Identification of Mechanical Parameters using Physics-Informed Neural Networks (PINNs)

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<u>Colleagues :</u>

Origin/math.: H. Boulenc, J. Monnier (IMT, Toulouse) & P.A. Garambois (INRAE, Aix-en-Pce)

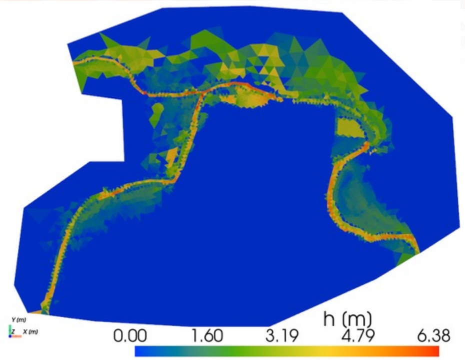
Mecha.: **E. Rabineau**, **R. Bonnet-Eymard** (ICA, Toulouse), **J. Réthoré** (GeM, Nantes)

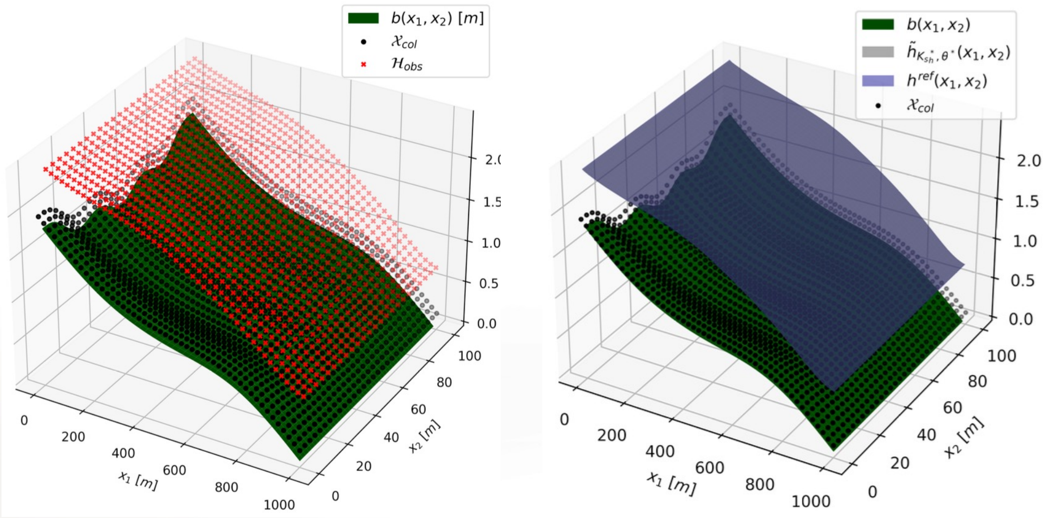
GTs Mécamat: *« Apprentissage profond »* & *« mesures de champs et identification »*
May 27, 2025 (from Toulouse)

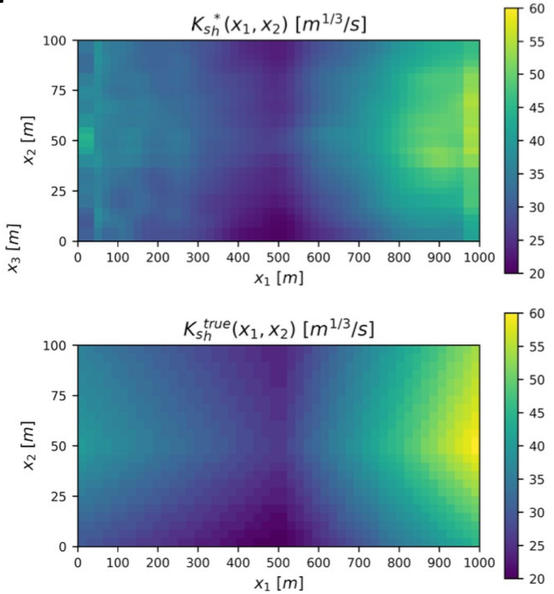
Motivation & inspiration

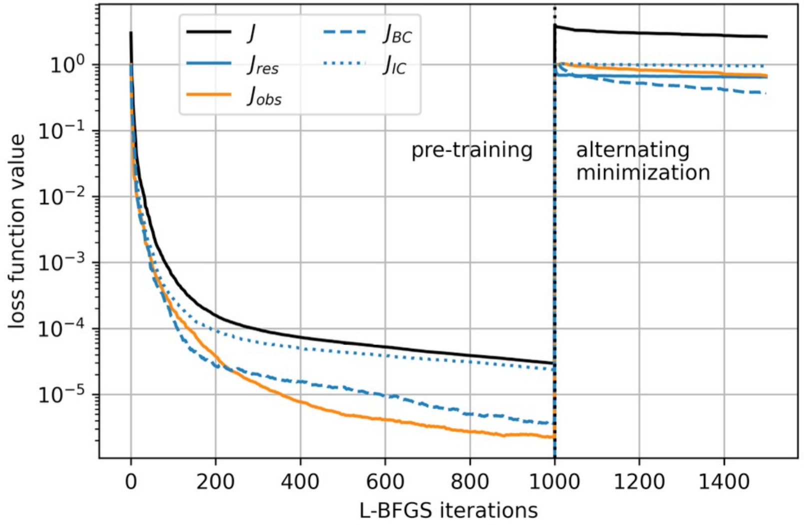
High-dimensional data assimilation using Neural Networks

- **Context:** Identification of the **spatially-distributed** friction **coefficient**
  - ➡ **Shallow-Water equations** (nonlinear, dynamic)
- **Method: Semi-Parameterized PINN (PINN** *[Raissi et al. 19]***)**
  where the physical parameters are treated as NN parameters









➡ **Parameter** space of **dim. 1000**; identification in **2 min** on a standard 6GB-GPU machine (compared to 5h with VDA methods).

Boulenc, H., Bouclier, R., Garambois, P.-A., and Monnier, J.(2025). Spatially-distributed parameter identification by physics-informed neural networks illustrated on the 2D shallow-water equations. *Inverse Problems,* 41, 035006.

Outline

1. **PINN-based material identification from classical FEMU**
   - ➡ Direct appli. of PINN, experimental mech., comparison with FEMU
2. **Appli. 1: synthetic beam with distributed *E(x)***
   - ➡ Fourier features, mechanical regularization, fixed point
3. **Appli. 2: real 2D case with cst *E* and ν**
   - ➡ Real experimental context, accuracy vs FEMU
4. **Appli. 3: synthetic 2D case with distributed *E(x,y)***
   - ➡ Mixed formulation, potential for efficient and accurate identif.

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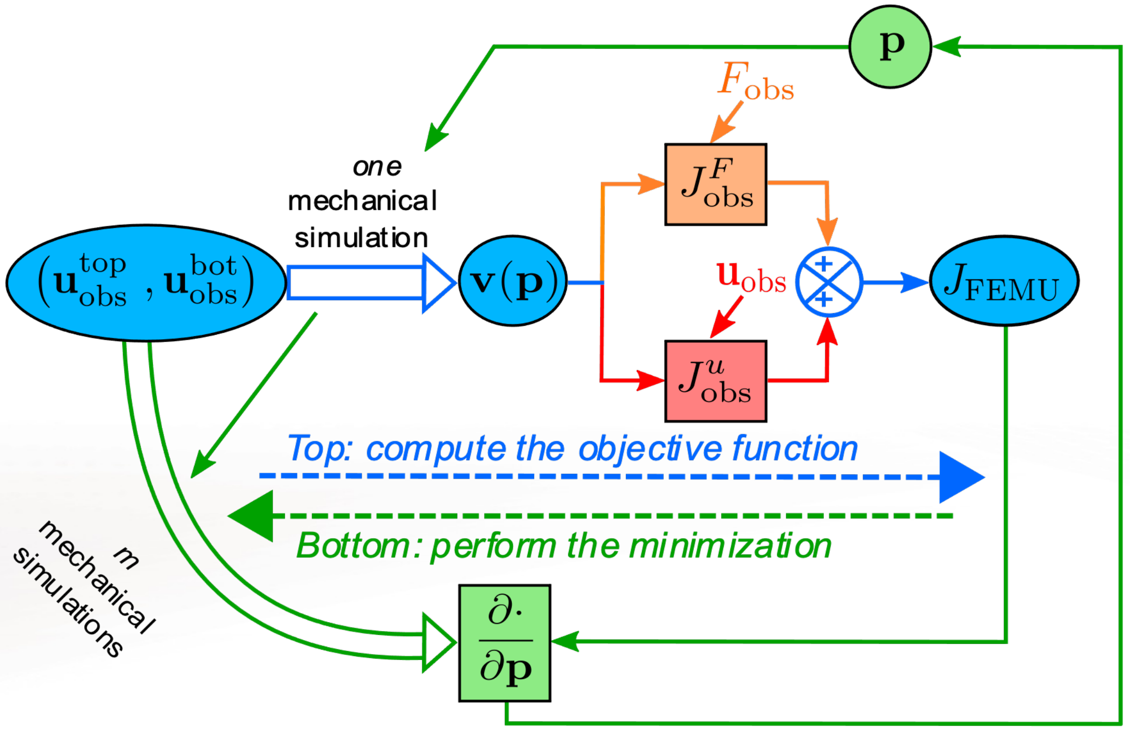
Brief overview of FEMU (3/3)

Concluding remarks on the FEMU approach



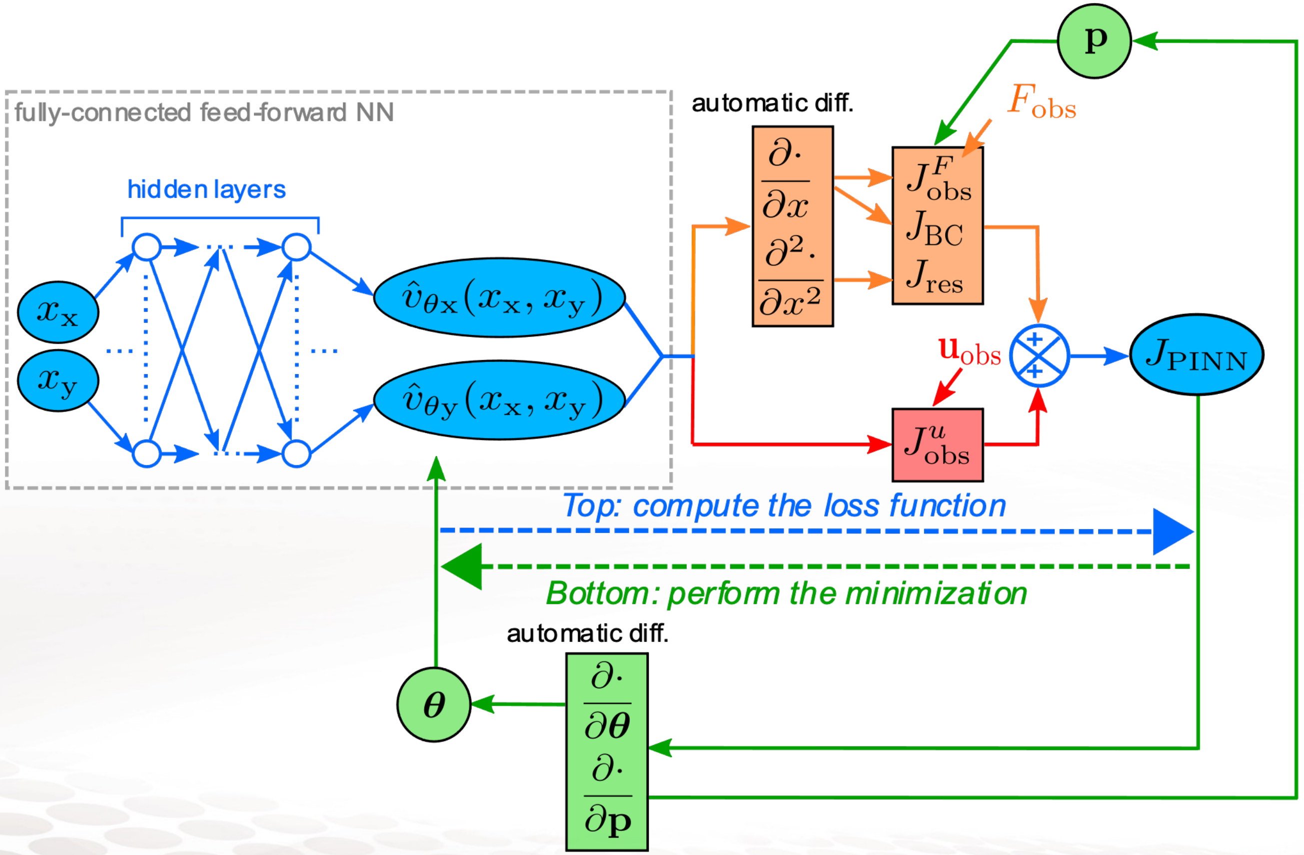
- **Mechanics strongly enforced** (best mechanics that fits data)
- May be **computationally expensive** when:
  - m is large (spatially varying properties)
  - Solving a single mechanical problem is costly
- Importance of the **measured disp. boundary**

In contrast, the PINN-based approach will somehow **bypass the direct solution** of the mechanical problem

Direct application of inverse PINN (3/3)

[Origin: Raissi et al. 19, Appli. Mech.: Wu et al. 23, Di Lorenzo et al. 23, Wei et al. 23, Motlagh et al. 25, etc]

Concluding remarks on the PINN approach (*w.r.t.* FEMU)

