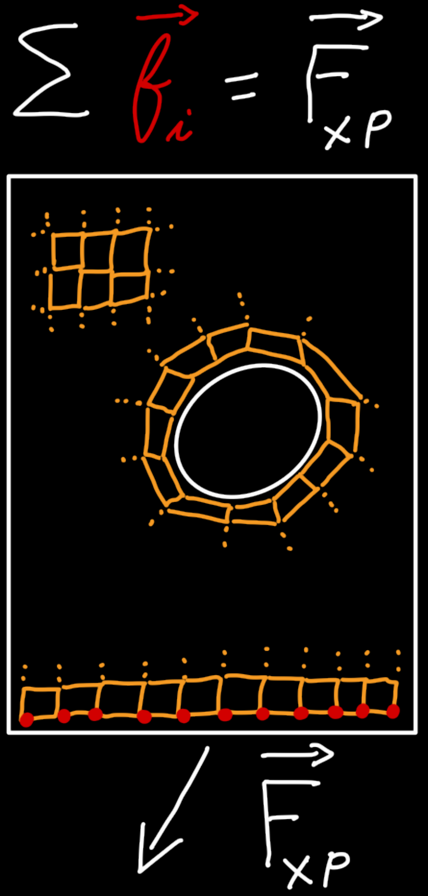
$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

Data Driven Identification (DDI)



Blind spot = Ker(D)

- Quantity of interest

$$\bar{\bar{\sigma}}$$

- Auxiliary field

$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

Data Driven Identification (DDI)

- Quantity of interest

$$\bar{\bar{\sigma}}$$

- Auxiliary field

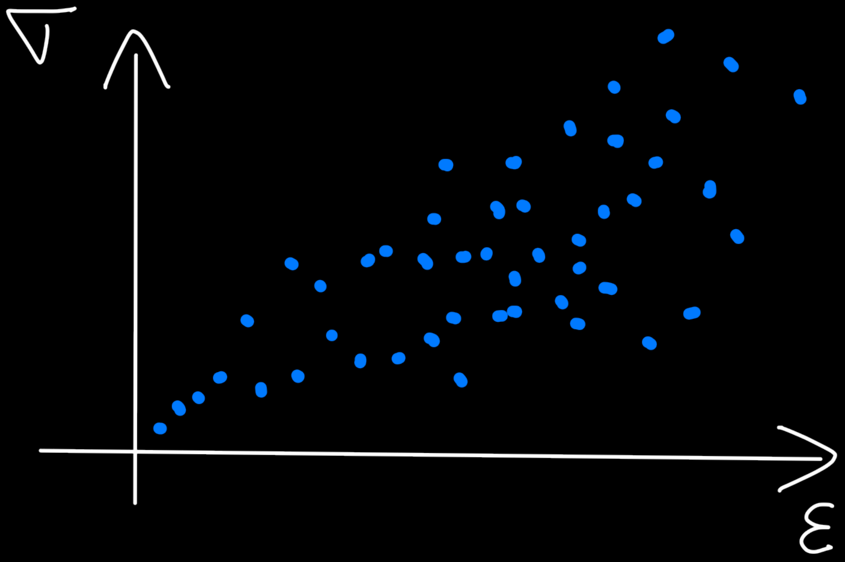
$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- Regularization

Data Driven Identification (DDI)



- Quantity of interest

$$\bar{\bar{\sigma}}$$

- Auxiliary field

$$\vec{f}(\vec{x})$$

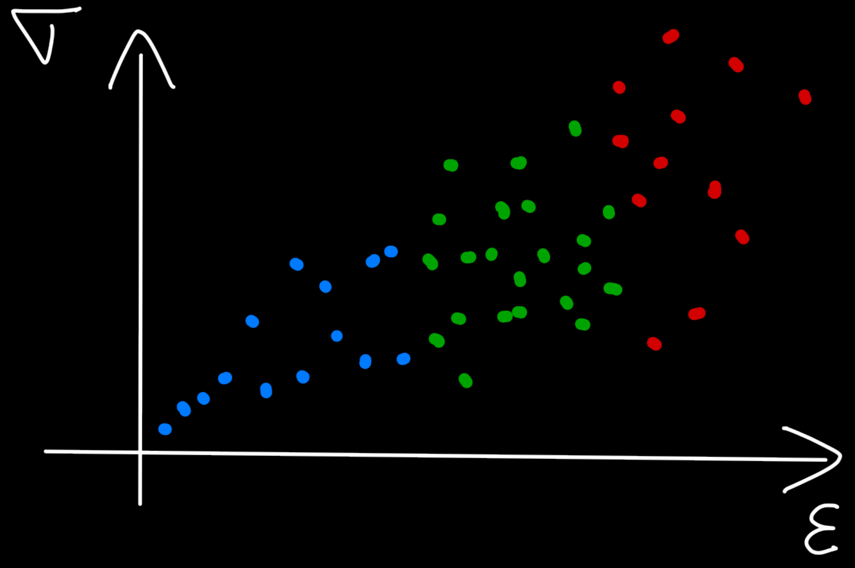
- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- Regularization

$$\bar{\bar{\sigma}}(\bar{\bar{\epsilon}}) \simeq \Phi_k(\bar{\bar{\epsilon}})\bar{\bar{\mathcal{S}}}_k$$

Data Driven Identification (DDI)



Introduce a clustering in constitutive space

- Quantity of interest

$$\bar{\bar{\sigma}}$$

- Auxiliary field

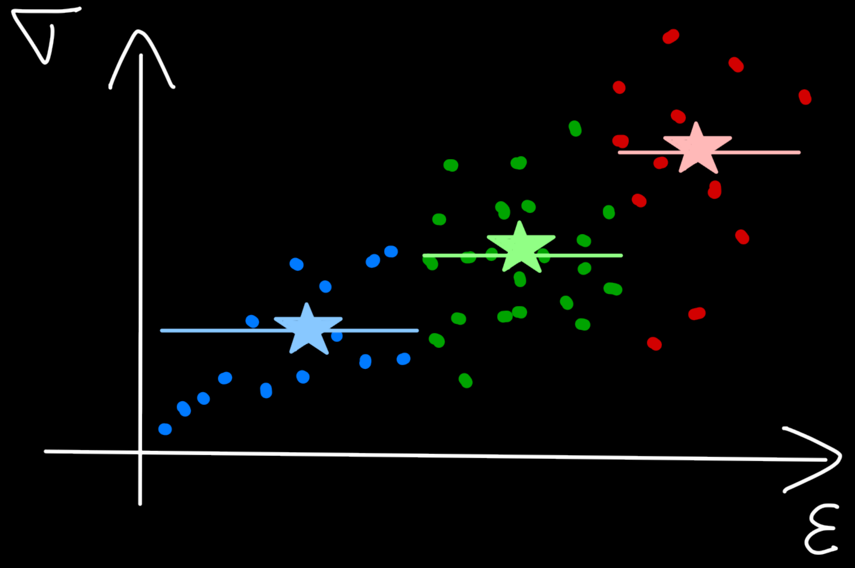
$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- Regularization

Data Driven Identification (DDI)



Piecewise constant regularization
in constitutive space

- Quantity of interest

$$\bar{\bar{\sigma}}$$

- Auxiliary field

$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

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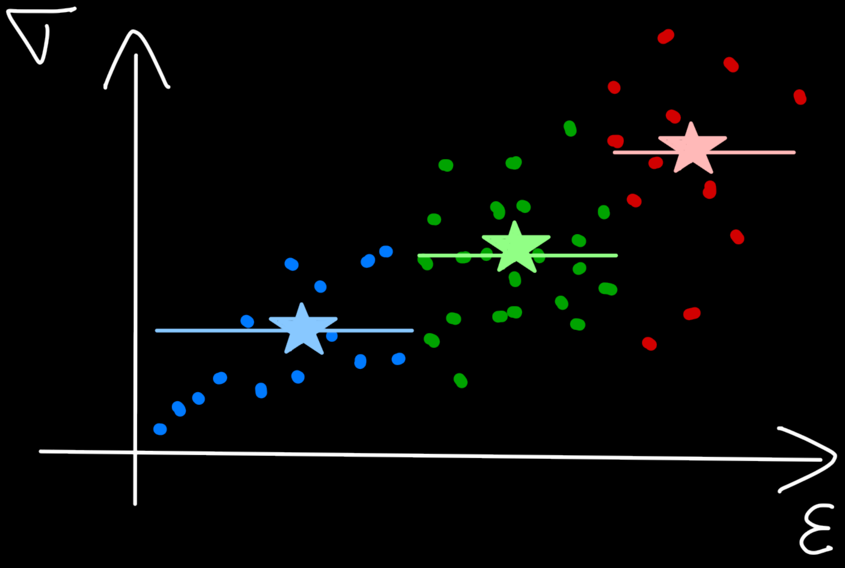
Data Driven Identification (DDI)



Piecewise constant regularization
in constitutive space

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- Auxiliary field

$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- Regularization

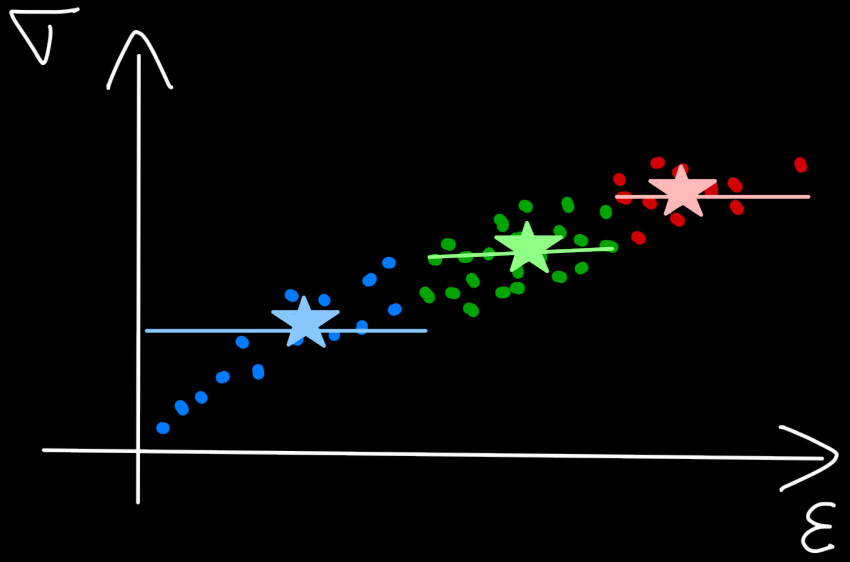
$$\bar{\bar{\sigma}}(\bar{\bar{\epsilon}}) \simeq \Phi_k(\bar{\bar{\epsilon}})\bar{\bar{S}}_k$$