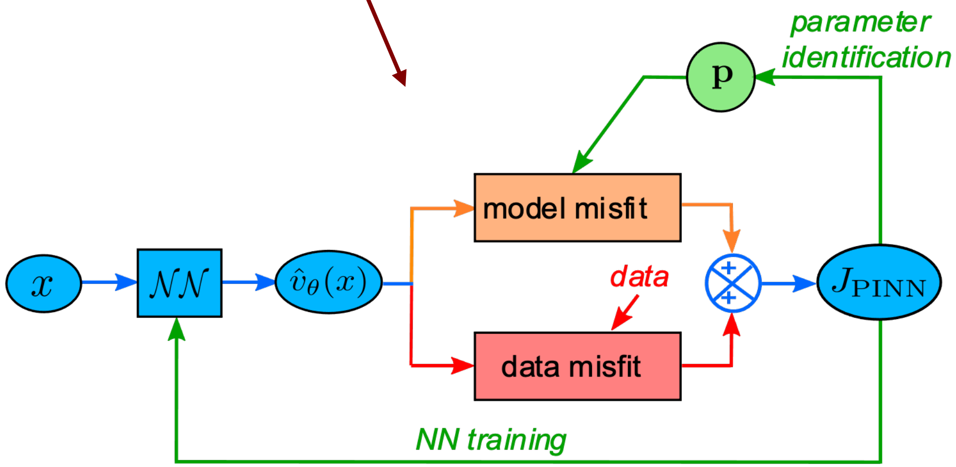


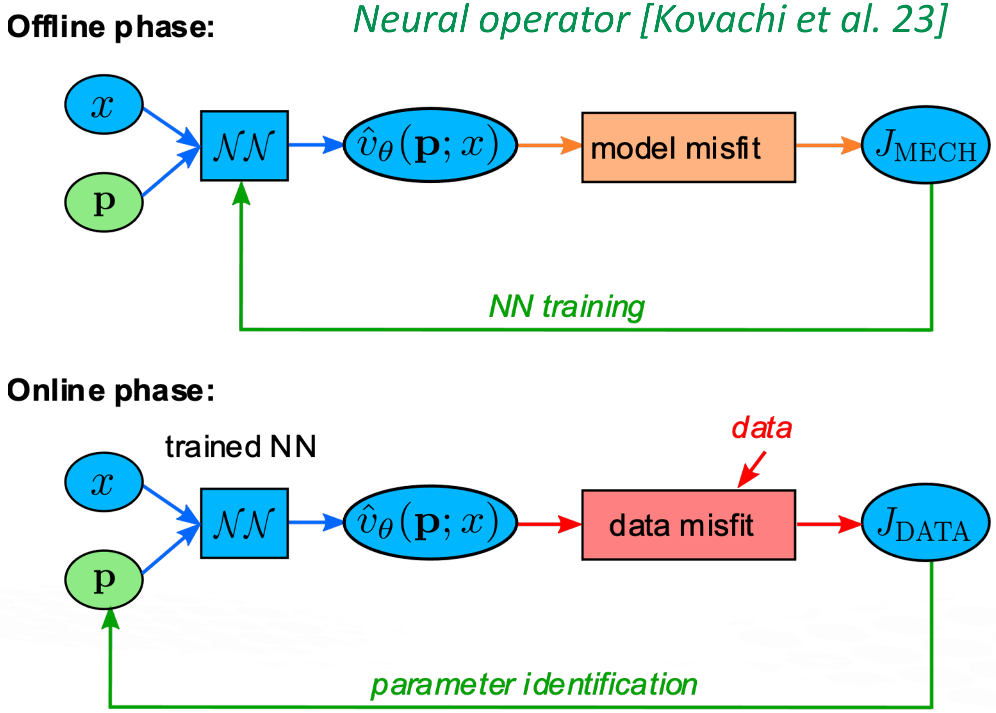
- The parameters ***p*** are treated as **trainable variables** of the **NN**
- Solution with **available robust optim.** algos. *(e.g., PyTorch)*
  - ⇒ Derivatives accurate and efficient with **automatic diff.**
  - **High-dimensional *p*** not a pb. (still $m << n_\theta$ & NN efficient)
- Solution of (multiple) **mechanical pbs replaced by learning a single function with a NN**
  - ⇒ May be **advantageous** when m **large**, & **complex mechanics**
  - ⇒ Requires **careful NN training** for good accuracy (*w.r.t.* FEMU)
- Reduce the impact of **boundary disp. measurements**

Comparison with *surrogate* modeling

Semi-Parameterized PINN VS Fully-Parameterized PINN

PI-DeepONet [Wang et al. 21, Goswami et al. 22]
Neural operator [Kovachi et al. 23]





We seek for a **single function** $\hat{v}_\theta$ associated with a **unique set** of $\mathbf{p}$, all within a **single training phase**

Offline: costly learning of the full mechanical operator $\mathbf{p} \mapsto \hat{v}(\mathbf{p}, x)$

Online: use the surrogate model for **rapid inference** of $\mathbf{p}$

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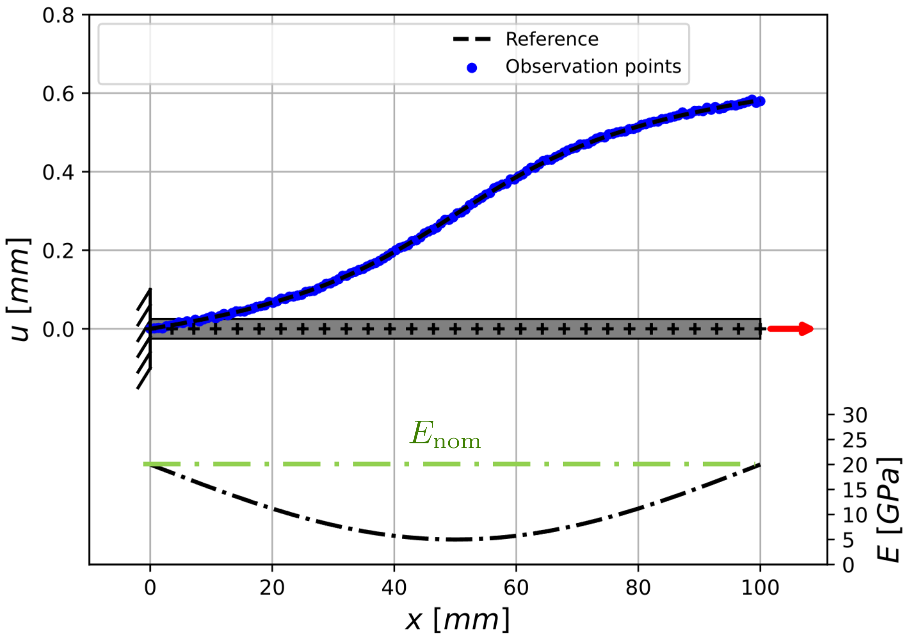
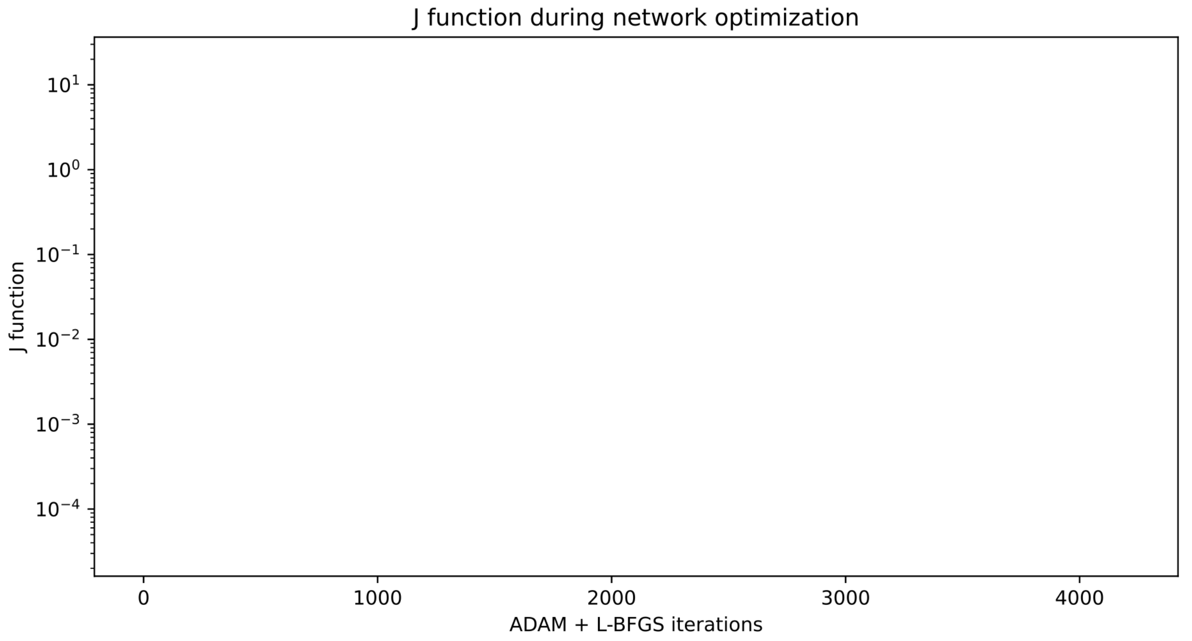
Results: beam with distributed E (1/4)

A rather smooth distribution: a sinusoidal *E(x)* ➡ $$E(x) = E_{\text{nom}}\left(1 - \frac{3}{4}\sin\left(\frac{\pi}{L}x\right)\right)$$

NN disp. solution *u(x)* and calibrated *E(x)*



Loss functions minimization during initialization and training



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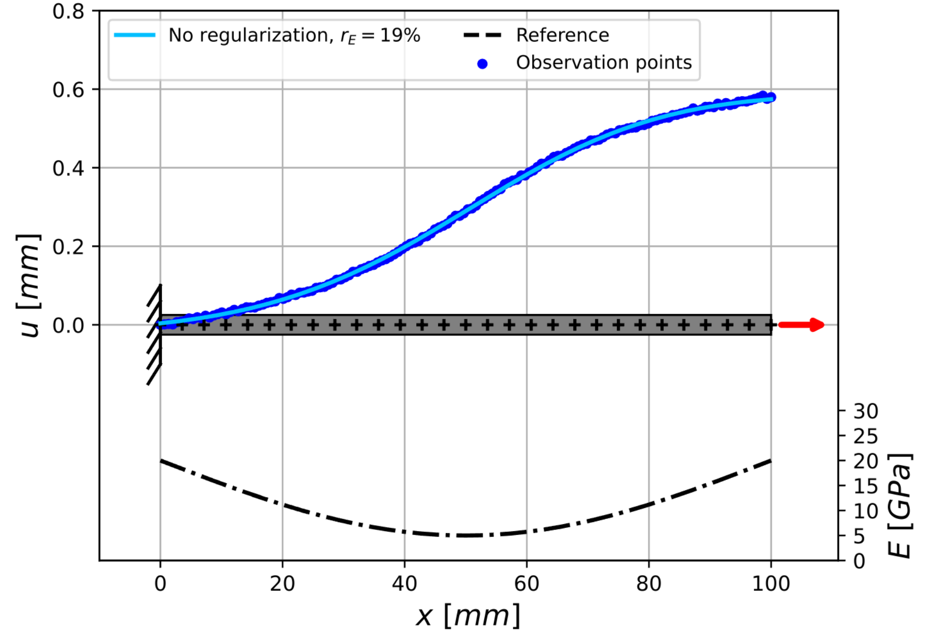

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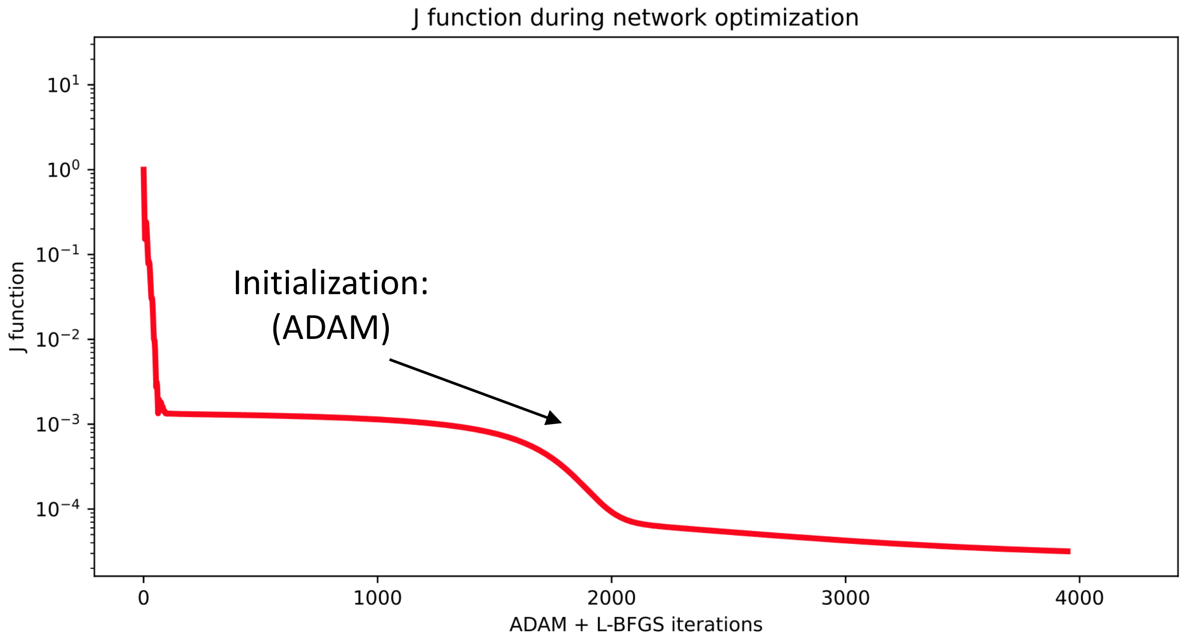

We are able to **capture the solution after initializing** even **without Fourier features**, because the latter is smooth.

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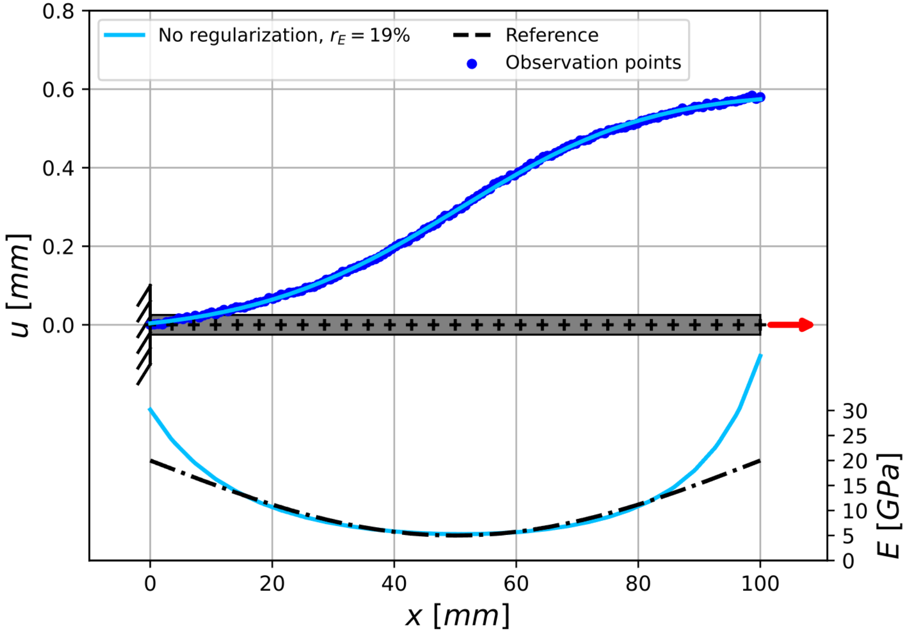

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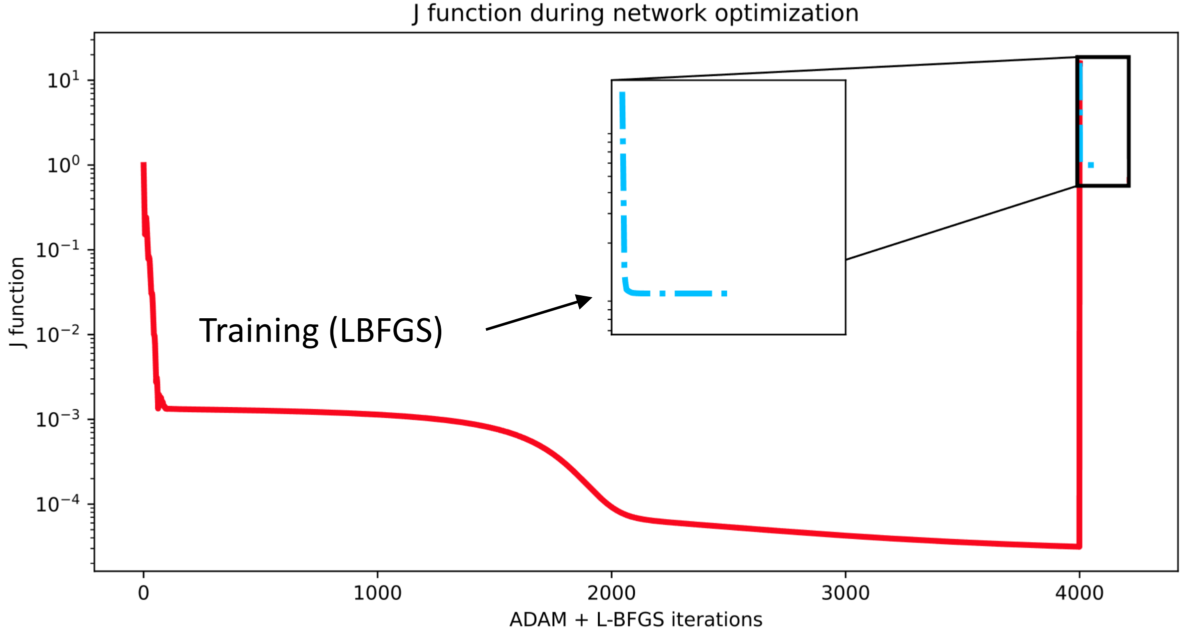

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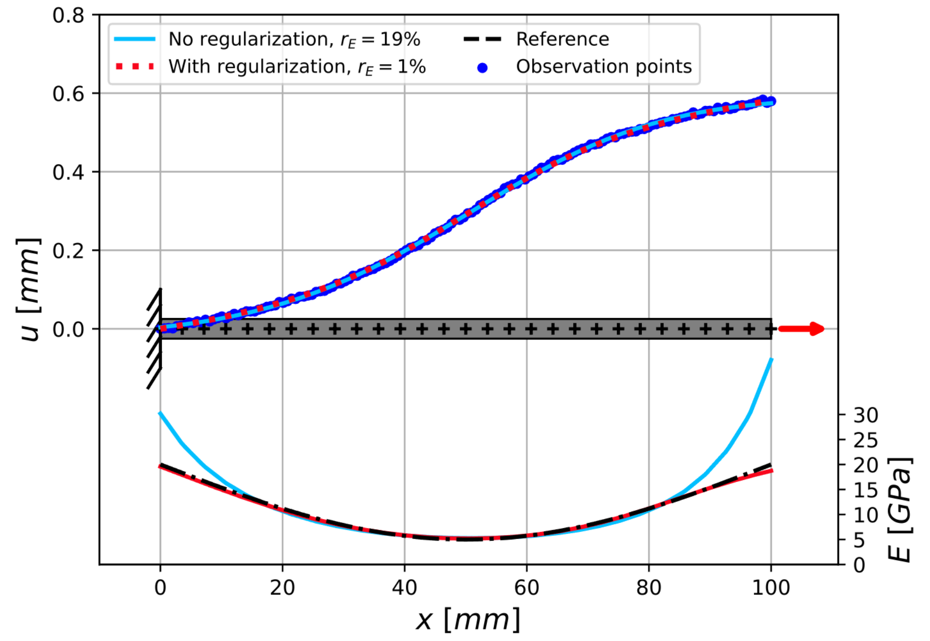

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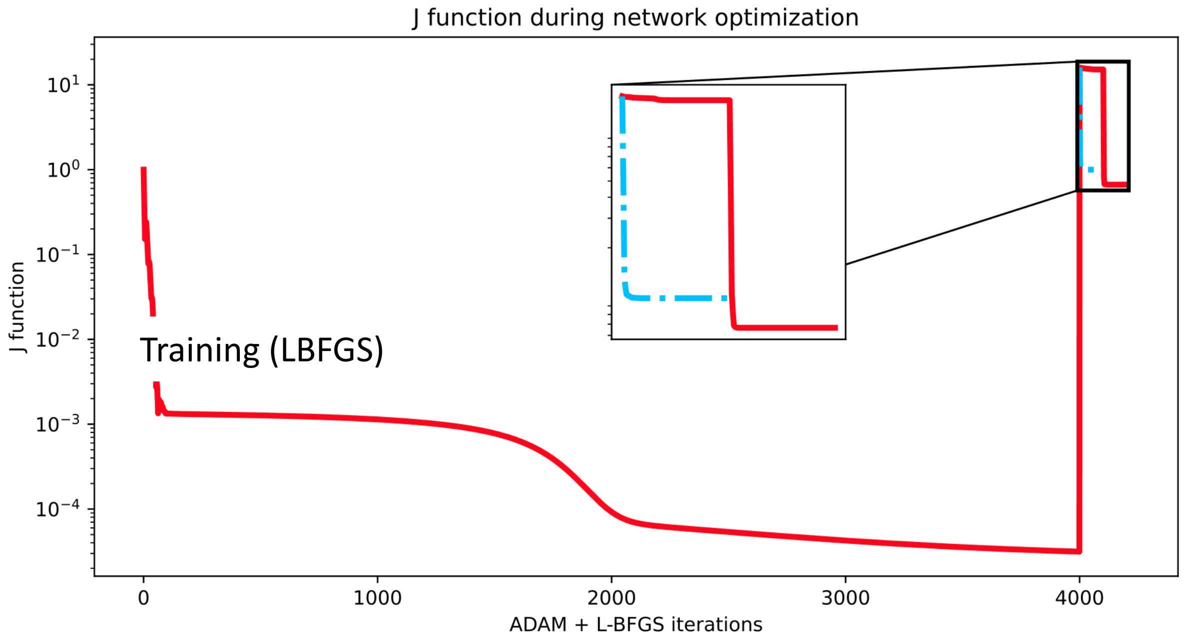


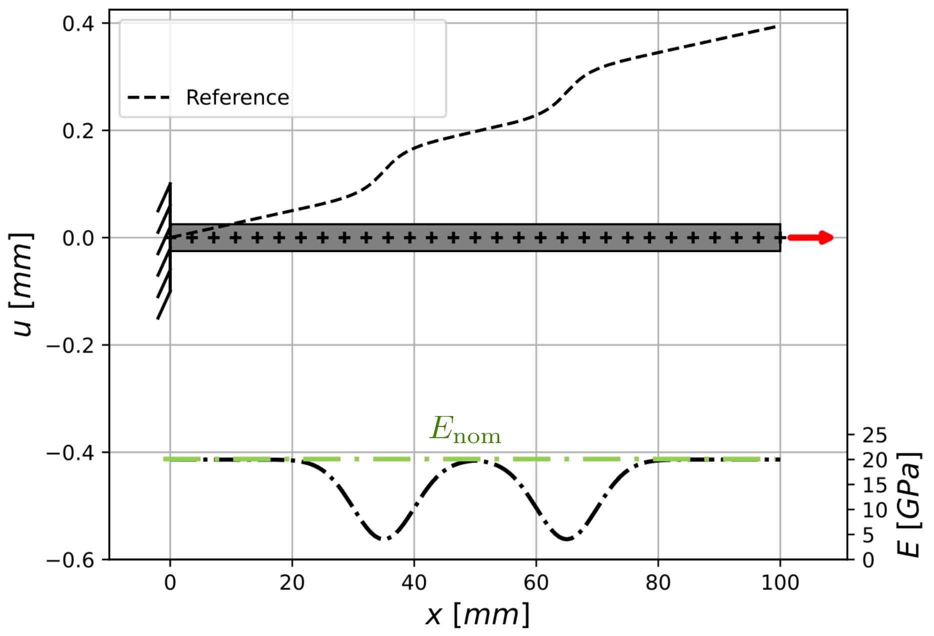
We are able to **capture the solution after initializing** even **without Fourier features**, because the latter is smooth.
We do **not manage to directly capture *E(x)***, especially near the boundaries.
With mechanical regularization, we are able to **capture properly *E(x)***, with only 1 fixed-point iteration.

Results: beam with distributed *E* (2/4)

A sharper distribution: two-Gaussian for *E(x)*

$$E(x) = E_{\text{nom}}\left(1 - \left(\eta_{[0,L/2]}\mathcal{N}(\overline{x} = 35, \gamma = 5) + \eta_{[L/2,L]}\mathcal{N}(\overline{x} = 65, \gamma = 5)\right)\right)$$

NN disp. solution *u(x)* and calibrated *E(x)*



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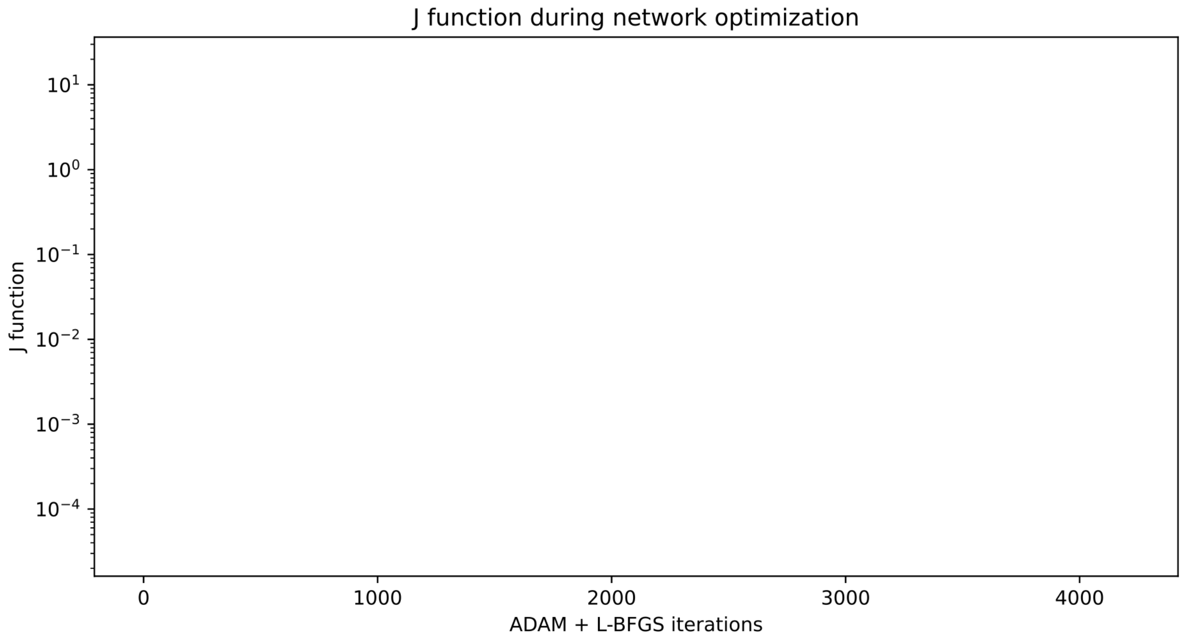

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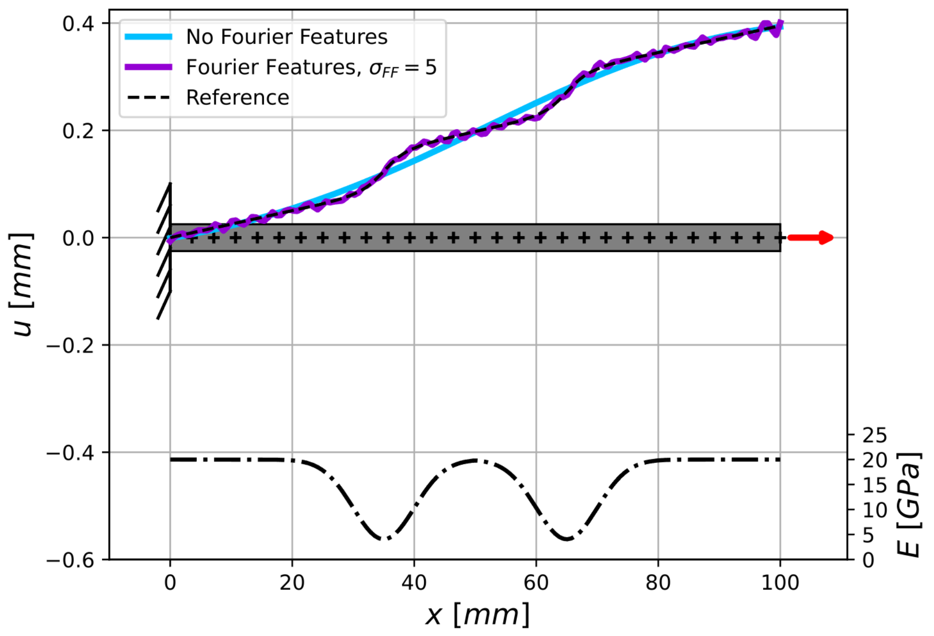

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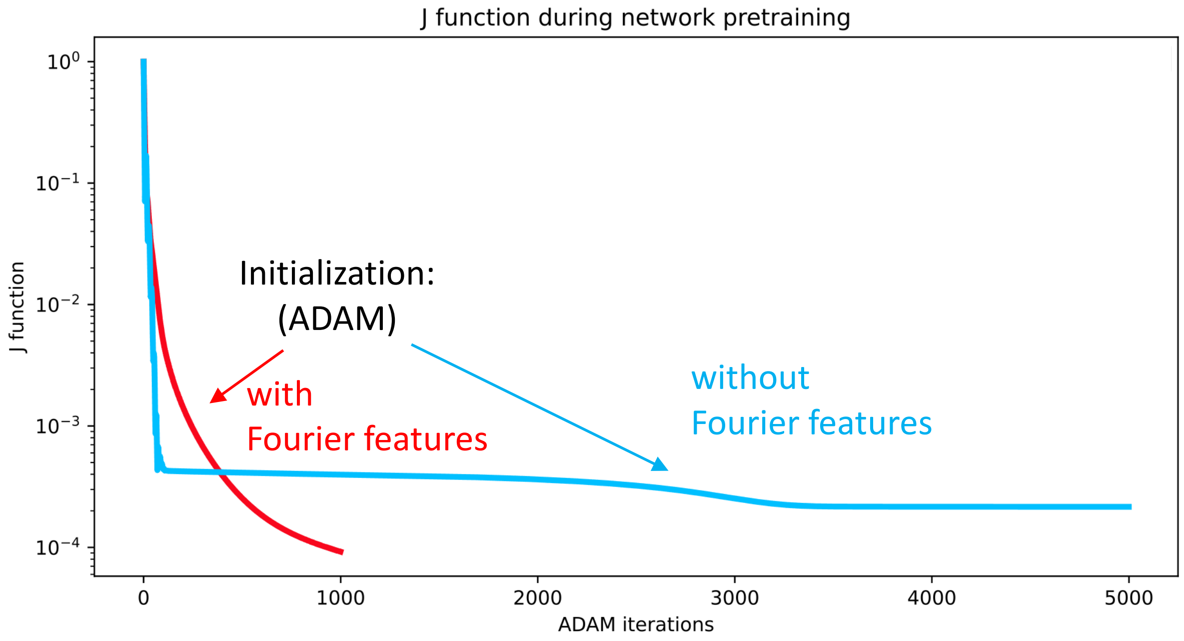