● mechanical states

★ material states database

Kirchdoerfer & Ortiz (2016)

Data Driven Identification (DDI)



Minimize the distance between a stress field at equilibrium and a regularized field

- Quantity of interest

$$\bar{\bar{\sigma}}$$

- Auxiliary field

$$\vec{f}(\vec{x})$$

- Balance principle

$$\vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

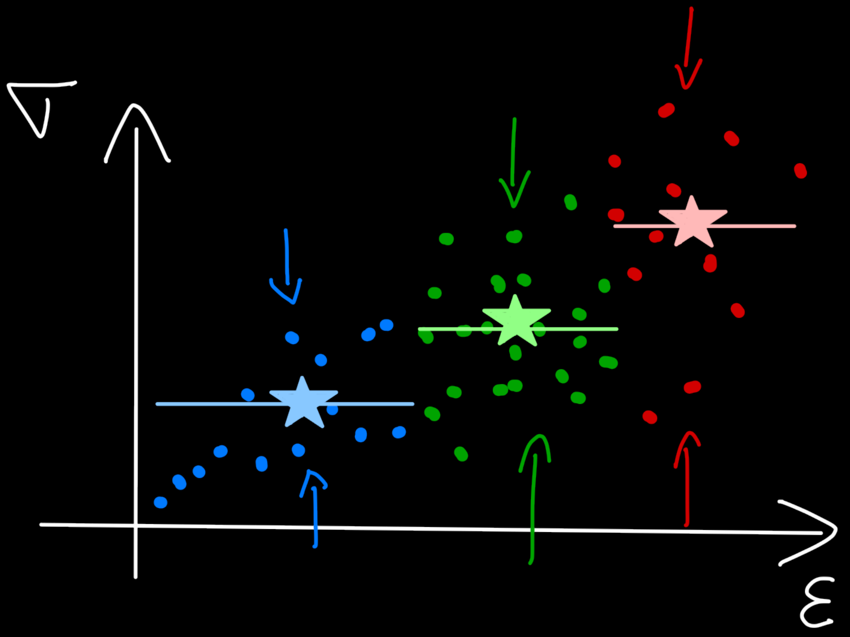
- Regularization

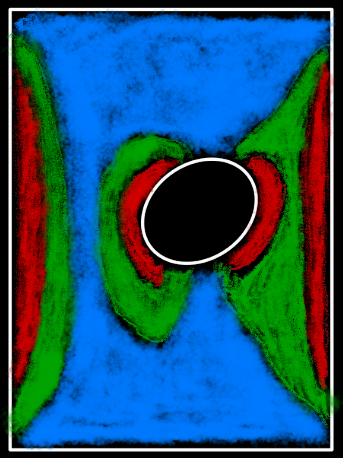
$$\bar{\bar{\sigma}}(\bar{\bar{\epsilon}}) \simeq \Phi_k(\bar{\bar{\epsilon}})\bar{\bar{\mathcal{S}}}_k$$

- Minimization

$$\min_{\bar{\bar{\mathcal{S}}}_i,\, \bar{\bar{\sigma}}} \left\| \bar{\bar{\sigma}} - \Phi_k \bar{\bar{\mathcal{S}}}_k \right\|_{\mathbb{C}}^2 \;\; \& \;\; \vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

DDI Blind Spot?





- Minimization problem

$$\min_{\bar{\bar{\mathcal{S}}}_i,\,\bar{\bar{\sigma}}} \left\| \bar{\bar{\sigma}} - \Phi_k \bar{\bar{\mathcal{S}}}_k \right\|_{\mathbb{C}}^2 \;\; \& \;\; \vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- Equivalent formulation

$$\sigma = \arg\min_{\sigma} \frac{1}{2}\, \sigma^T\, H\, \sigma$$

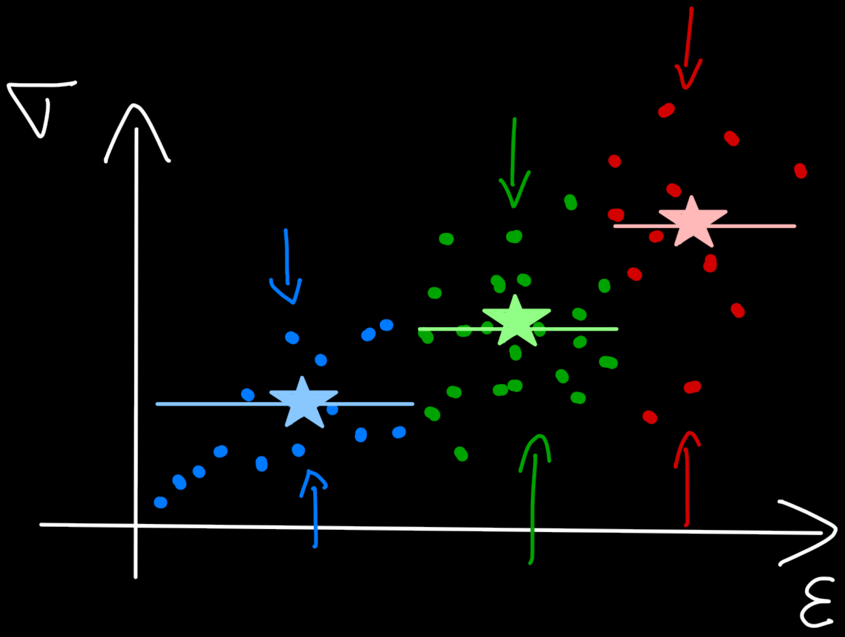
$$\& \qquad D\sigma = f$$

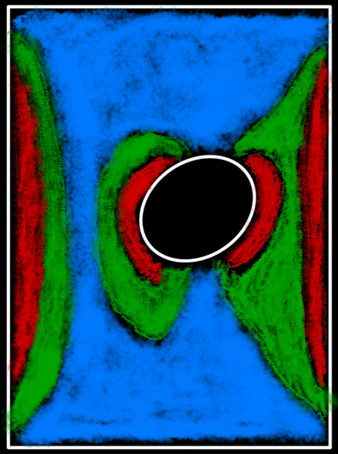
- Solution is unique if

$$\mathrm{Ker}\, H \;\cap\; \mathrm{Ker}\, D \;=\; \{0\}$$

A. Leygue (CR-MECA 2025)

Some results





- Equivalent formulation

$$\sigma = \arg\min_{\sigma} \frac{1}{2}\, \sigma^T\, H\, \sigma$$

$$\& \qquad D\sigma = f$$

- Loss of uniqueness $\quad \mathrm{Ker}\, H \;\cap\; \mathrm{Ker}\, D \;\neq \{0\}$
    - The geometry is too simple
    - The clustering is too fine
    - The strain data is not rich enough
- Efficient alternated projection algorithms

N. Hachem et al (submitted)

Learning *from* the data

Experimental setup

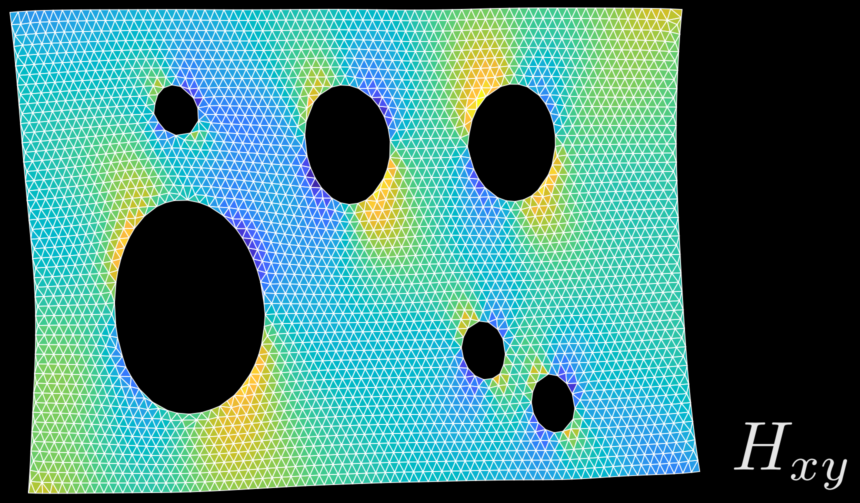
- Membrane with holes
- Carbon black-filled polychloroprene (Joreau elastomers)
- New vs. <u>aged</u> 21 days at 100°C

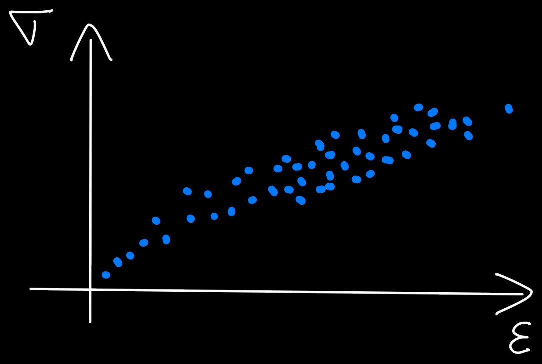
- Symétrie hexapod
- Quasi-static “monotonous” in-plane loading
- Incompressibility & plane stress assumed

- DIC with Ufreckles (J. Réthoré)
- 622 snapshots
- ~6000 elements mesh → over 3.5 million points

H. Madeira thesis (2024)

Kinematics



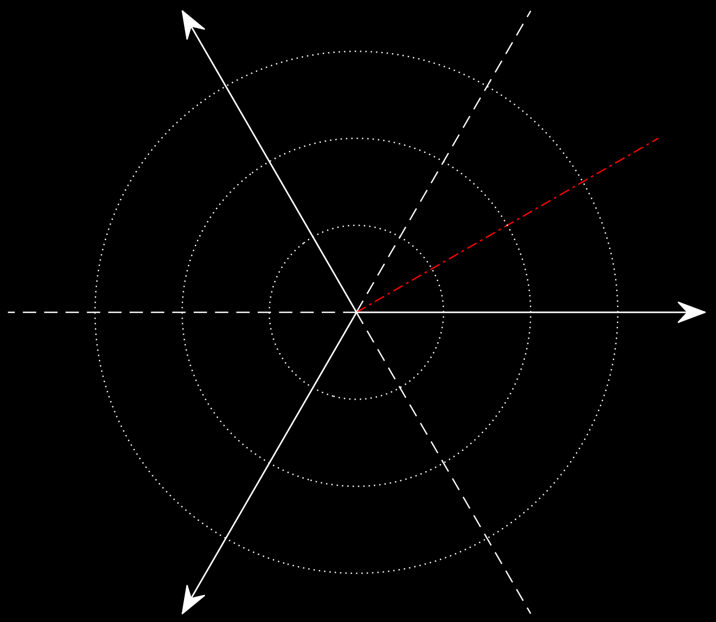


Representation in 3D(strain) x 3D (stress) ?