In this case, **Fourier features** are required **to capture the locally varying solution.**

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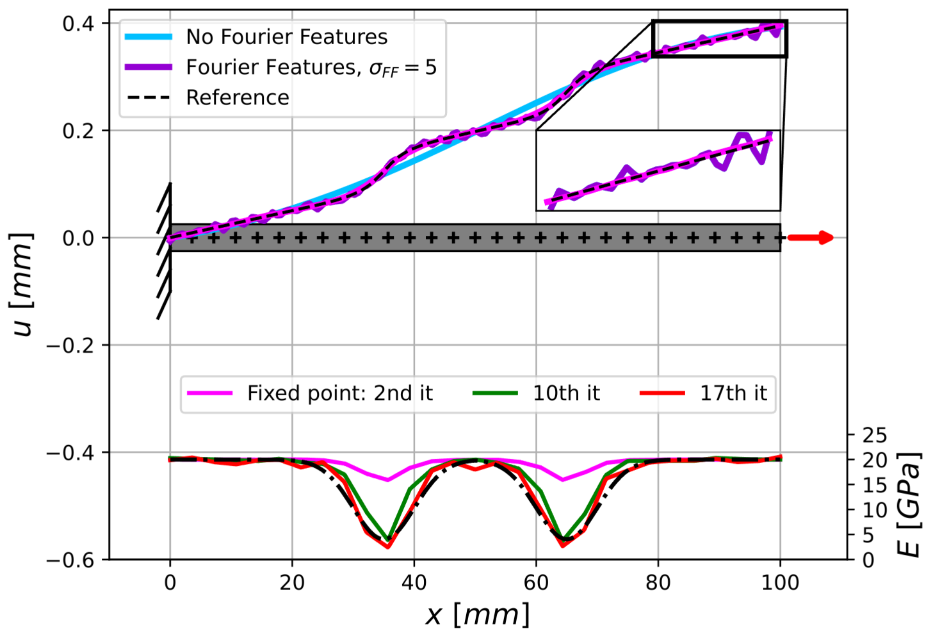

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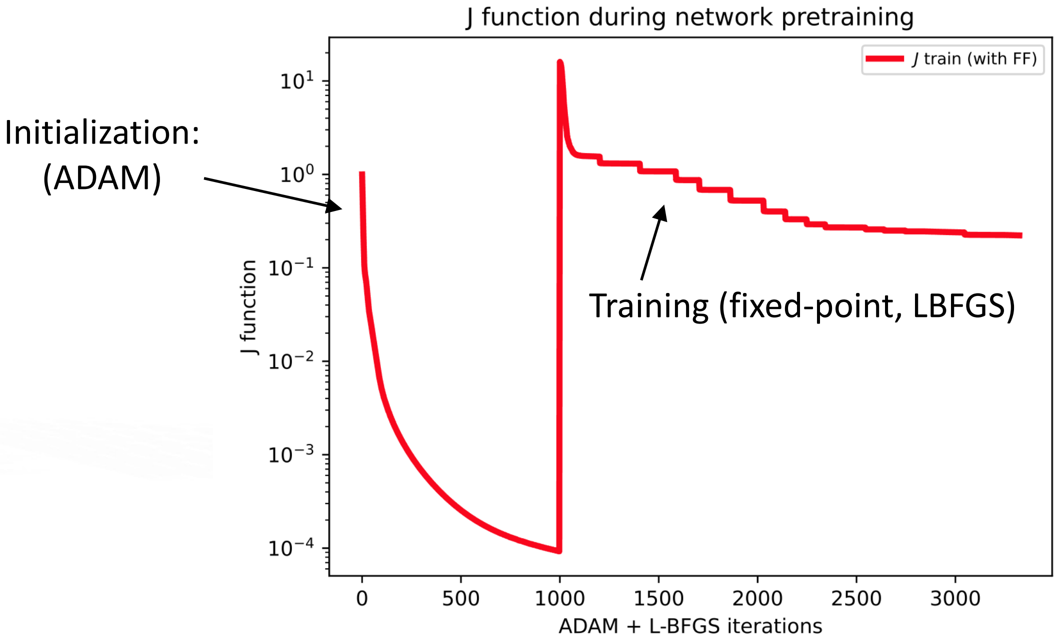


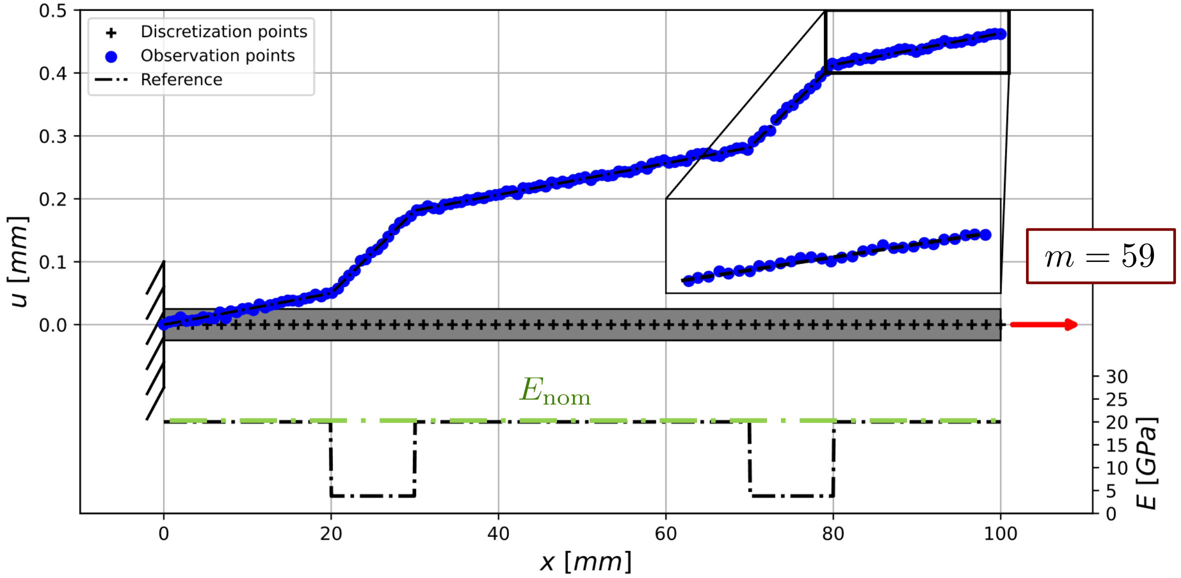
In this case, **Fourier features** are required **to capture the locally varying solution.**
The **oscillations** introduced by the Fourier features **can be mitigated through mechanical regularization.**
This allows to **correctly capture *E(x)* during the fixed-point iterations.**

Results: beam with distributed E (3/4)

A singular distribution: two-step for *E(x)*

$$E(x) = E_{\text{nom}}\Big(1 - \big(\eta_{[0.2L,0.3L]} + \eta_{[0.7L,0.8L]}\big)\Big)$$

NN disp. solution *u(x)* and calibrated *E(x)*



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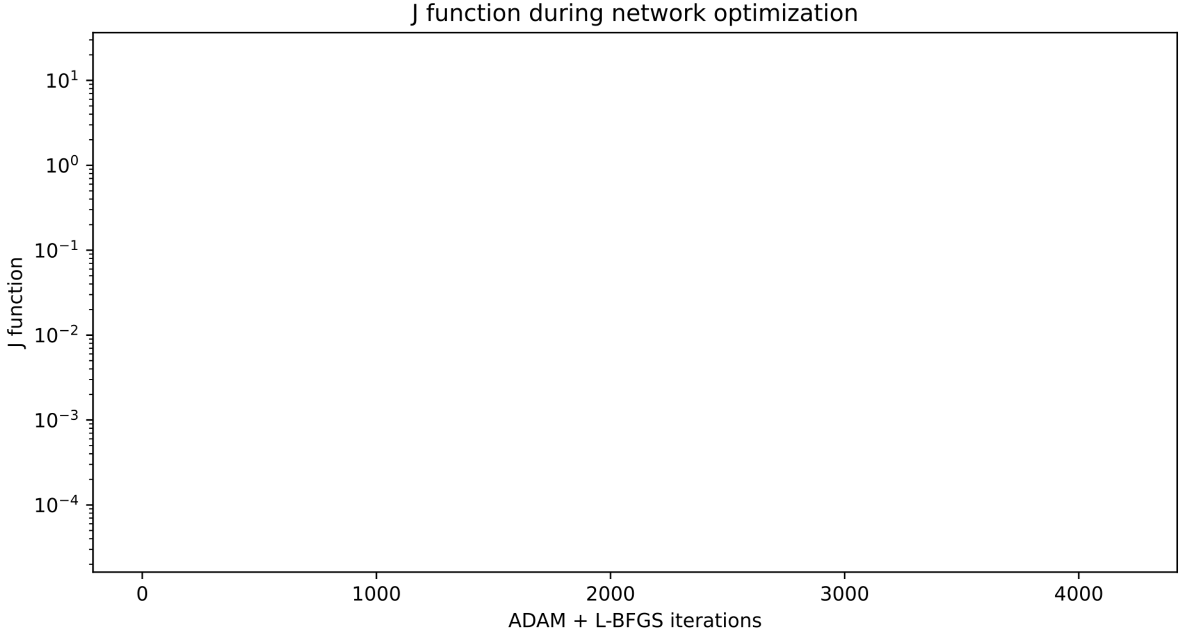

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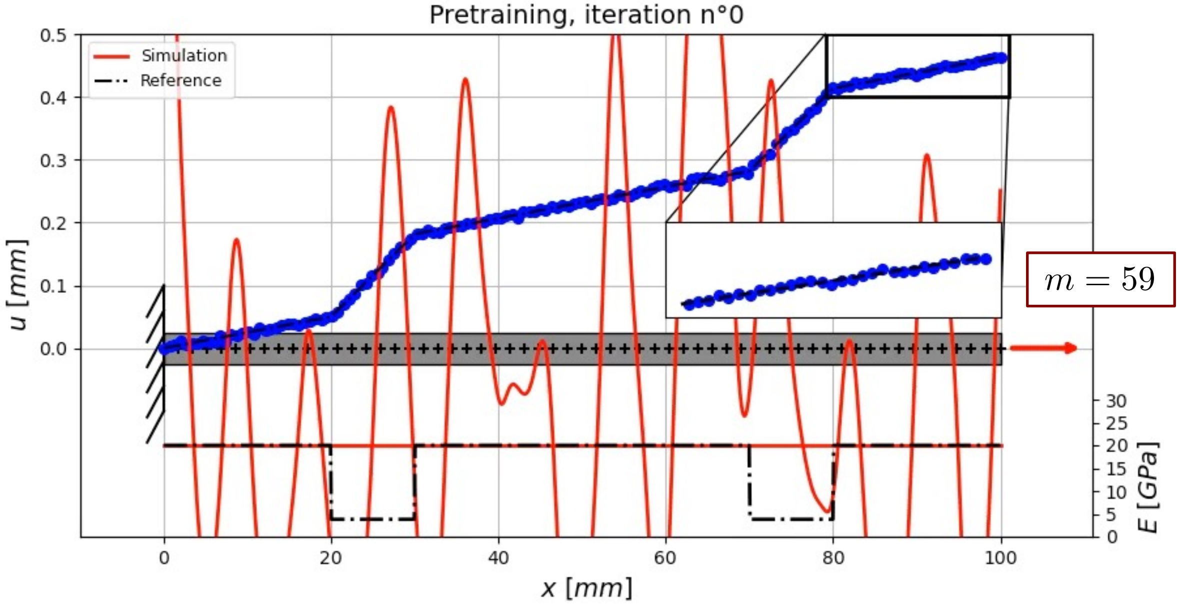

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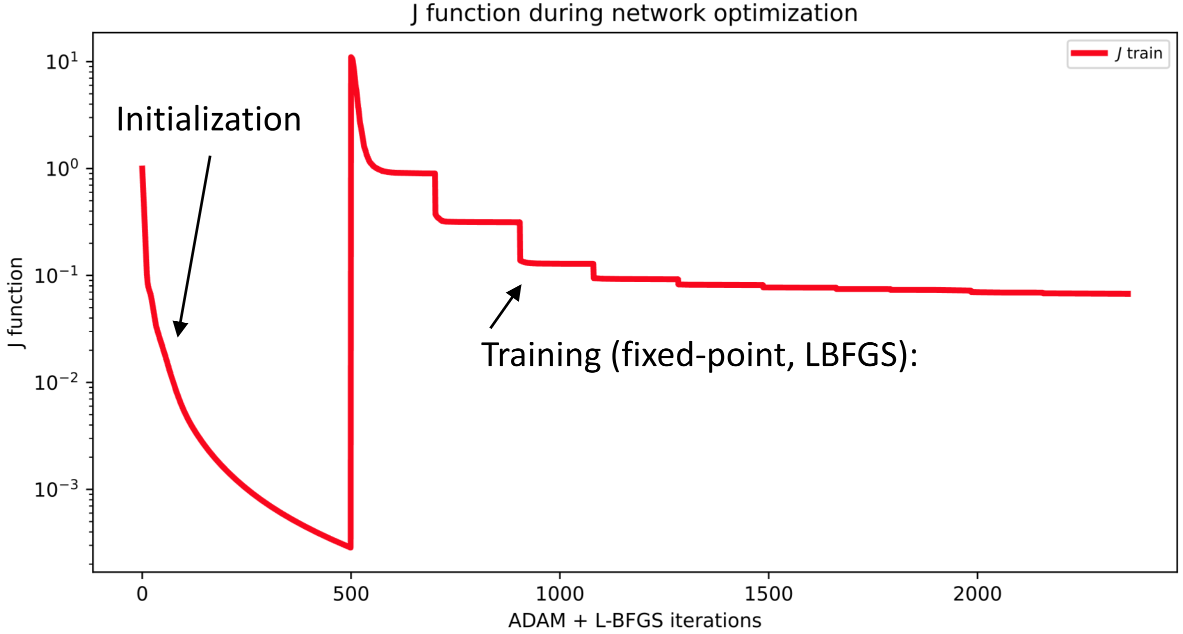


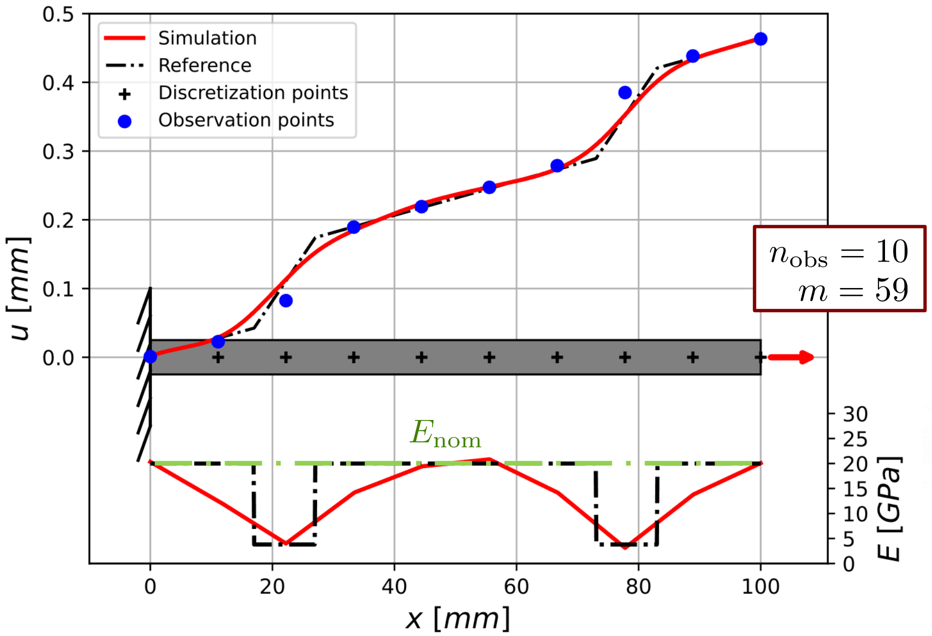
Same key components: **Fourier features** + **mechanical regul.** + **fixed point**.