- Strain tensor

$$\bar{\bar{H}} = \frac{1}{2} \ln \left(\bar{\bar{F}} \cdot \bar{\bar{F}}^T \right)$$

Kinematics



- Strain tensor

$$\bar{\bar{H}} = \frac{1}{2} \ln \left(\bar{\bar{F}} \cdot \bar{\bar{F}}^T \right)$$

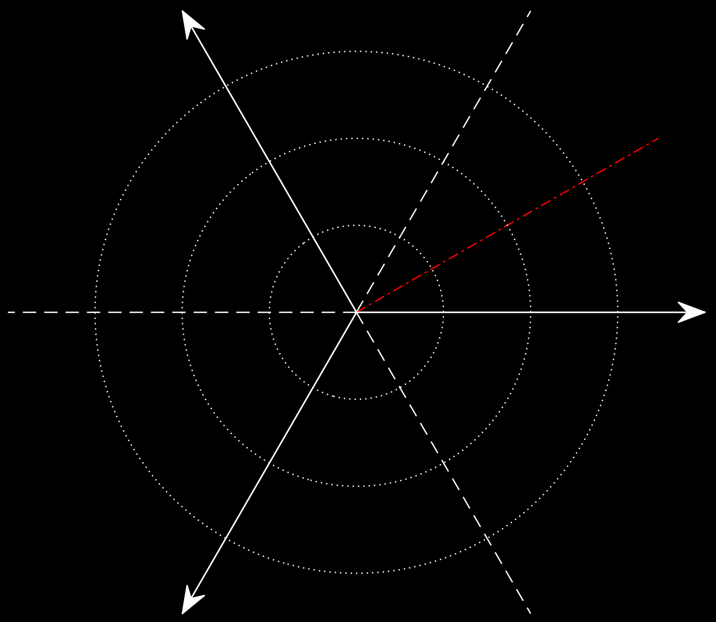
- Haigh-Westergaard representation

$$z = \mathrm{tr} \bar{\bar{H}}$$

$$r = \sqrt{\mathrm{dev} \bar{\bar{H}} : \mathrm{dev} \bar{\bar{H}}}$$

$$\cos(3\theta) = \frac{3\sqrt{6}}{r^3} \det \left(\mathrm{dev} \bar{\bar{H}} \right)$$

Kinematics



- Strain tensor

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- Haigh-Westergaard representation

$$z = \mathrm{tr} \bar{\bar{H}}$$

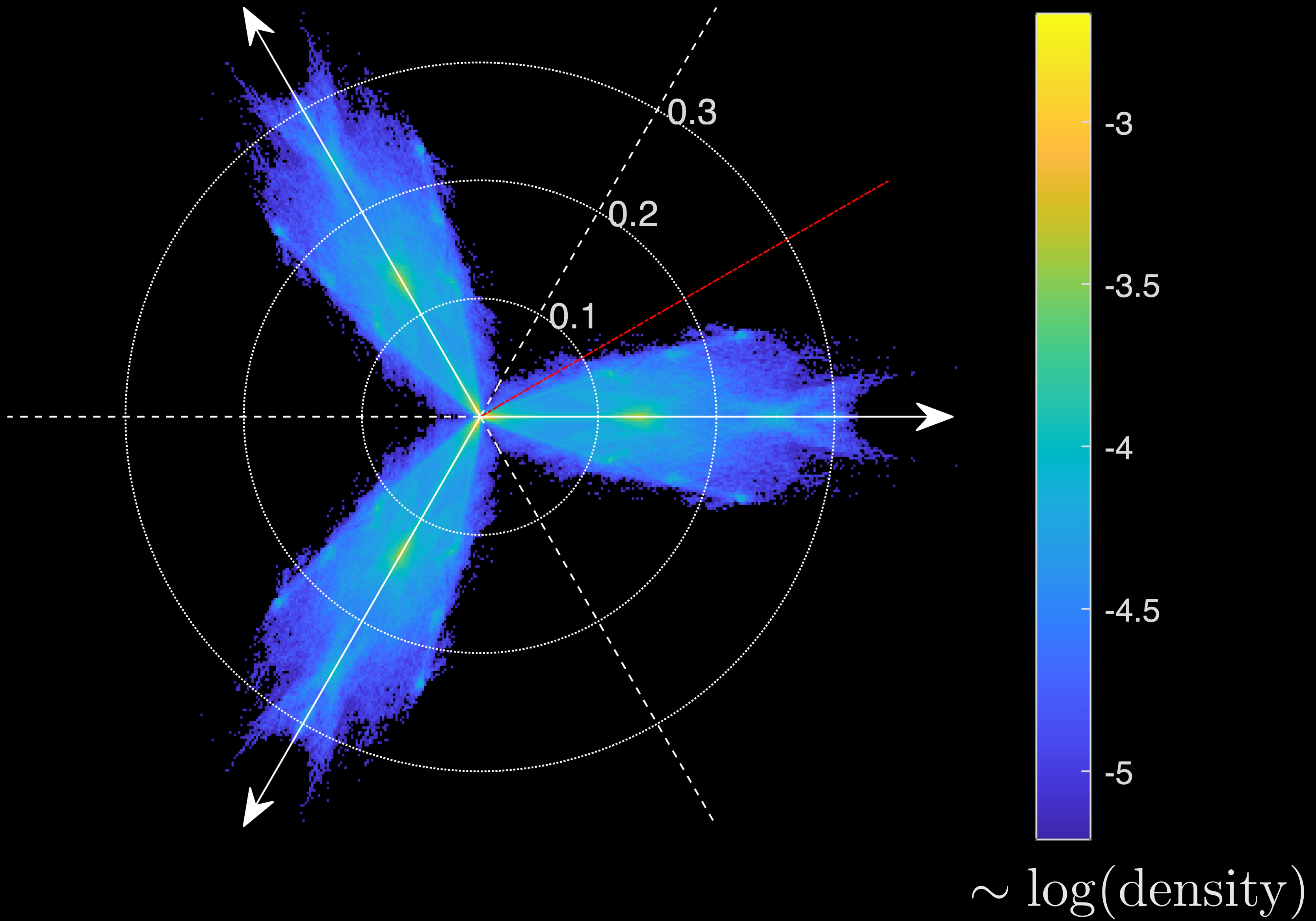
$$r = \sqrt{\mathrm{dev} \bar{\bar{H}} : \mathrm{dev} \bar{\bar{H}}}$$

$$\cos(3\theta) = \frac{3\sqrt{6}}{r^3} \det \left(\mathrm{dev} \bar{\bar{H}} \right)$$

- Interpretation

$\theta = 0$	uniaxial tension
$\theta = \pi/6$	pure shear
$\theta = \pi/3$	equibiaxial tension

Kinematics



DDI details

- Minimization problem:

$$\min_{\bar{\bar{\mathcal{S}}}_i\,,\,\bar{\bar{\sigma}}} \left\| \bar{\bar{\sigma}} - \Phi_k \bar{\bar{\mathcal{S}}}_k \right\|_{\mathbb{C}}^2 \;\; \& \;\; \vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- 2-norm:

$$\mathbb{C} = \mathbb{I}$$

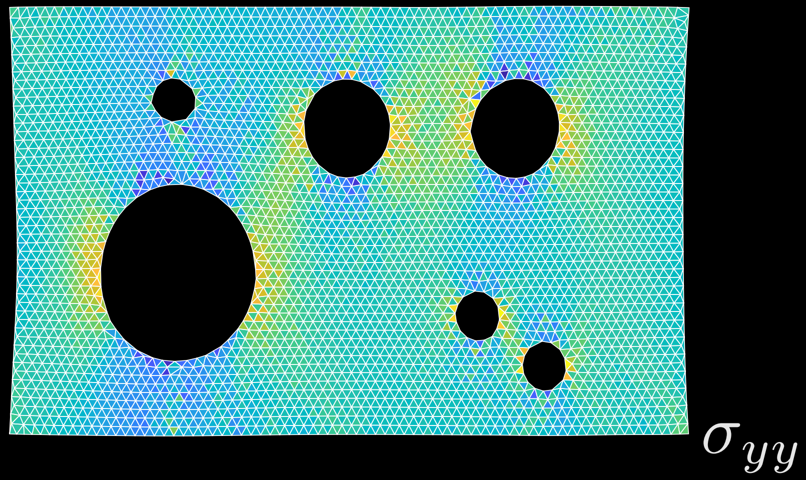
- Clustering

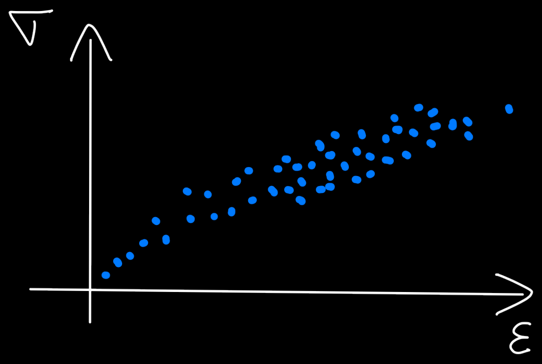
k-means

$\bar{\bar{H}}$- based

3.5k clusters (ratio=1000)

Stress field





Representation in 3D(strain) x 3D (stress) ?

- Stress tensor

$$\bar{\bar{\sigma}}$$

- Orientation

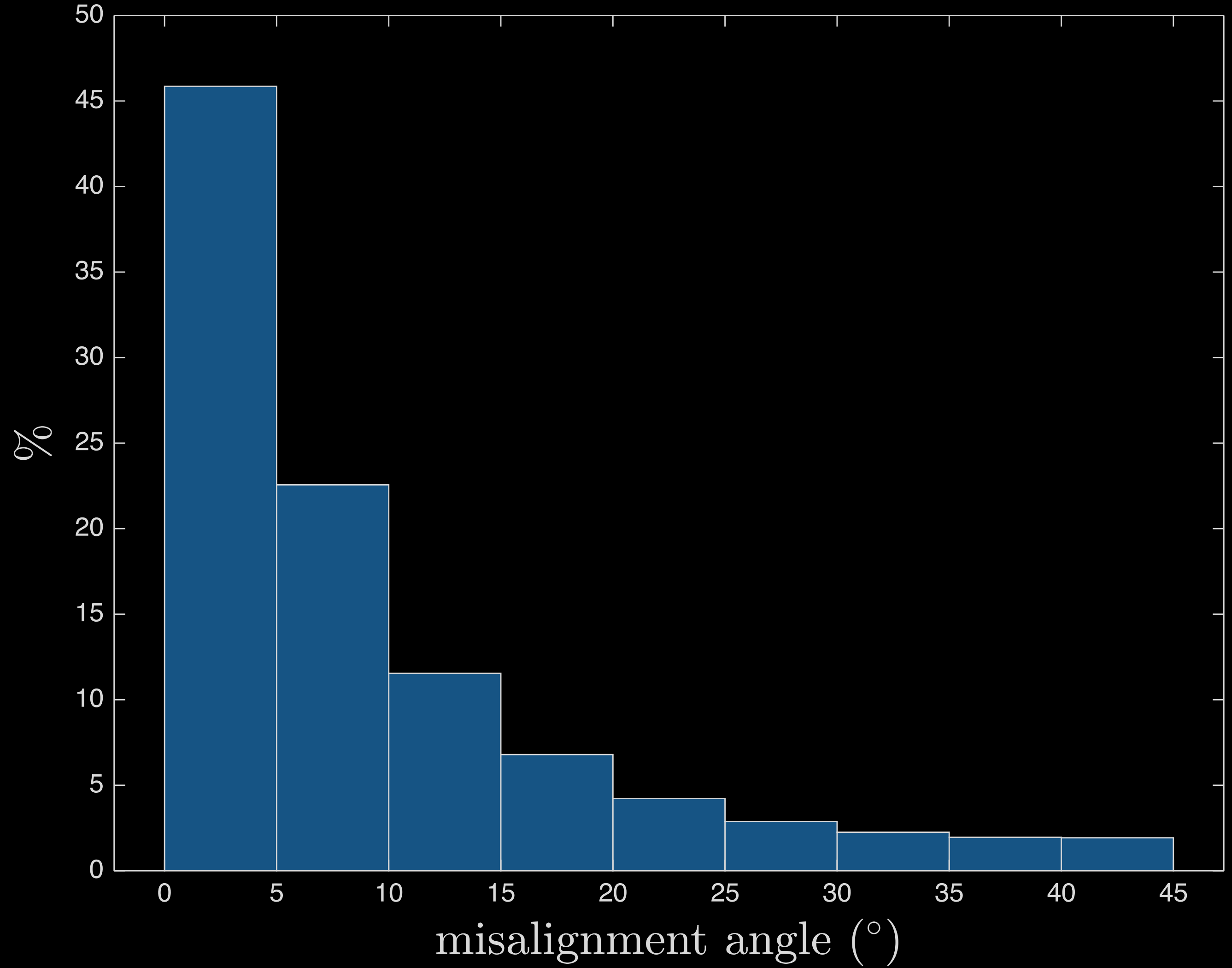
Misorientation w.r.t. strain

- Strain energy density (hyperelastic hypotheis)

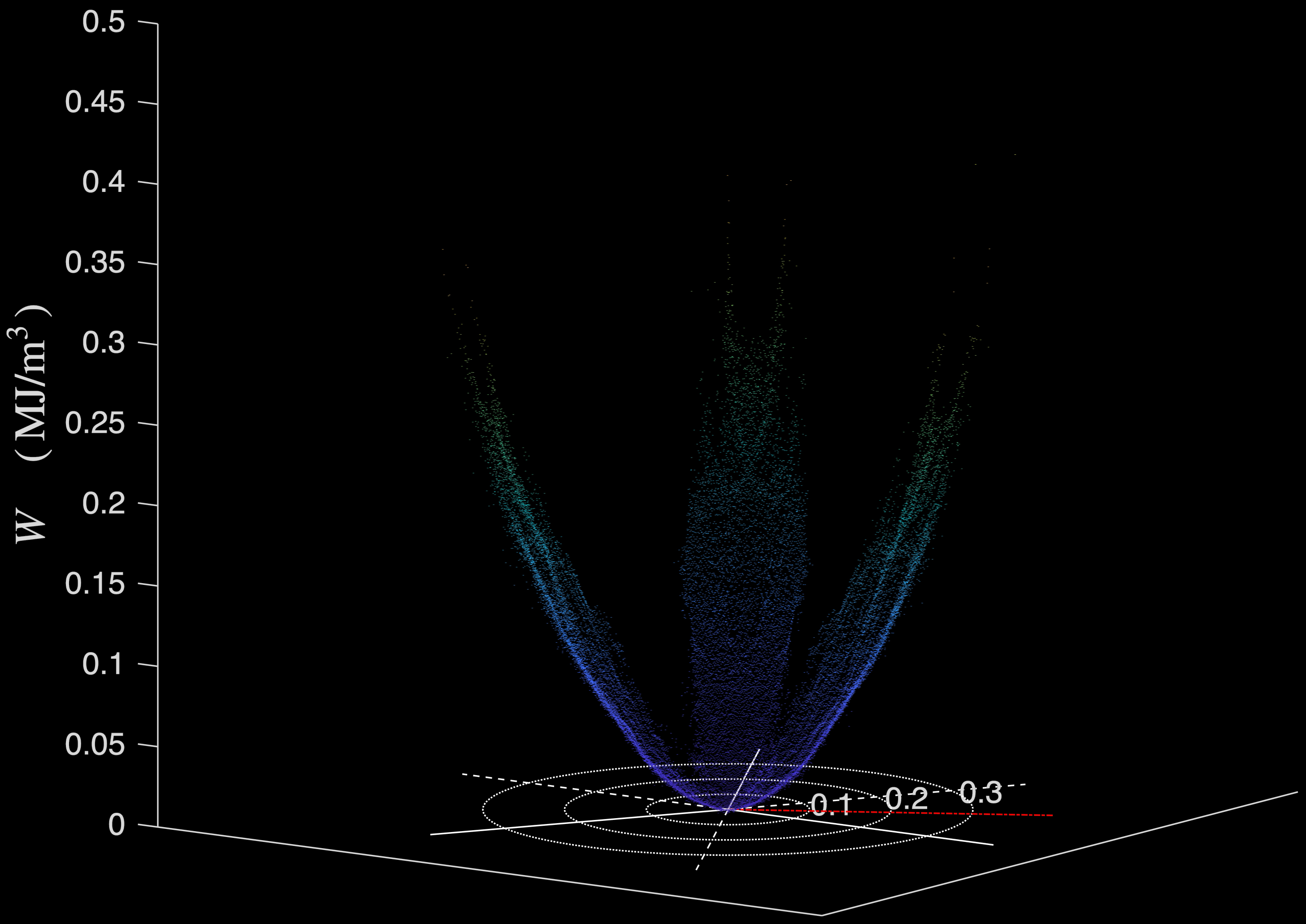
$$W\left(\bar{\bar{H}}\right) = \int_0^t \bar{\bar{\sigma}} : \bar{\bar{D}}\,\mathrm{d}t'$$

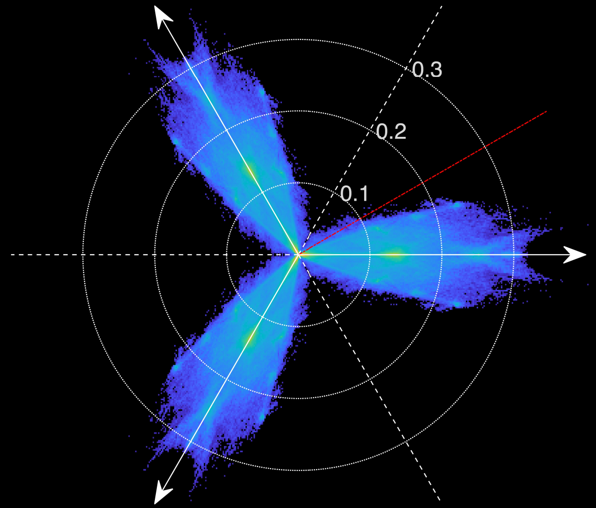
Isotropy

Strain-stress alignment?