Results: beam with distributed E (4/4)

Study of the robustness: two-step for $E(x)$

$$E(x) = E_{\text{nom}}\Big(1 - \big(\eta_{[0.2L,0.3L]} + \eta_{[0.7L,0.8L]}\big)\Big)$$

- Only one obs. and one element per step



We are able to **identify the weakened region of E** using a **single obs. & just one element**.

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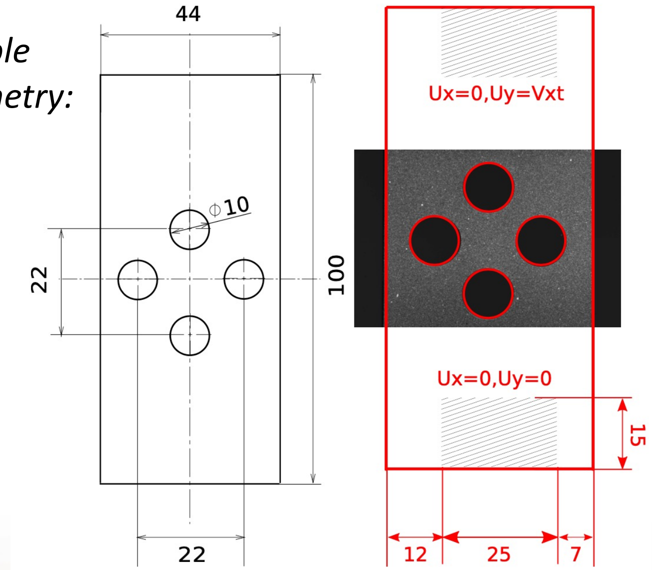
Appli. 2: real 2D case with cst E and ν (1/2)

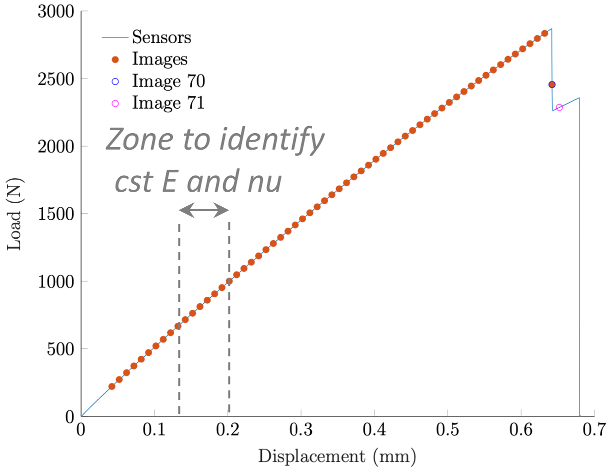
Example: traction test on a plate with several holes

Experimental setup

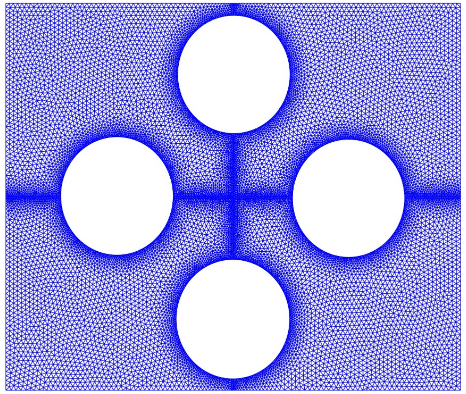
Sample geometry:



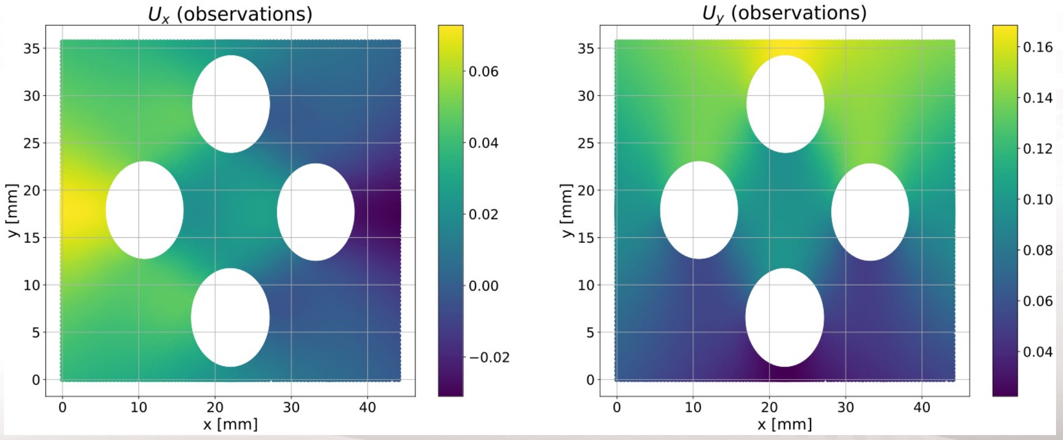
Loading



FE mesh



Measurements



- Material: PMMA
- Image resolution: 28.8 Mpix (6576x4384)
- Image scale: 1 Pix = 8,59 micron
- DIC software: Ufreckles *[Réthoré 18]*
- mesh: 50 Pix/el (refined: 16 Pix/el), 13,426 nodes)
- Data processing: Tikhonov regul. (cut-off of 50 Pix)

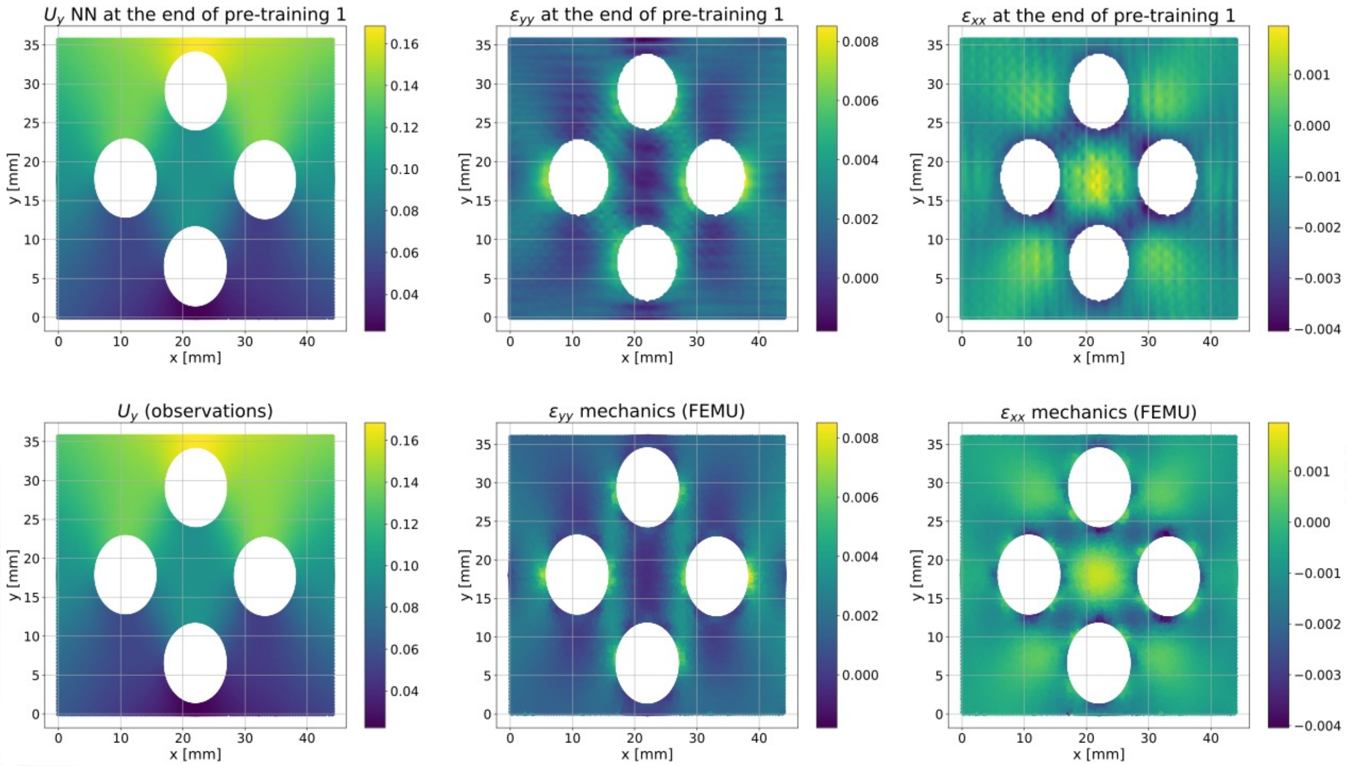
Results: real 2D case with cst E and ν (1/5)

Results at the end of initialization

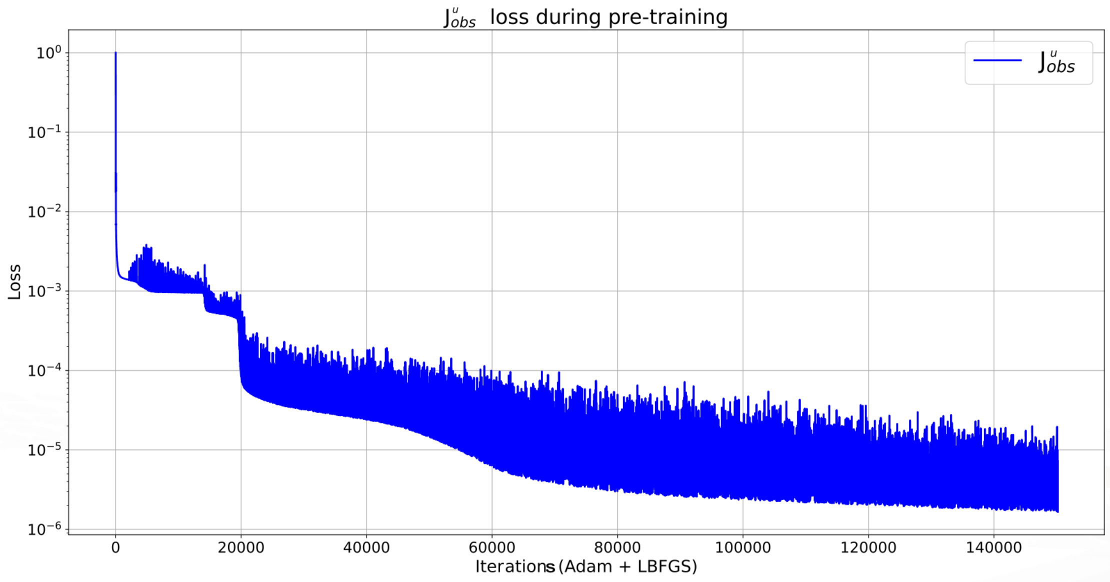
Contour plots of the solution

NN:

ref:



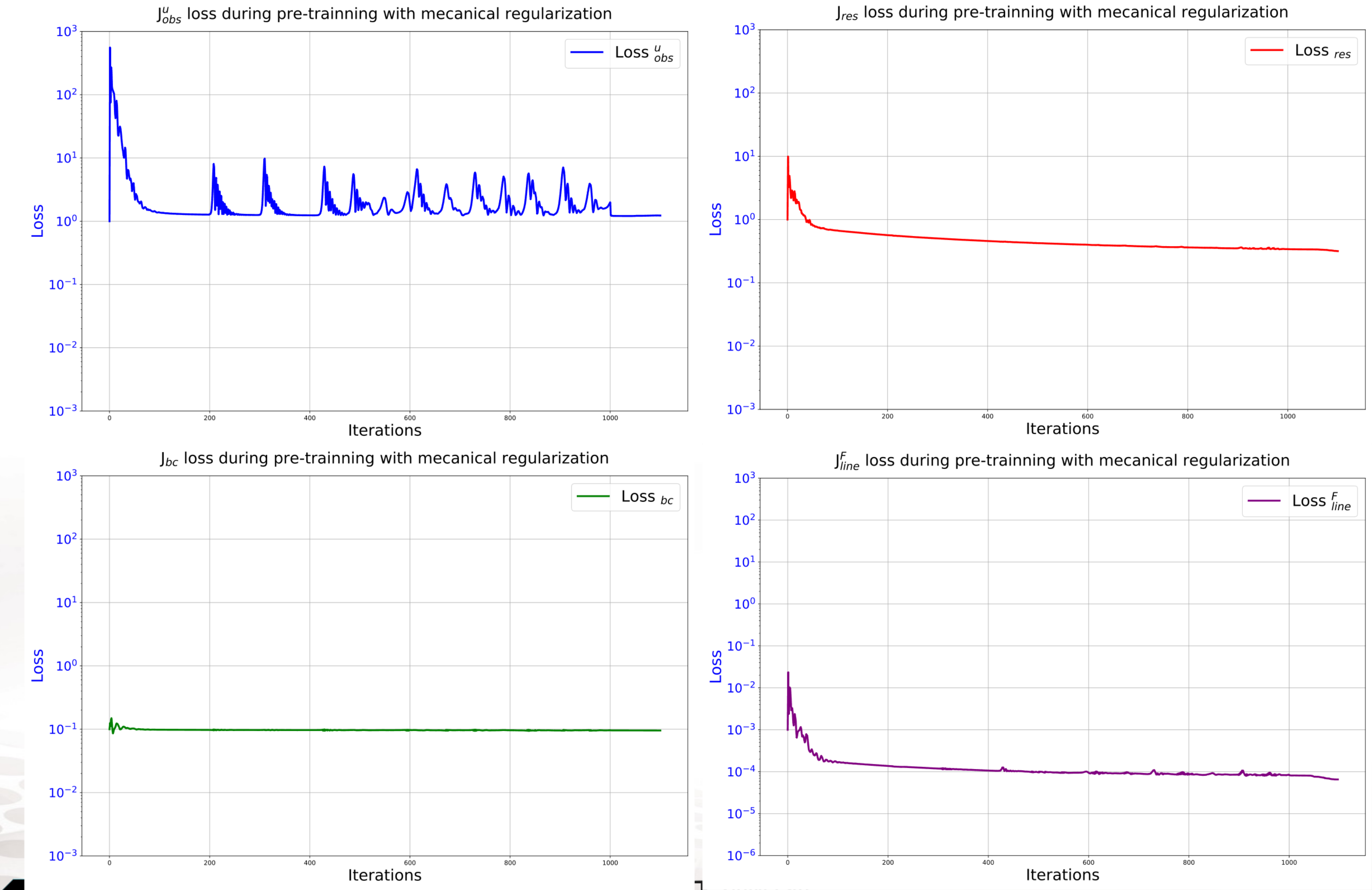
Loss functions minimization



The solution is **accurately learned by the NN** but exhibits **oscillations in its derivatives** (strain fields)