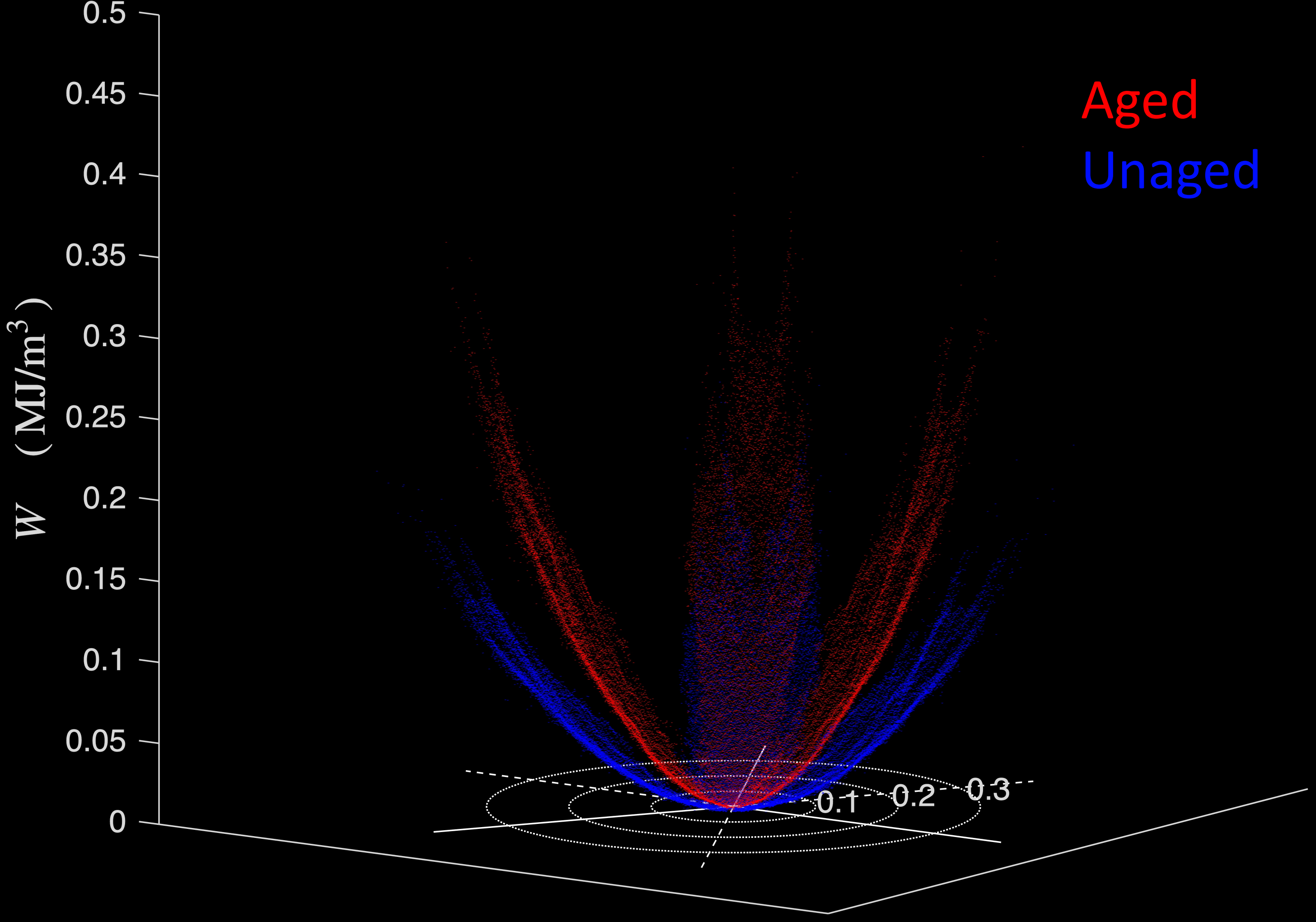


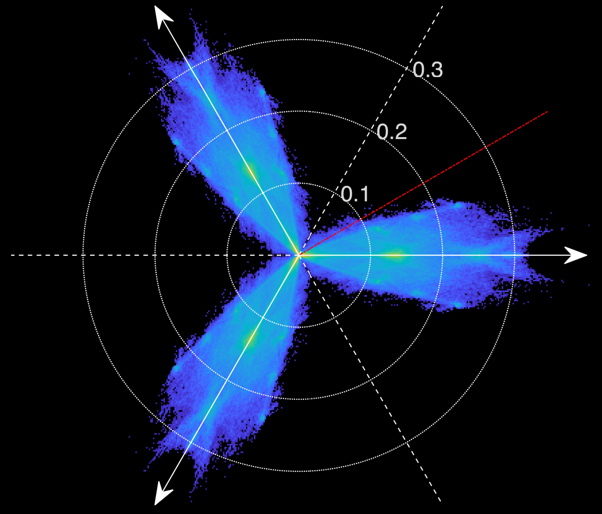
Strain energy density



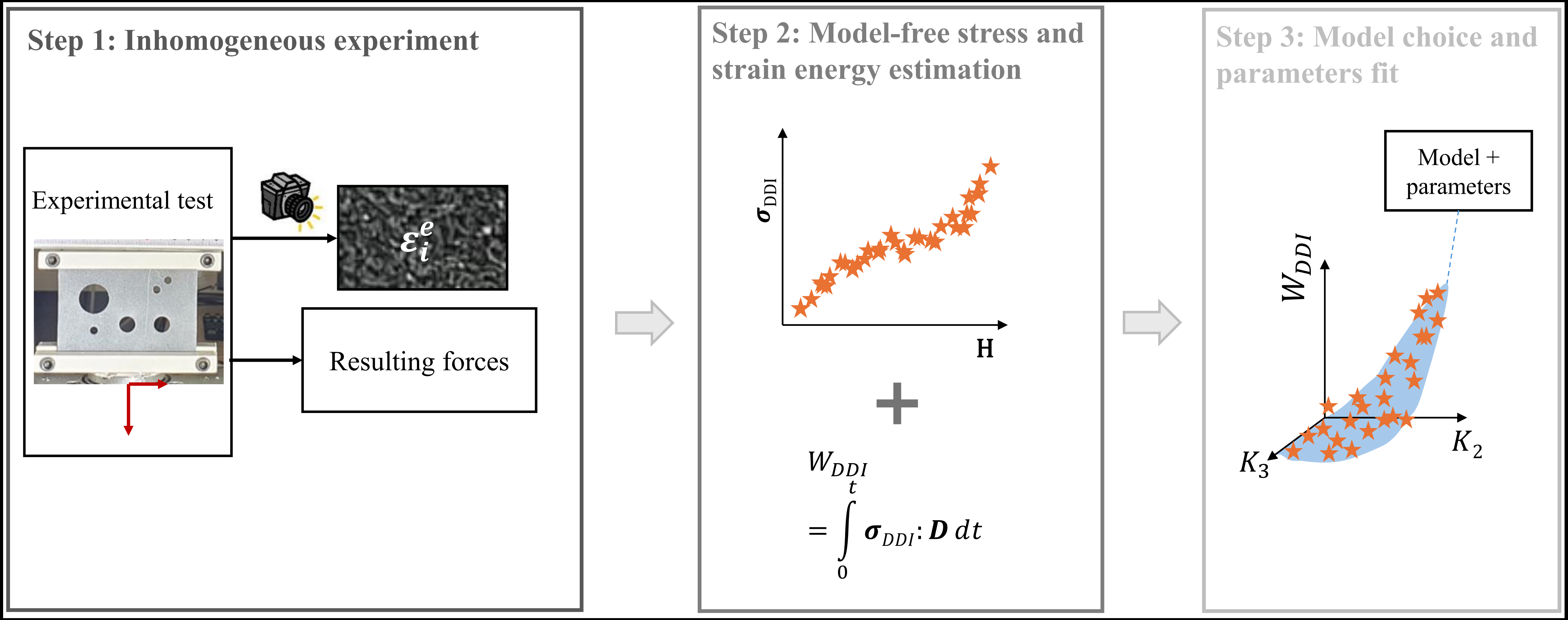


Aged vs. unaged





Data-Driven (Model) Identification



L. Costecalde Thesis (2024)

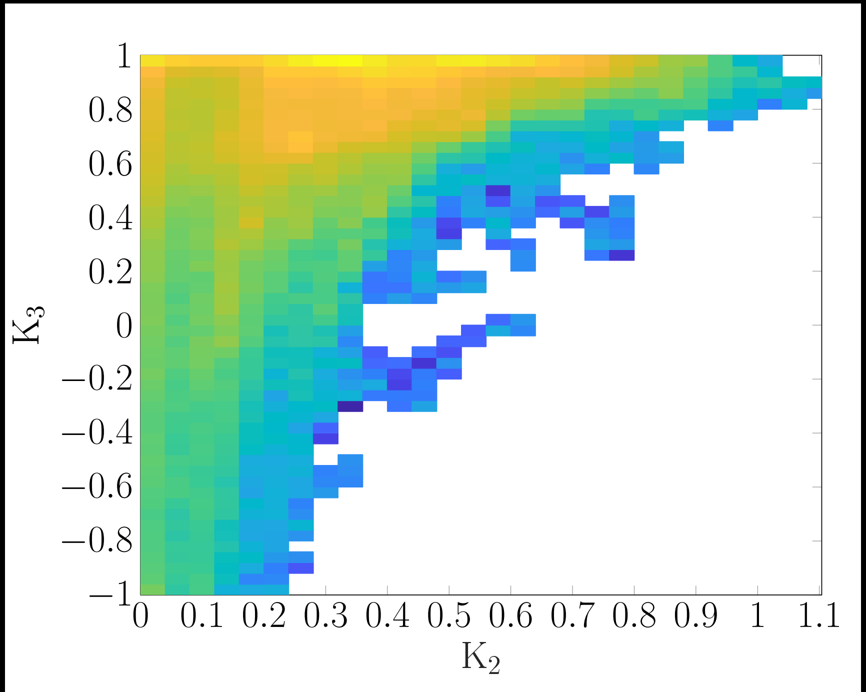
Experimental setup

- Membrane with holes (65 x 100 x 1.6 mm)
- Carbon black-filled SBR (Michelin)

- Symétrie hexapod
- Quasi-static “monotonous” in-plane loading
- Incompressibility & plane stress assumed

- DIC with Ufreckles (J. Réthoré)
- 1000 snapshots
- ~3400 elements mesh → over 3.4 million points

Kinematics



- Strain tensor

$$\bar{\bar{H}} = \frac{1}{2} \ln \left(\bar{\bar{F}} \cdot \bar{\bar{F}}^T \right)$$