Results at the end of regularization

Loss functions minimization



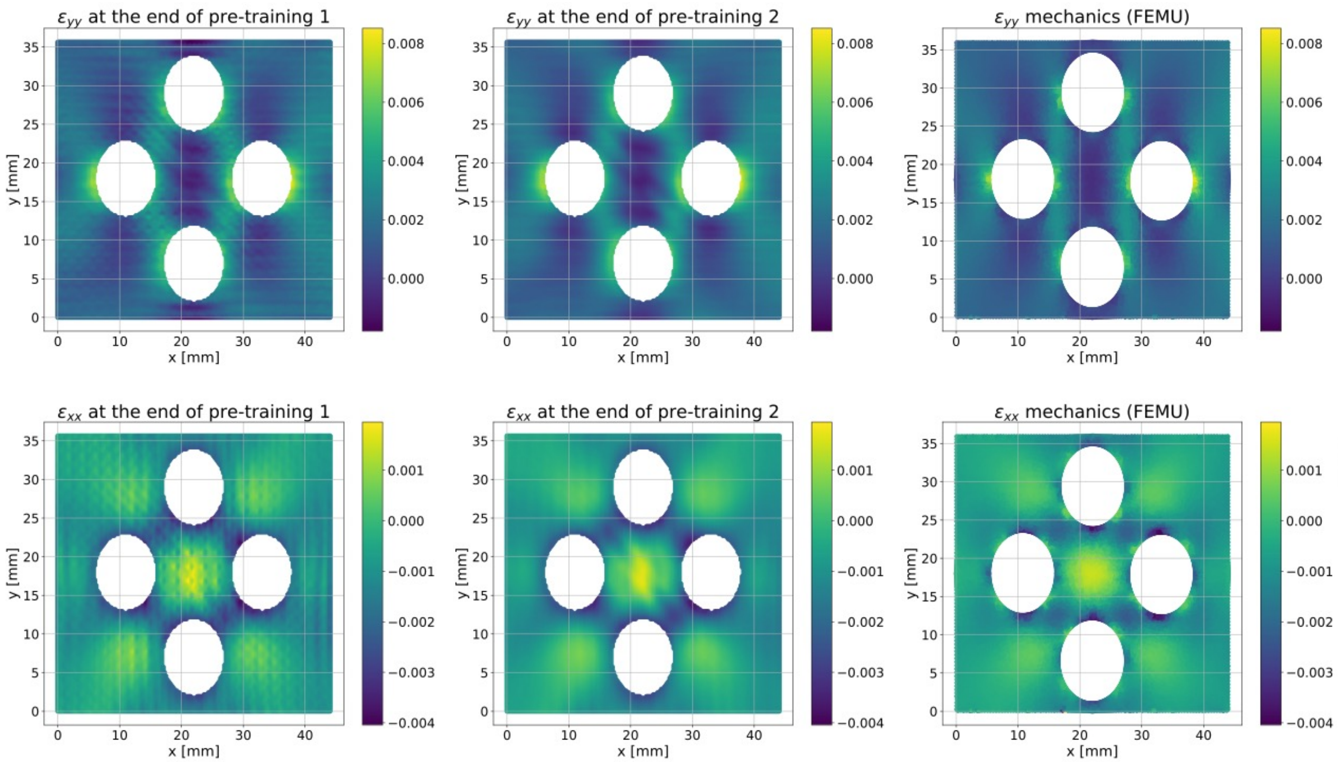
With the addition of mechanical loss terms, we are able to **enforce mechanical equilibrium** $(J_{\text{res}}, J^F_{\text{line}})$ without compromising the accuracy *w.r.t.* the observations.

Results: real 2D case with cst E and ν (3/5)

Results at the end of regularization

Contour plots of the solution

initialization *regularization* *mechanics*



Mechanics serves as regularization: it reduces oscillations in the strain fields

Results: real 2D case with cst E and ν (3/5)

Results at the end of regularization

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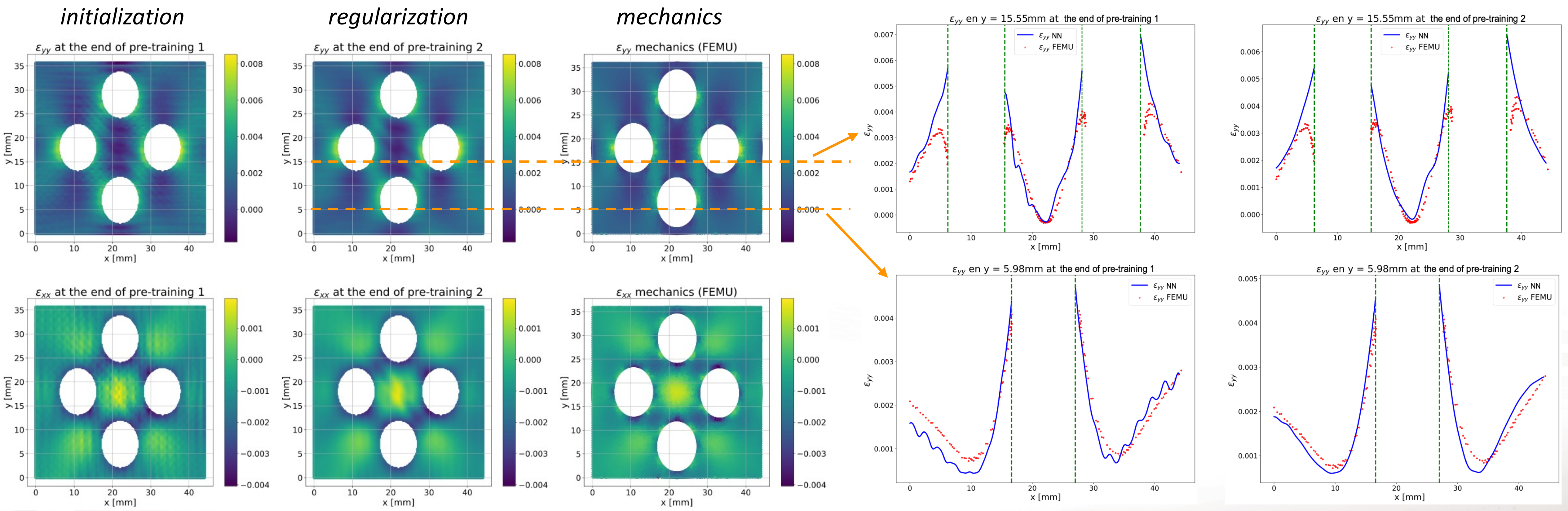
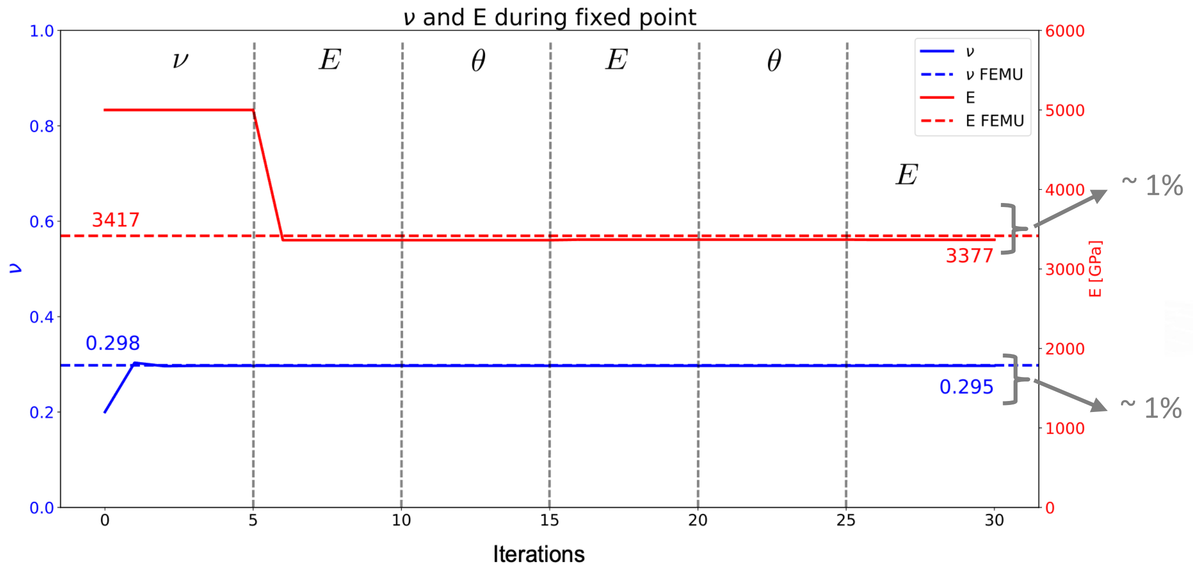


Mechanics serves as regularization: it reduces oscillations in the strain fields (but: hard to capture **very local behavior**)

Results: real 2D case with cst E and ν (4/5)

Results at the end of training

Convergence of the material parameters



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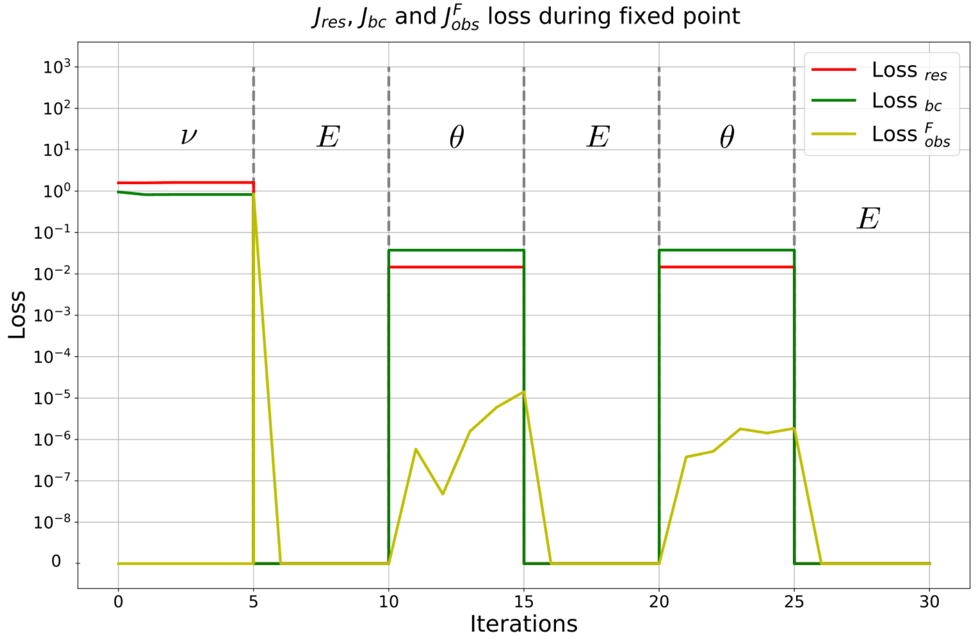


One iteration is sufficient **for *E***, and **two for** ν + fixed-point is not required here
The discrepancy with FEMU is on the **order of 1%**

Results: real 2D case with cst E and ν (5/5)

Results at the end of training

Results for each image and comparison with FEMU

We manage to obtain **very accurate results** *w.r.t.* **FEMU** (slight difference may be due to the **difficulty to capture very local stress**)
Appears more **stable through the loading** than FEMU: may be due to the **absence of disp. BC for mechanics**

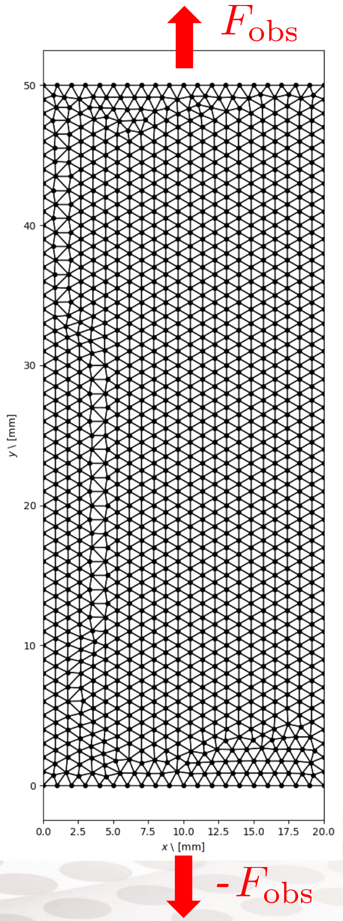
Outline

Appli. 3: synthetic 2D with *E(x,y)* (1/2)