- Invariants by Criscione (2000)

$$K_1 = \mathrm{tr}\bar{\bar{H}}$$

$$K_2 = \sqrt{\mathrm{dev}\bar{\bar{H}} : \mathrm{dev}\bar{\bar{H}}}$$

$$K_3 = \frac{3\sqrt{6}}{K_2^3} \det \left(\mathrm{dev}\bar{\bar{H}} \right)$$

- Interpretation

$K_3 = 1$ uniaxial tension

$K_3 = 0$ pure shear

$K_3 = -1$ equibiaxial tension

DDI details

- Minimization problem:

$$\min_{\bar{\bar{\mathcal{S}}}_i\,,\,\bar{\bar{\sigma}}} \left\| \bar{\bar{\sigma}} - \Phi_k \bar{\bar{\mathcal{S}}}_k \right\|_{\mathbb{C}}^2 \;\; \& \;\; \vec{D}\left(\bar{\bar{\sigma}}\right) + \vec{f} = \vec{0}$$

- 2-norm

  $\mathbb{C}$ pseudo elasticity tensor with $\nu = 0.5$

- Clustering

  k-means

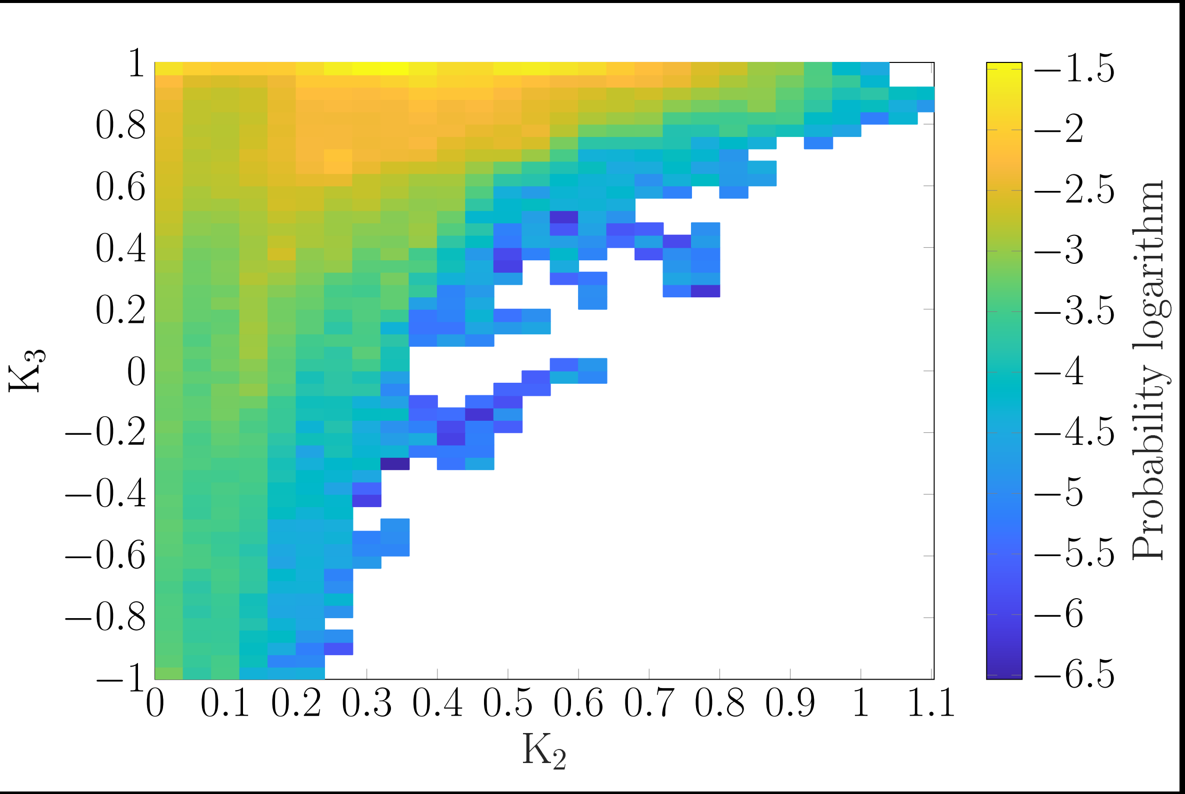
  $\bar{\bar{H}}$ - based

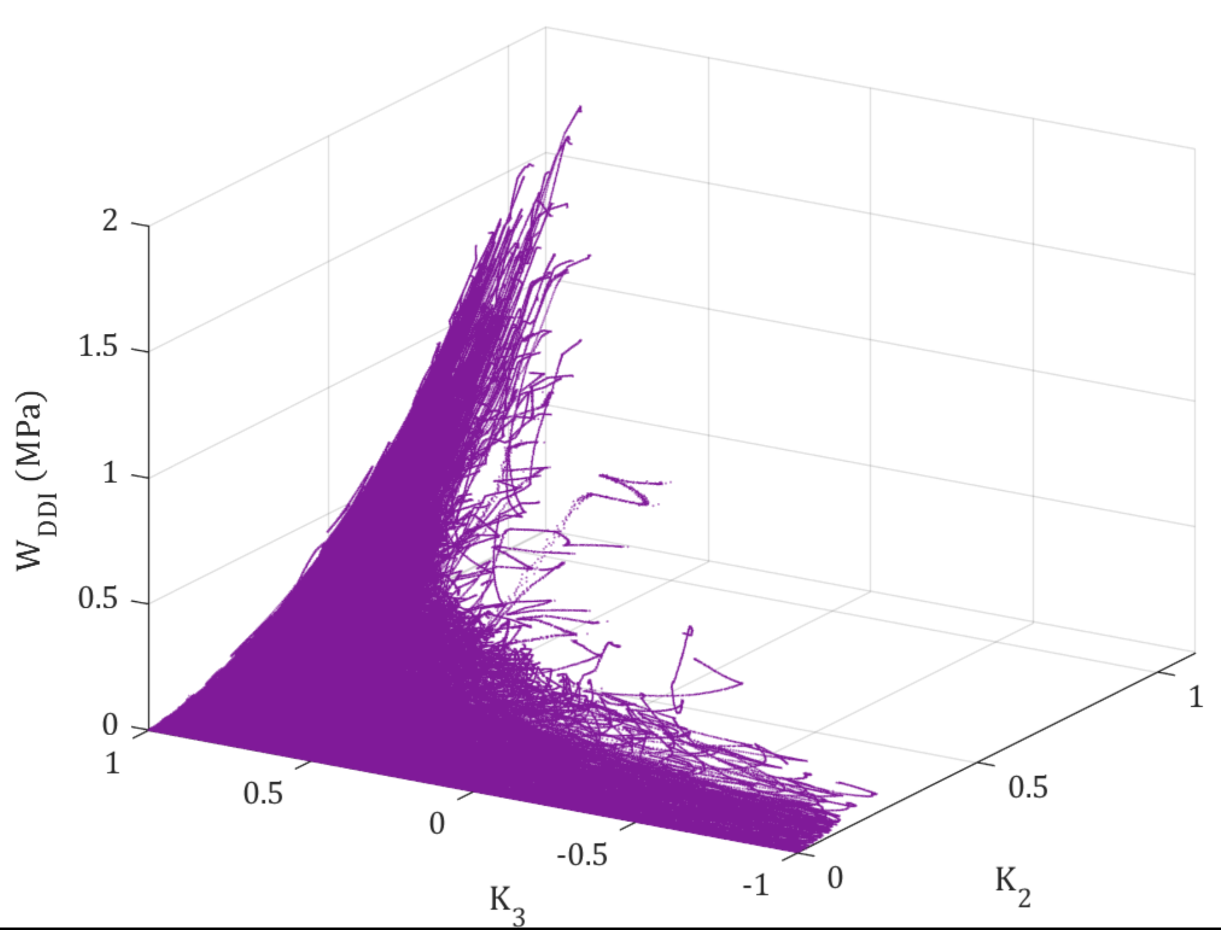
  ~3.5k clusters (ratio=1000)

- Ogden model

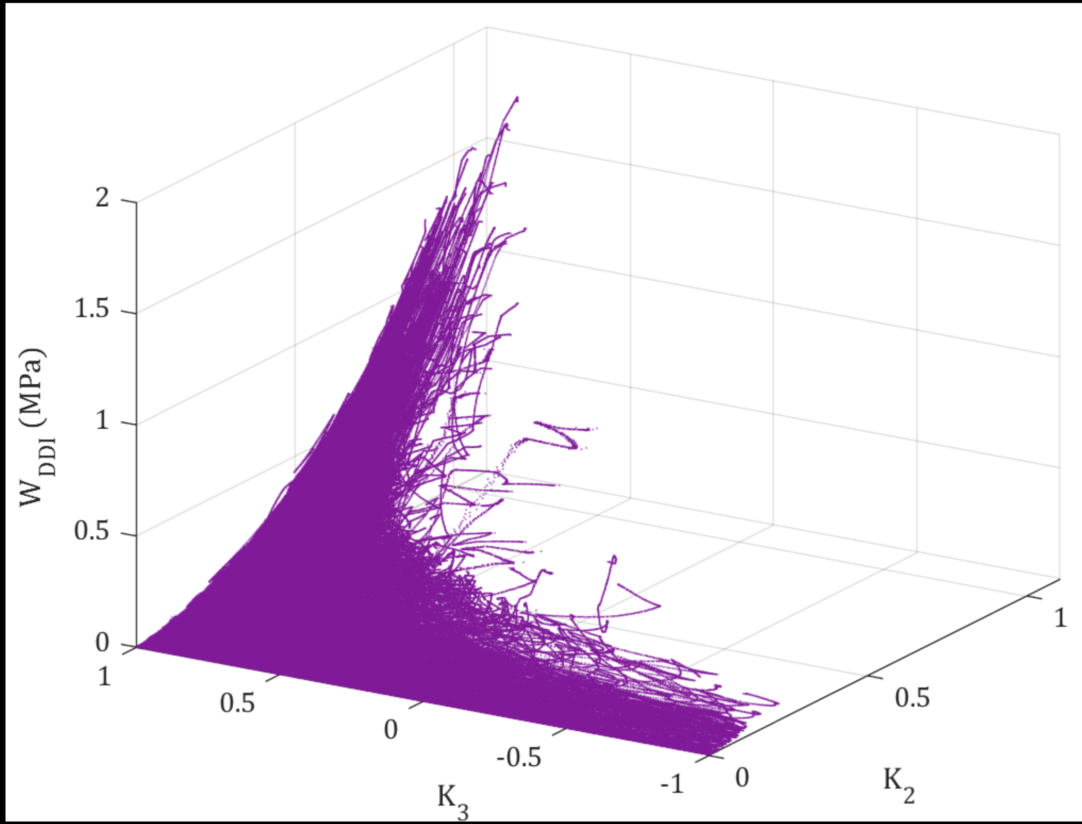
$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_p \frac{\mu_p}{\alpha_p} \left(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3 \right)$$