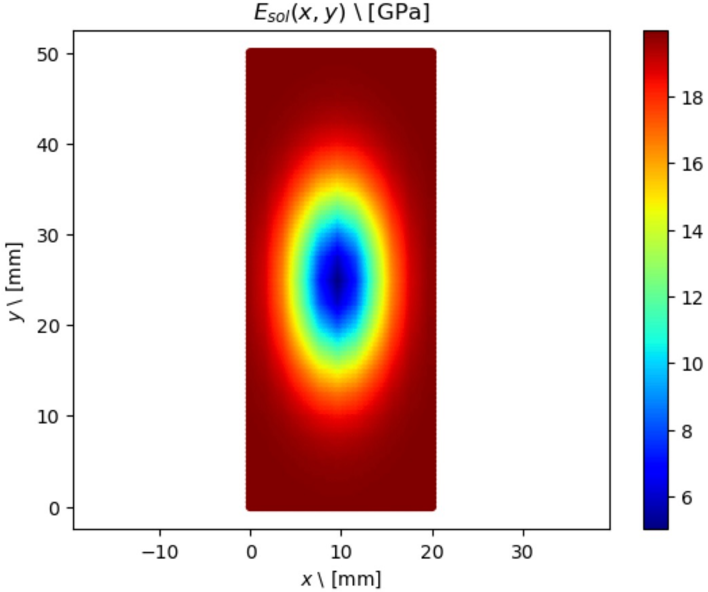
Example: traction test on a plate with a Gaussian inclusion

Synthetic test case

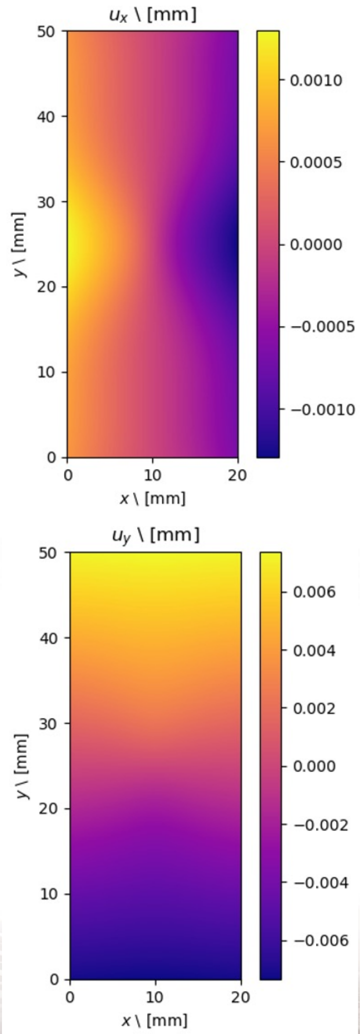
FE mesh and loading



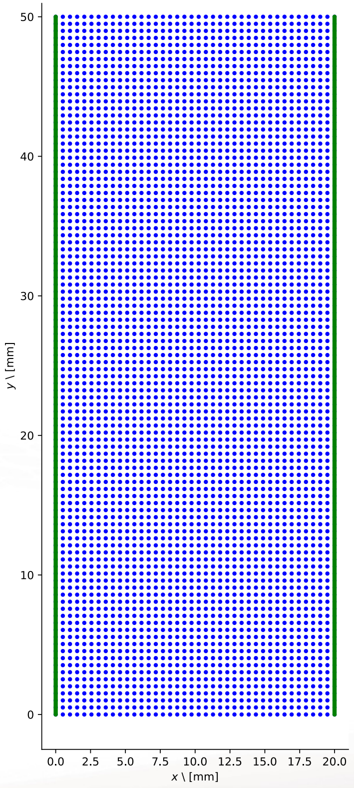
Reference E(x)



Generated measurements



Collocation points



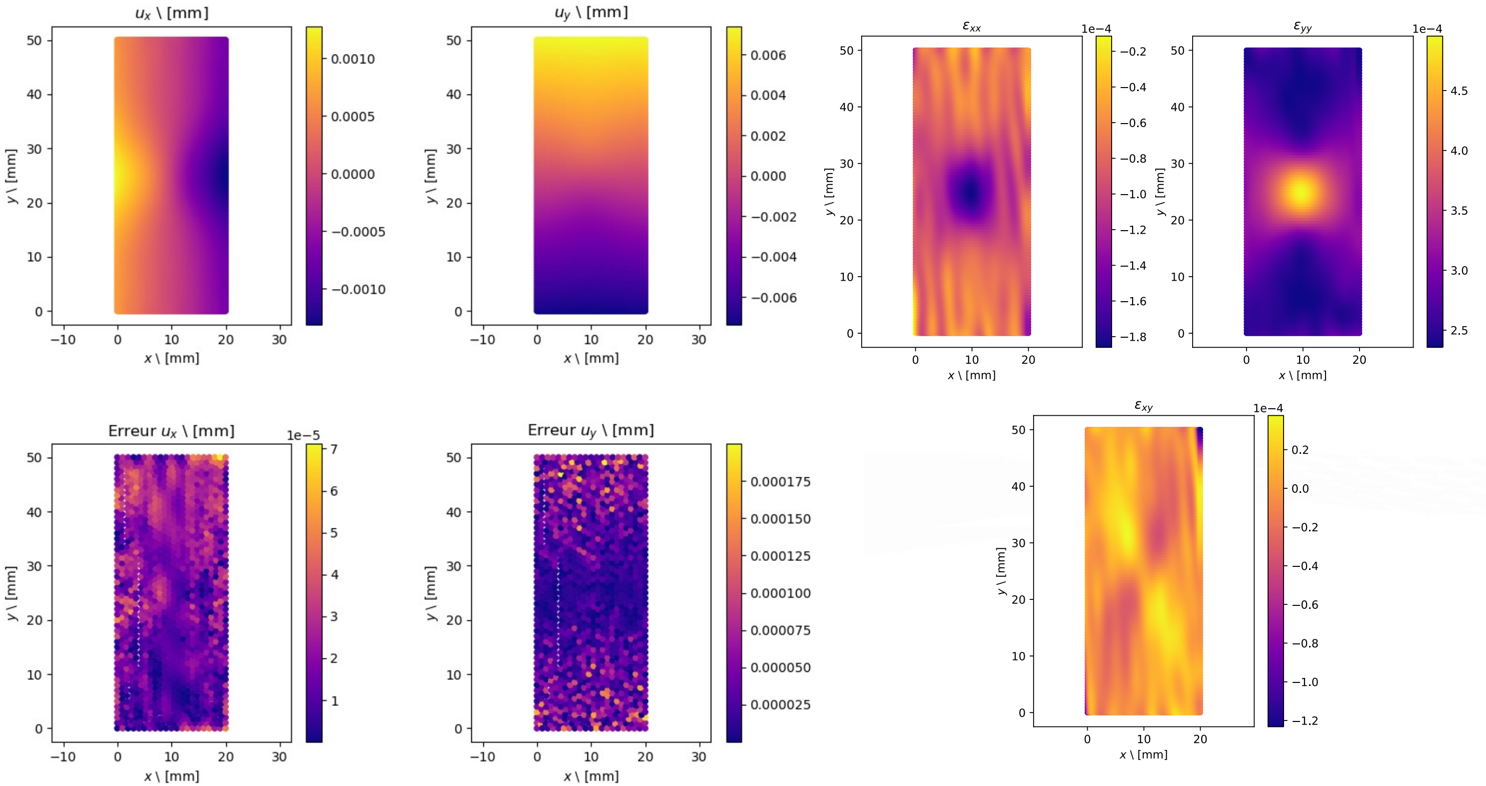
$$
\begin{aligned}
L &= 20 \text{ mm} \\
H &= 50 \text{ mm} \\
E_{\mathrm{nom}} &= 20 \text{ GPa} \\
\nu &= 0.3 \quad \text{(fixed)} \\
F_{\mathrm{obs}} &= 100 \text{ N.mm}^{-1} \\
\gamma_u &= 1\% \\
n_{\mathrm{obs}} &= 1236 \\
n_{\mathrm{col}} &= 40 \times 100 = 4,000 \\
n_{\mathrm{col_{BC}}} &= 500 + 500 = 1000 \\
n_{\mathrm{col_{line}}} &= 500
\end{aligned}
$$

Results: synthetic 2D with *E(x,y)* (1/5)

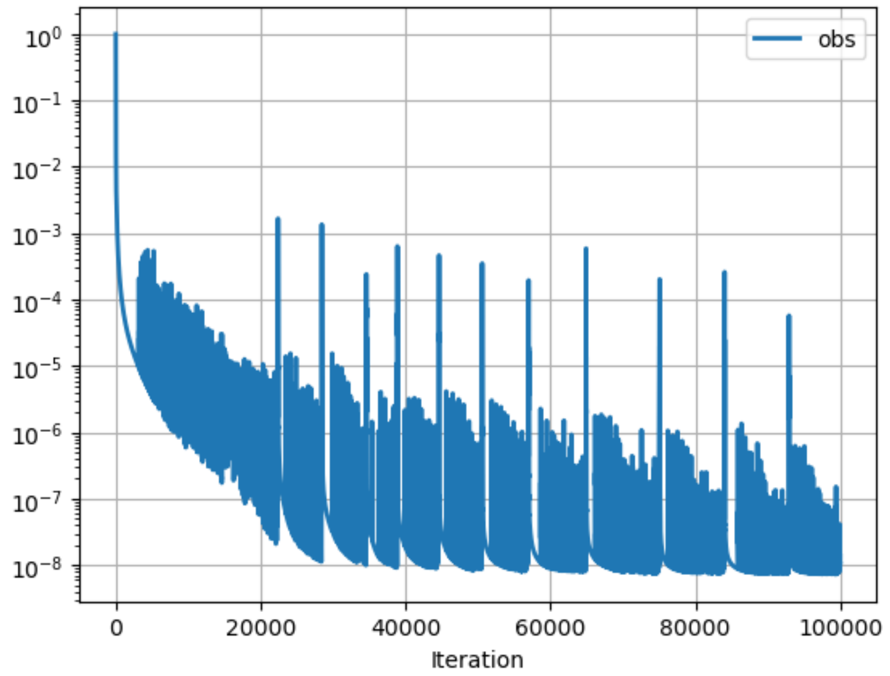
Results at the end of initialization

Contour plots of the solution

Displacement *Strain*



Loss functions minimization



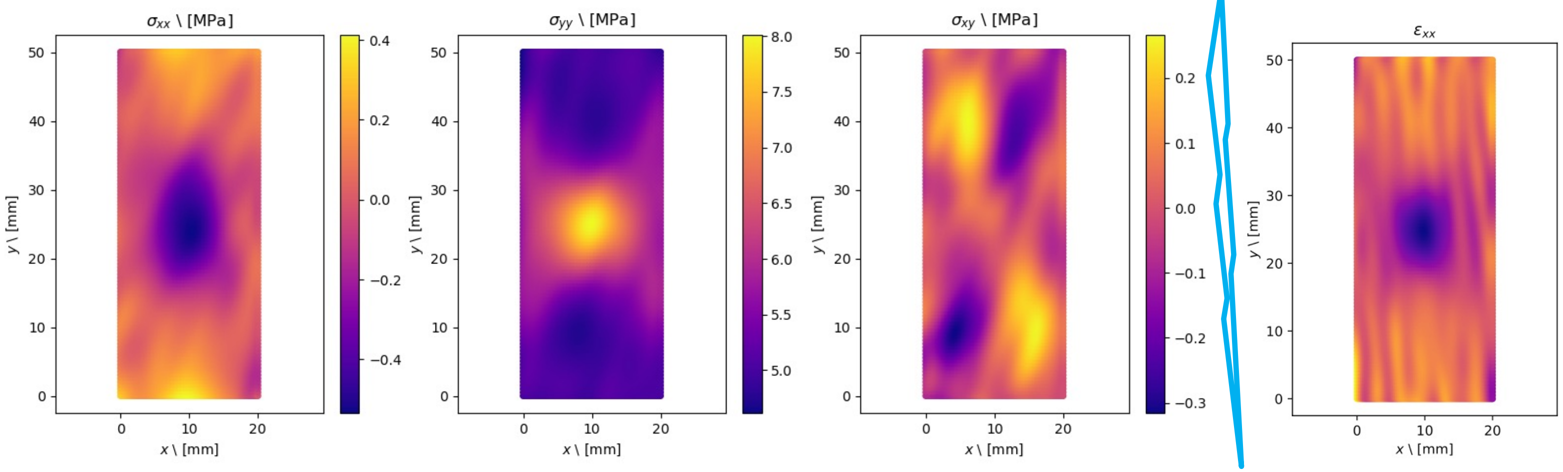
Again: solution **accurately learned by the NN** but exhibits **oscillations in its derivatives** (strain fields)

Results: synthetic 2D with *E(x,y)* (2/5)

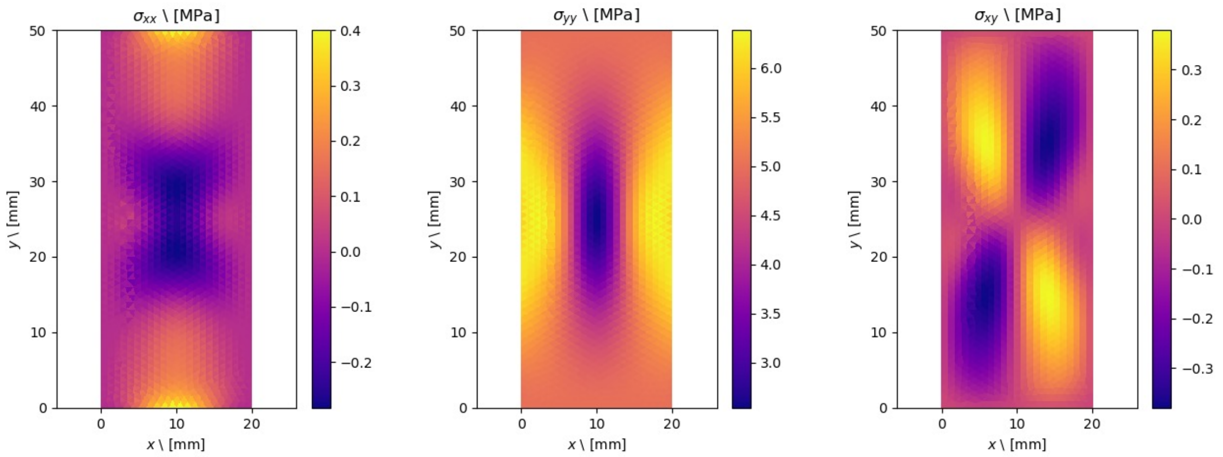
Results at the end of regularization

Contour plots of the solution

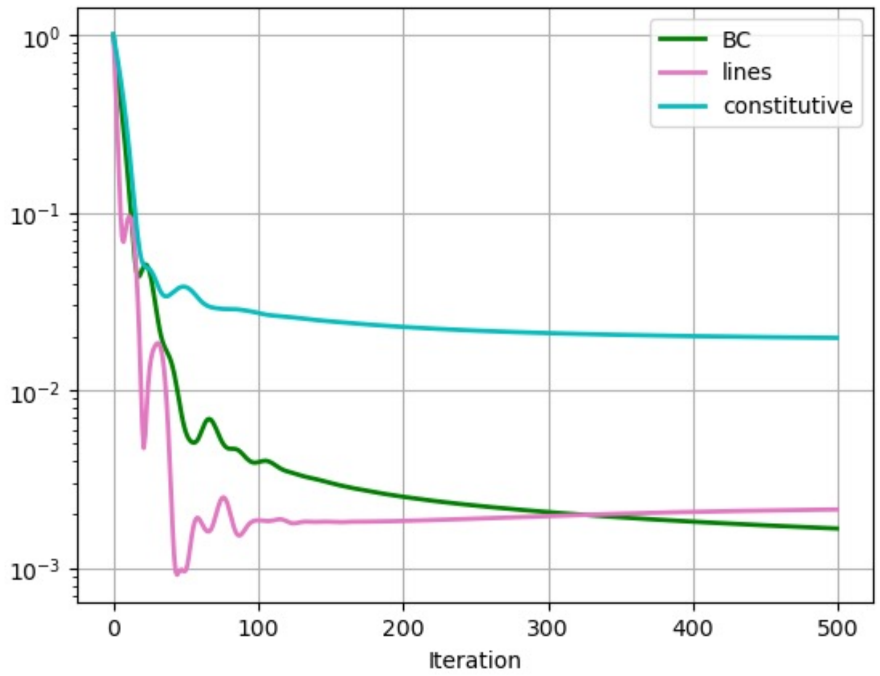
NN:



ref:



Loss functions minimization

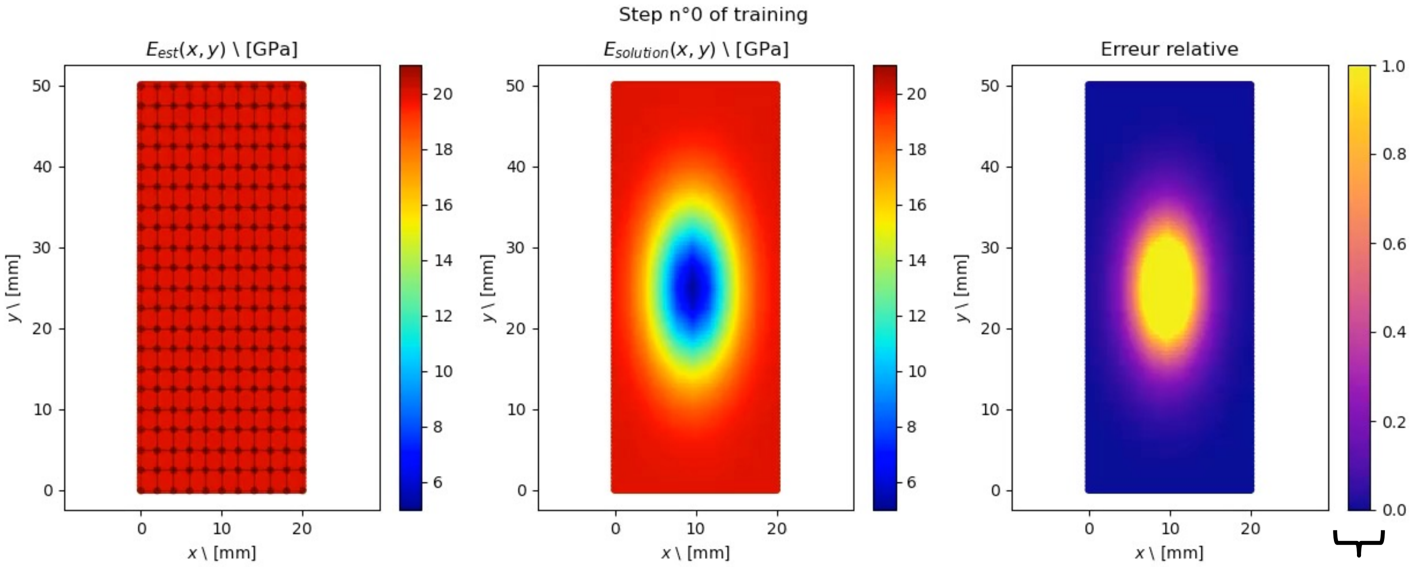


$\hat{\sigma}_\theta$ plays the role of the **mechanical regulator** here

Results: synthetic 2D with *E(x,y)* (3/5)

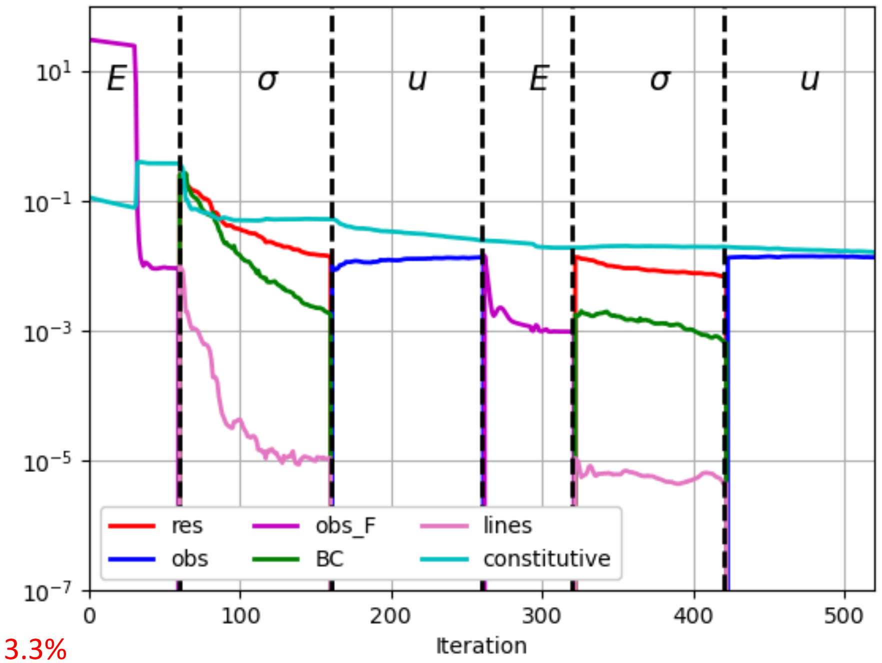
Results at the end of training

Convergence of the *E(x)*



Relative error of 4.4%
with a standard deviation of 3.3%

Loss functions minimization



With the **fixed-point,** we are able to **capture the distributed *E(x,y)* with good accuracy**

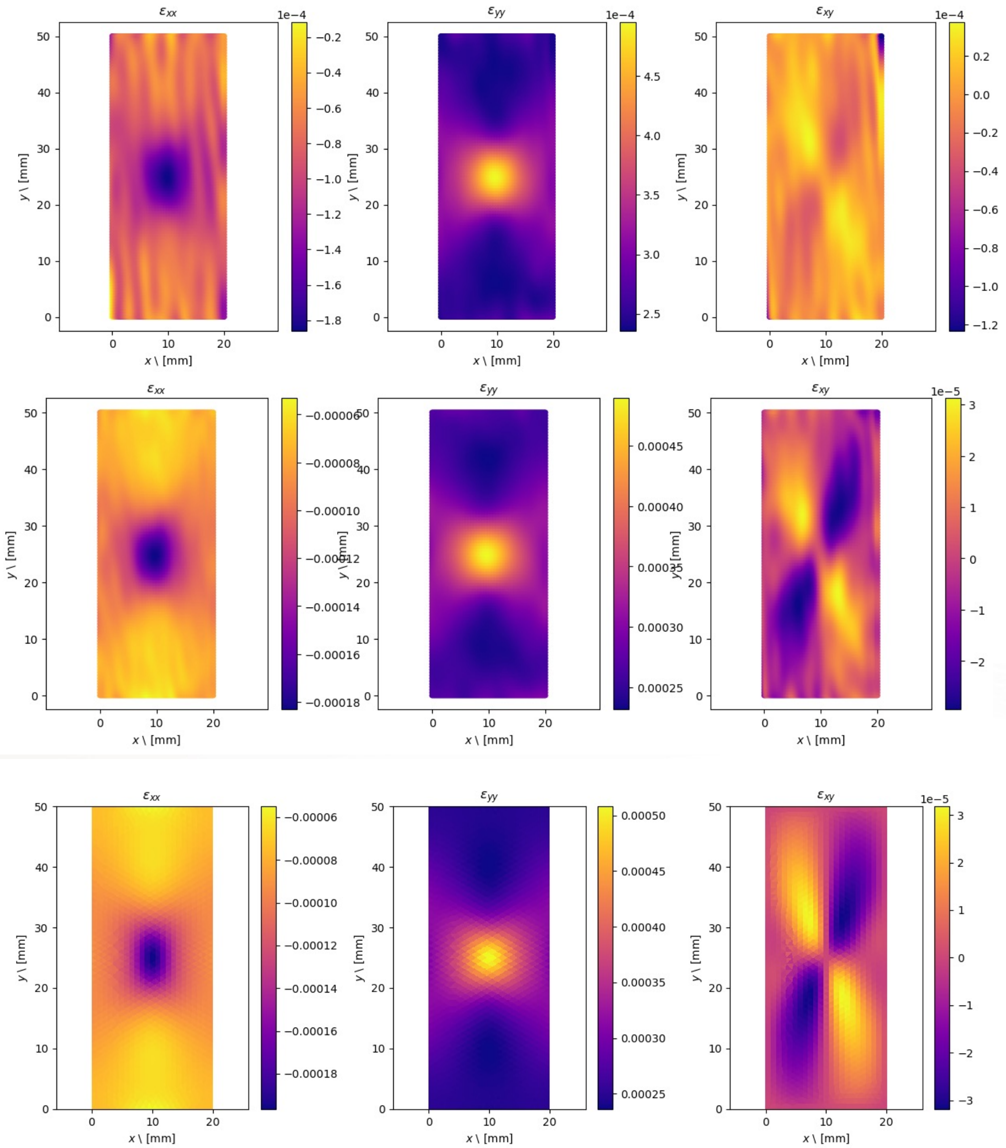
Results: synthetic 2D with *E(x,y)* (4/5)

Strain

NN at init.:

NN at the end:

Reference:

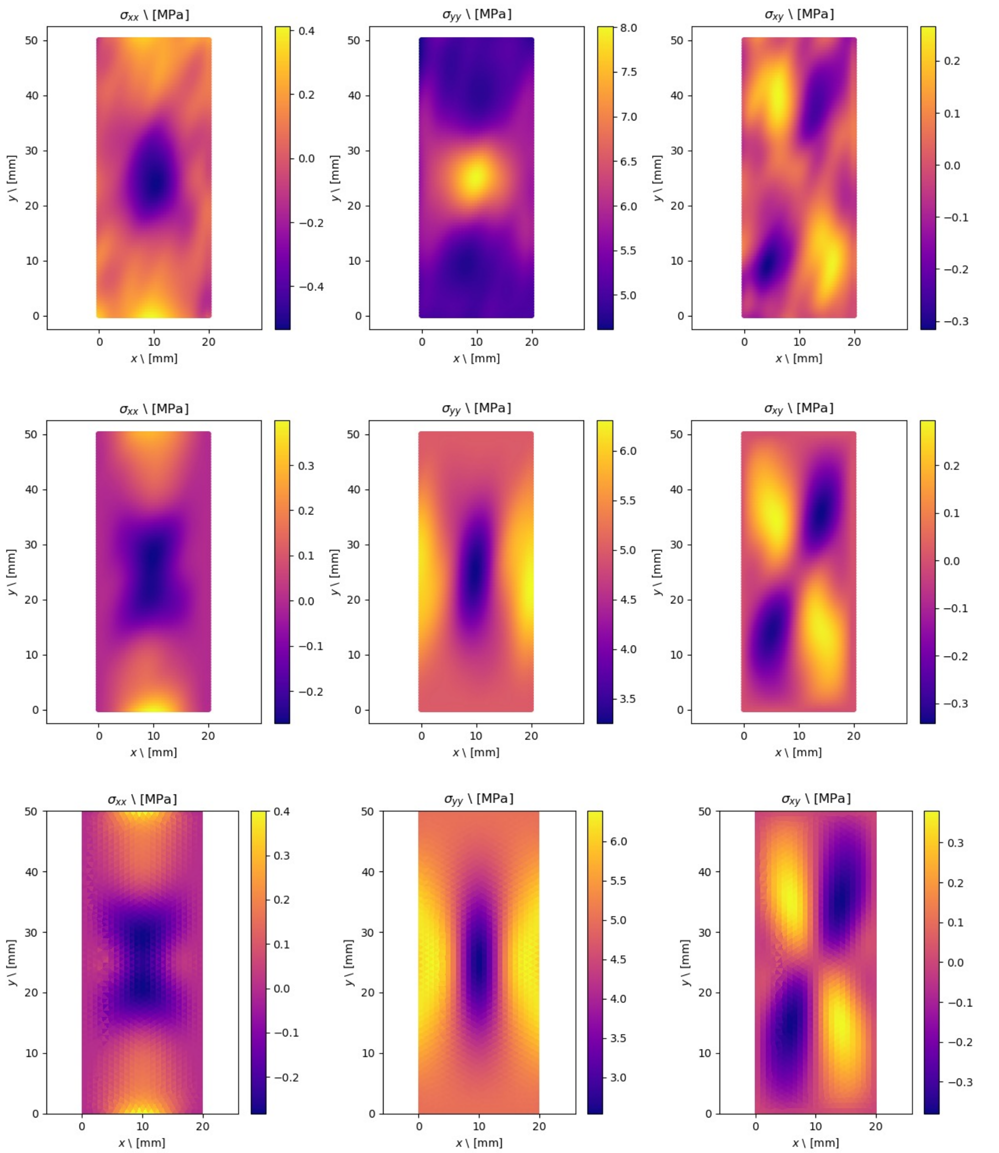


Stress

NN at init.:

NN at the end:

Reference:



Conclusion

Key points

- **Attempt to apply the inverse (Semi-Parameterized) PINN method to the identification of material properties**

  Solution of (multiple) **mechanical pbs replaced by learning a single function with a NN**
  May be **advantageous** when m **large**, & **complex mechanics**
  May be **simple**: solution with **available robust optim.** algos. *(e.g., PyTorch)*
  Requires **careful NN training** for good accuracy

- **Application to identify spatially-distributed Young modulus and on concrete experimental data**

  Able to identify **constant *E* and** ν on **experimental data:** 1% of $\neq$ w.r.t. FEMU
  Able to identify **distributed *E(x,y)*** on synthetic data

Prospects

Unify the formulations, **identify *E(x,y)* on real data**
Perform **multi-level PINNs** *[Aldirany et al. 24, Dolean et al. 24]*
Extend to **nonlinear material behaviors**

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Boulenc, H., Bouclier, R., Garambois, P.-A., and Monnier, J. (2025). Spatially-distributed parameter identification by physics-informed neural networks illustrated on the 2D shallow-water equations. *Inverse Problems,* 41, 035006.

Bouclier, R., Bonnet-Eymard, R., Rabineau, E., and Réthoré, J. PINN-based material identification of spatially varying elastic properties from experimental full-field displacement data. *In preparation.*

Thank you for your attention.