Kinematics & Identified energy





Fitting a model



Model: Ogden Model (3 terms)

$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_p \frac{\mu_p}{\alpha_p} \left(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3\right)$$

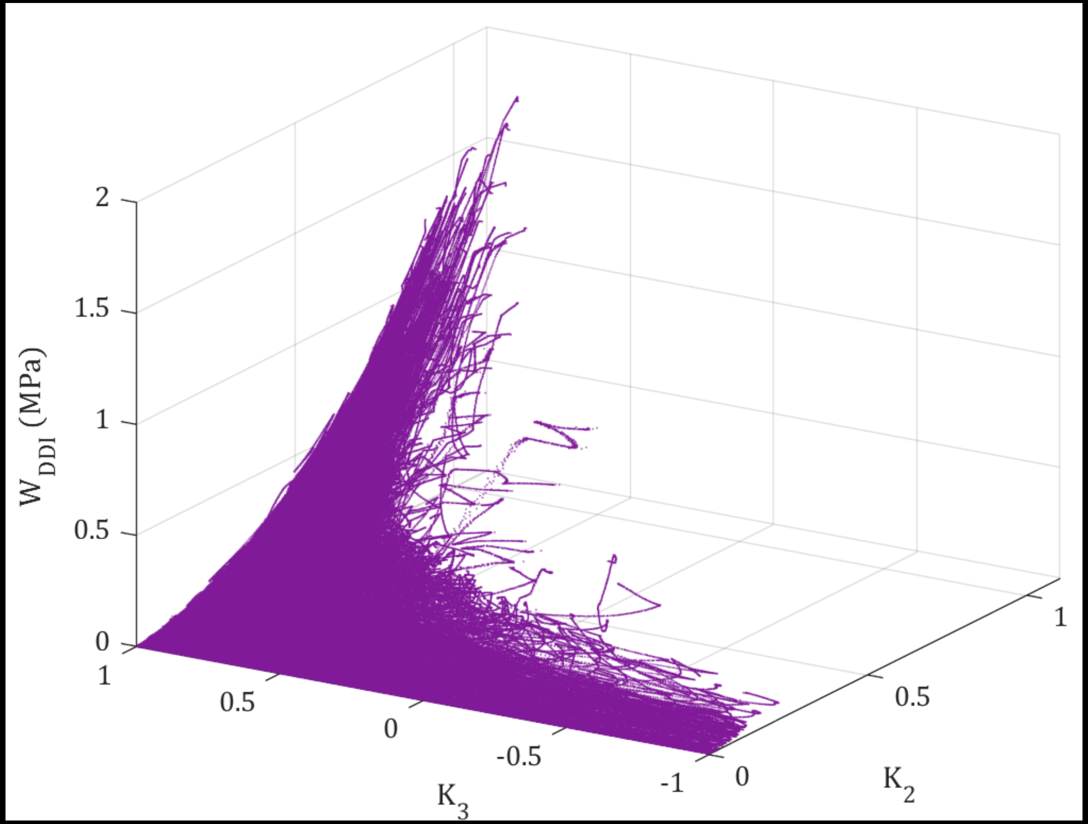
Fitting procedure:

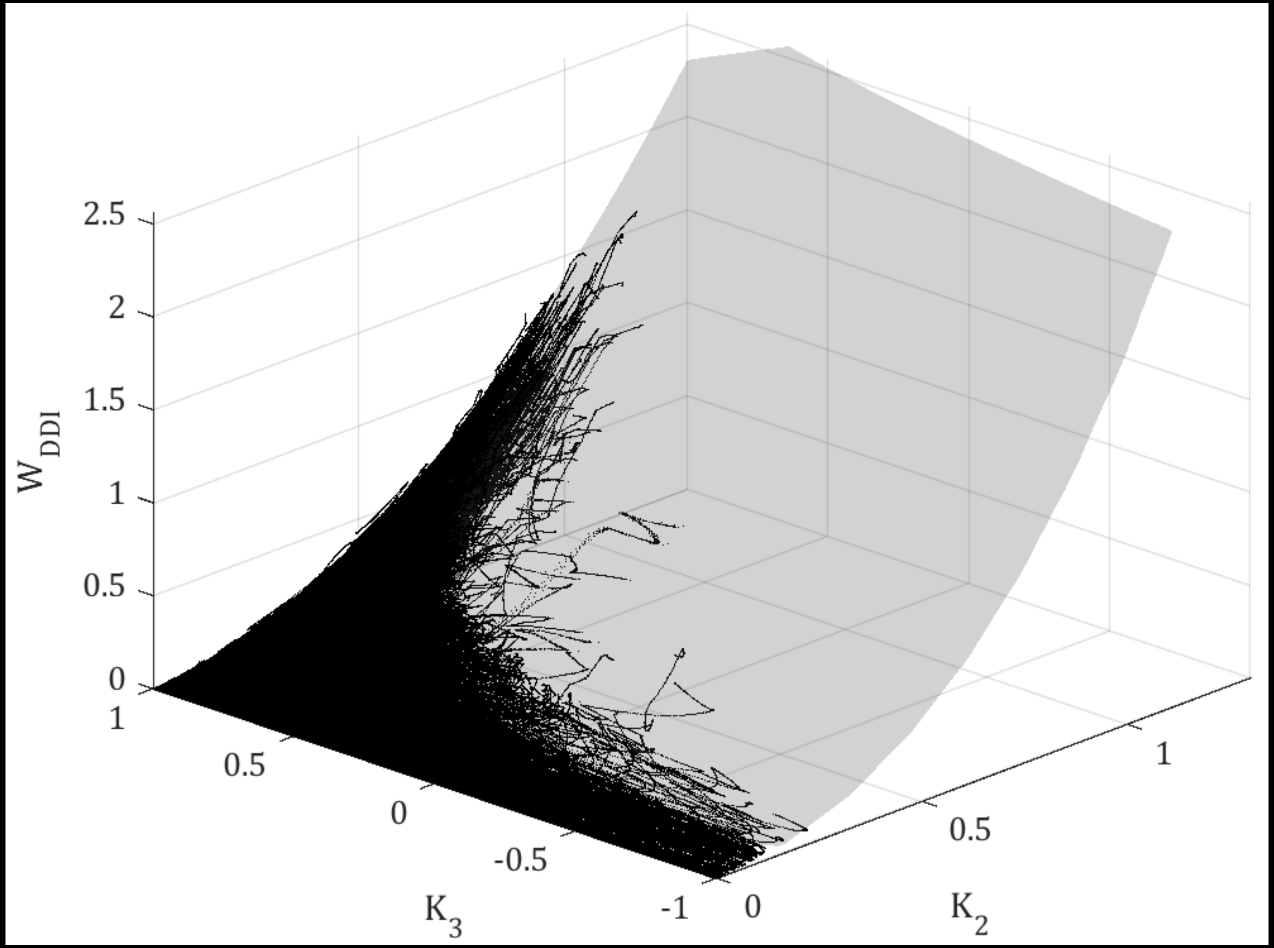
least square on all mechanical points

$$\alpha = \arg\min_{\alpha'} \int_\Omega \int_t \left(W_{DDI} - \Psi(\alpha')\right)^2$$

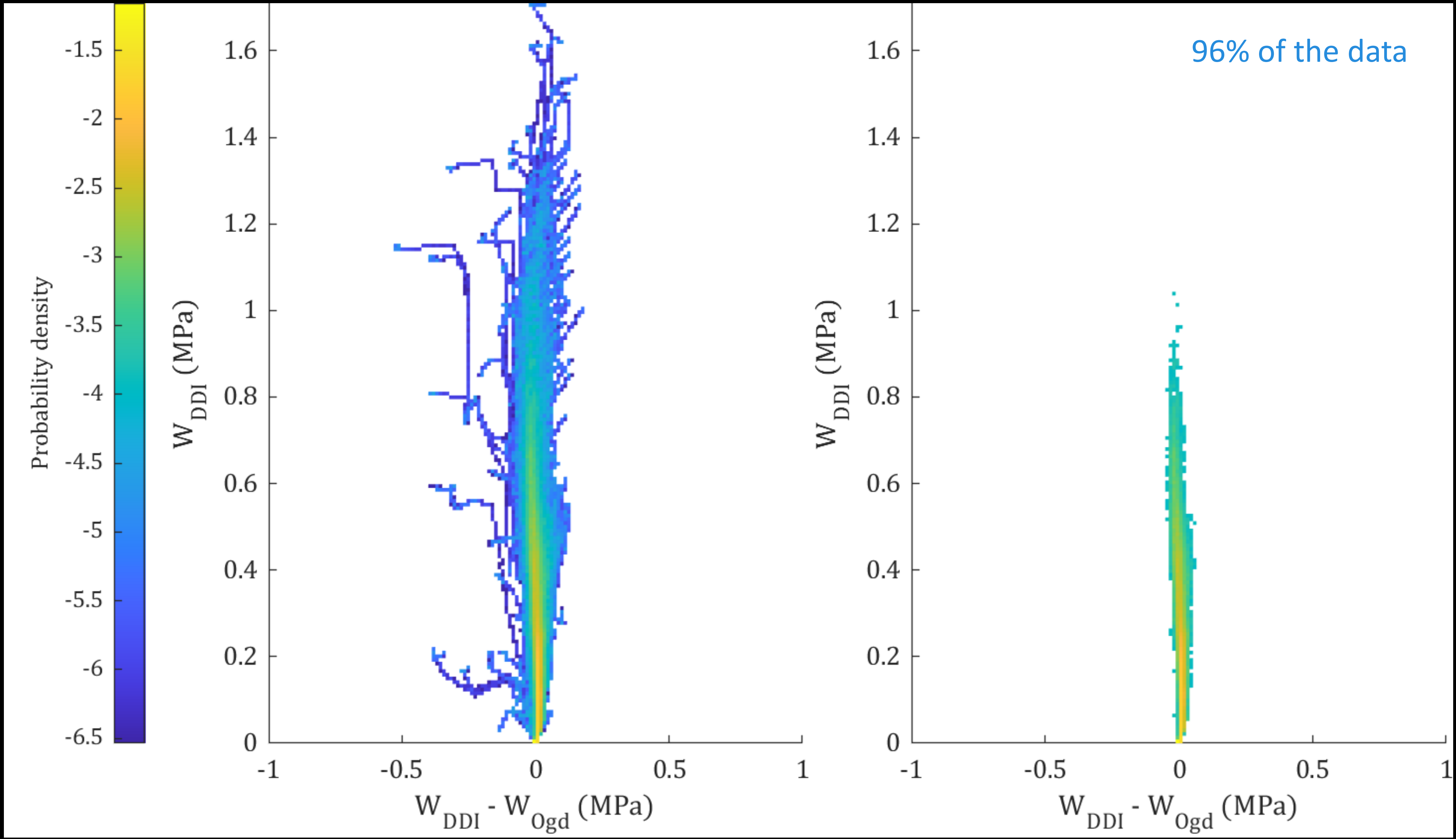
Fitting a model

$$\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_p \frac{\mu_p}{\alpha_p}\left(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3\right)$$

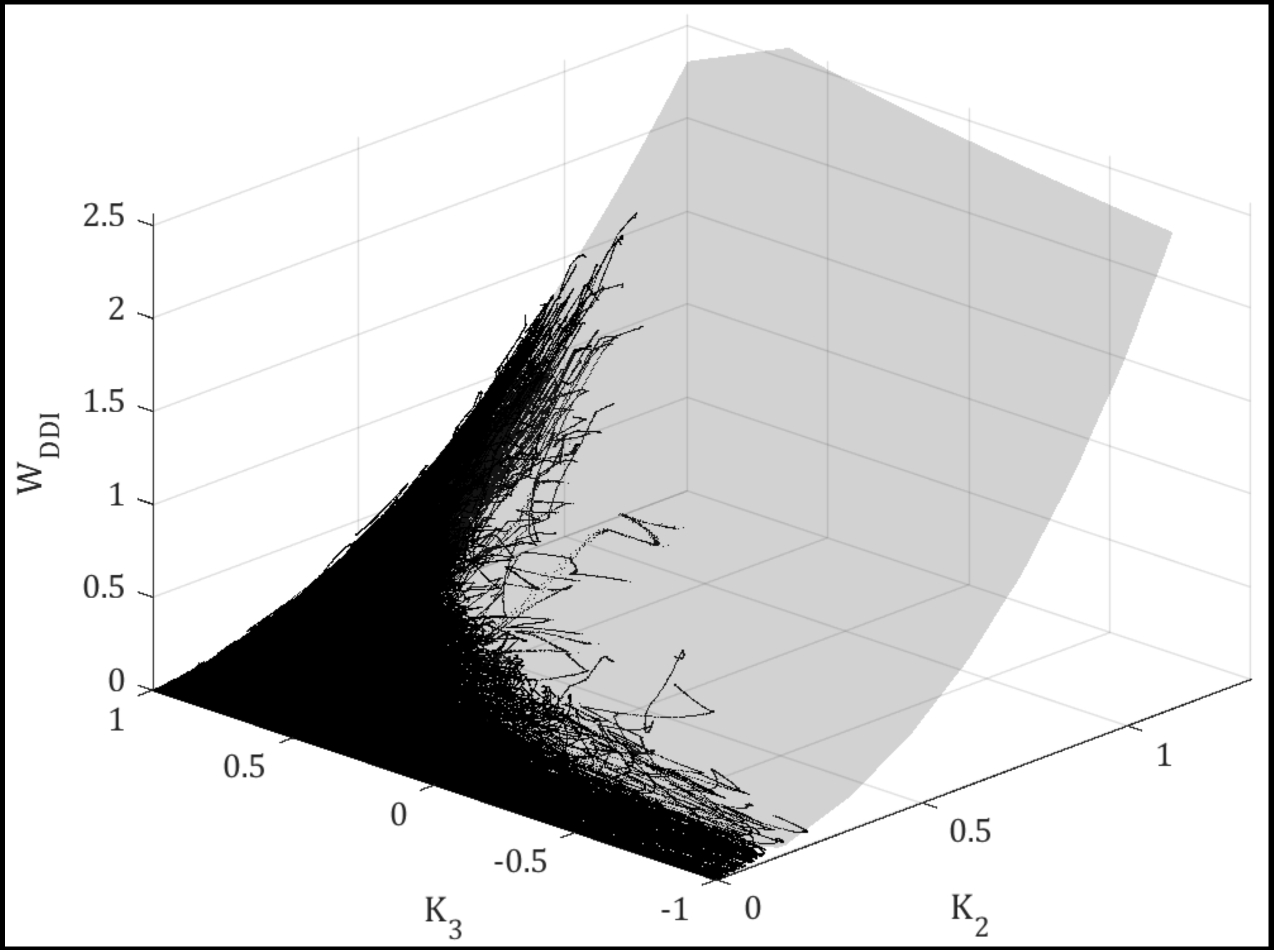


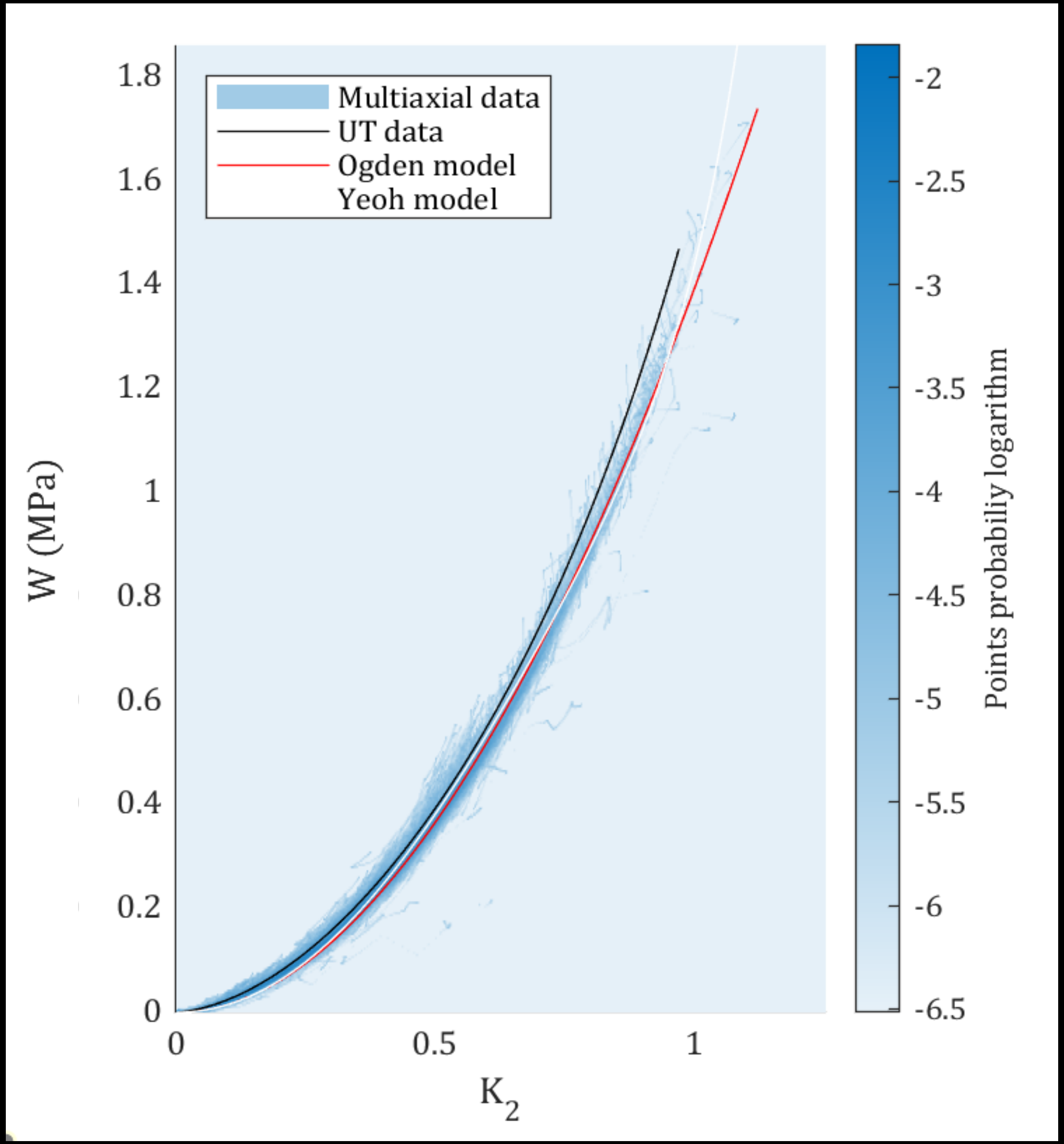


Fitting a model



Data-Driven (Model) Identification





Conclusions & perspectives

- Methods for estimating strain & stress in data-rich experiments:
  - Digital Image Correlation
  - Data-Driven Identification

- Preliminary theoretical results on model-free stress estimation
  - Efficient algorithms
  - Guidelines for optimal (hyper)parameters & experiment design

- Classical material models can easily be adjusted a posteriori
  - Cost !
  - Capitalizing on data

Collaborators

- Christophe Binetruy
- Michel Coret
- Léna Costecalde
- Marie Dalémat
- Héloïse Dandin
- Elie Eid
- Nour Hachem
- Raphaël Langlois
- Pierre-Yves Le Gac
- Maelenn Le Gall
- Hugo Madeira
- Annie Morch
- Auriane Platzer
- Julien Réthoré
- Rian Séghir
- Laurent Stainier
- Gabriel Valdes-Alonzo
- Erwan Verron
- Adrien Vinel